

## 1. Introduction

Calculation of a tensor product of irreducible representations for a higher rank Lie algebra is one of the group-theoretic problems, which came to the fore with the development of the 'grand unified model' for fundamental particles and forces in theoretical physics [1, 2, 3, 4, 5, 6]. For the explicit calculation of an SO(9) tensor product, which is the main purpose of this paper, the author got his motivation from a study of an SO(9) triplet structure [7, 8]. The structure proved to be a kernel subspace of the Kostant operator over the 16-dimensional coset space of  $F_4/SO(9)$  [9]. The kernel subspace is described by the tensor product of two SO(16) spinor representations and an  $F_4$  representation in SO(9) basis. A computational study of the SO(9) tensor product by explicit calculations will improve understanding the detailed structure of the kernel subspace. Moreover, an immediate yield of this study is that one can instantly use the obtained results in constructing the components of a coupled tensor operator from those of the basic tensors. In return, the coupled tensor operator may also be useful in studying the dynamics of supergravity, string/brane theories, or even of the M-theory.

This paper is organized as follows. In section 2, relevant basic facts of SO(9) are summarized to make the paper self-contained and to introduce the necessary notations and conventions. Explicit forms are presented of the SO(9) simple root and of Cartan subalgebra generators for vector and spinor representations. In section 3, the SO(9) tensor product to produce the other irreducible representations is explicitly calculated by using a straightforward algebraic method. In section 4, after introducing the Schwinger's bosonic oscillators for the SO(9) vector and spinor irreducible representations, a coupled tensor operator is constructed. For a special purpose, the abstract forms of four tensor operators that produce the SO(9) triplet pattern are also presented. The final section contains summary and remarks.