5. Summary and remarks

The matrix representations of the simple root and the Cartan subalgebra generators of SO(9) for the vector and spinor irreps are extracted directly from the 9-vector and 16-spinor weight diagrams. A higher SO(9) irrep can be obtained from a tensor product of the 9-vector and 16-spinor irreps. Acting on the highest weight of the irrep in the weight product basis by just only four matrix generators of the negative simple roots, one gets the weight vectors of the irrep, level by level, and finally the complete weight diagram. For any higher irrep, it is hard and mostly impossible to obtain the complete weight diagram by directly subtracting the simple roots from the weights. When one obtains the same weights after simple root subtractions and if one does not know the multiplicity formulas, there is no way to tell whether the new weights are degenerated.

An advantage of using these matrix generators is that the difference of the degenerated weights will be clearly seen. Therefore, this method can solve the

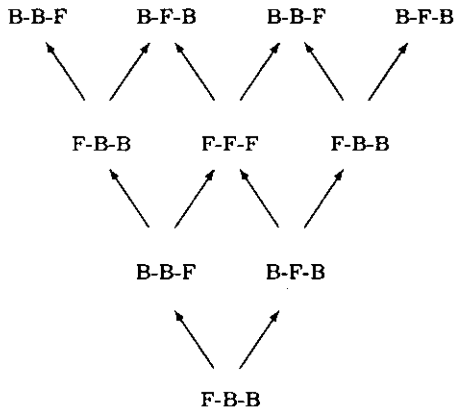

Figure 3. The SO(9) triplet pattern. There are four different higher spin triplet families, F-B-B, F-F-F, B-B-F, and B-F-B, where F means fermionic (odd a_4) and B bosonic (even a_4) degrees of freedom.

degeneracy problem. Another advantage is that this method can be applied to a tensor product of more than two irreps. Moreover, this method is simple and works in general for any compact Lie algebra (possibly including a non-compact one). However, there is a disadvantage. It is quite tedious to obtain the weight products of the higher irrep for a higher rank Lie group without writing a computer program to execute such a calculation.

The study of the SO(9) tensor products guides the ways to generate the SO(9) states and construct the SO(9) coupled tensor operators by using the Schwinger's bosonic oscillators. Their commutation relations are the twisted version of those normal bosonic oscillators used in Fulton's paper [13].

The Schwinger's creation and annihilation oscillators for the 9-vector and 16-spinor irreps are introduced corresponding to the weights in their diagrams. In generating the states of all SO(9) irreps, at most three copies of the 9-vector creation oscillators and two copies of the 16-spinor creation oscillators acting on the vacuum state are linearly combined in accordance with the SO(9) weight products.

The transformations of the 9-vector and 16-spinor operator components are seen through their commutation relations with the simple root generators. Then, the SO(9) coupled tensor operators are constructed from these operator components, and these operators are always the irreducible ones. As in [14], the 36 components of $A^{[2]+}$ and 16 components of $\Psi^{(1/2)+-}$ can be combined to form the 52 generators for the adjoint irrep of F_4, where the $A^{[2]+}$ generators transform the vector (or spinor) weights into vector (or spinor) weights and the $\Psi^{(1/2)+-}$ generators transmute the vector (or spinor) weights into spinor (or vector) weights.

It deserves further investigation to find out whether the constructed tensor operators are useful to study the dynamics of higher massless or massive spin fields in the supergravity and superstring theories or even in the M-theory.

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