Modelling the Volatility and Assessing the Performance of the Model: Case Study in Some Indonesia Stock Prices

Prince of Songkla University
Pattani Campus

Subhan Ajiz Awalludin

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science in Applied Mathematics

Prince of Songkla University

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Thesis Title	Modelling the Volatility and Assessing the Performance of the Model: Case Study in Some Indonesia Stock Prices
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This is to certify that the work here submitted is the result of the candidate's own investigations. Due acknowledgement has been made of any assistance received.

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I hereby certify that this work has not been accepted in substance for any degree, and is not being currently submitted in candidature for any degree.

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ชื่อวิทยานิพนธ์ การจำลองความผันผวนและการประเมินผลของตัวแบบ: กรณีศึกษาราคาหุ้นอินโดนีเซียบางตัว

ผู้เขียน Mr. Subhan Ajiz Awalludin

สาขาวิชา คณิตศาสตร์ประยุกต์

ปีการศึกษา 2558

บทคัดย่อ

การประมาณความผันผวนของผลตอบแทนราคาหุ้นให้แม่นตรงมากเท่าที่จะทำได้เป็นสิ่งจำเป็น เพราะความผันผวนมีความสำคัญทั้งในทางทฤษฎีและปฏิบัติ วิทยานิพนธ์ฉบับนี้นำเสนอตัวแบบ GARCH(1,1) สำหรับการประมาณความผันผวนของผลตอบแทนรายวันของราคาหุ้นบางตัวของอินโดนีเซีย ในช่วงเวลาจาก 12 กรกฎาคม 2550 ถึง 29 กันยายน 2558 ค่าพารามิเตอร์ของตัวแบบถูกประมาณโดยการประมาณภาวะน่าจะเป็นสูงสุด ลำดับความผันผวนถูกกระชับด้วยการประมาณค่าในช่วงด้วยเส้นโค้งกำลังสามอย่างธรรมชาติเพื่อศึกษาพฤติกรรมของความผันผวนในช่วงเวลานั้น จากนั้นทำการประเมินความสามารถในการจับความผันผวนของตัวแบบ GARCH(1,1) โดยใช้การจำลองมอนติคาร์โล ผลการศึกษาแสดงให้เห็นว่า GARCH(1,1) สามารถกระชับความผันผวนได้ใกล้เคียงกับความผันผวนที่กำหนดขึ้น นั่นแสดงว่าตัวแบบ GARCH(1,1) สามารถจับความผันผวนได้ค่อนข้างดี

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Author Mr. Subhan Ajiz Awalludin

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ABSTRACT

Estimating volatility of stock returns as accurate as possible is needed since the importance of volatility in theory and practice. Aim of the study is to show the process of assessing the performance of volatility model. This study presented GARCH(1,1) model for estimating volatility of daily returns of some stock prices of Indonesia over the period from 12 July 2007 to 29 September 2015. Parameters of the model were estimated by Maximum Likelihood Estimation. The fitted volatility series were estimated by using natural cubic spline in order to study the behavior of the volatility over the period. The performance of how good the GARCH(1,1) can capture the volatility is assessed by using Monte Carlo Simulation. The result shows that the GARCH(1,1) gives fitted volatility which is close to assumed volatility. This indicates that the GARCH(1,1) is able to capture the volatility quite well.

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Contents

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Contents (Continued)

Prince of Songkla University
Pattani Campus

List of Tables

Prince of Songkla University
Pattani Campus

List of Figures

Prince of Songkla University Pattani Campus

Chapter 1

Introduction and Literature Reviews

1.1 Background

Market stands an important role in operating an economy. Particularly, capital market, which is used to raise finance for long term. There are financial investments traded in the capital market, such as bond, stock, mutual fund and derivative instruments. Stock is one of the popular financial investments for investors because the stock offers interesting returns for its investors. The stock can be defined as a sign of capital participation in a company. Investors in the stock market have both the benefits and the risks. Basically, having stock will get dividend from the company's profit and capital gain from stock price increases, while the risks are suffering capital loss in case stock price decreases and liquidity risk when the company stops operating.

In general, the investor can consider either open, high, low or close price of the successive trading day. In this study we consider close price. It is well known that the stock price changing over time were essentially independent of each other (Malkiel, 1999). The prices can be affected by many factors such as political instability, natural disasters, economic crises, or wars (Posedel, 2005). Consequently, the stock prices movement is unpredictable. Understanding how stock prices change and forecasting their movement are considered by investors, so that they can make an appropriate decision to sell, buy, or save the stock.

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The stock price has high correlation with itself in the previous time and its volatility is not stationary, that indicates the mean and the variance of price change over time (Taylor, 2008). It leads to a difficulty of the investigation. Finding another financial random variable is needed. In fact, working with the change in price is more convenient since the result of analysis can be used to give an appropriate result for price (Taylor, 2008). The change in price corresponds to returns of stock price. The returns are computed by differencing the log of the price from one day to the next. Their values can be either negative or zero or positive. Positive returns reveal gaining a profit, while zero and negative returns reveal stagnant return and suffering a loss, respectively.

However, the problem arises while investigating the returns distribution. Most of the time, the returns distribution is not normal which contains leptokurtic (fat tail) (Arowolo, 2013). Fat tail indicates that the returns deviate from linear line which corresponds theoretical quantiles.

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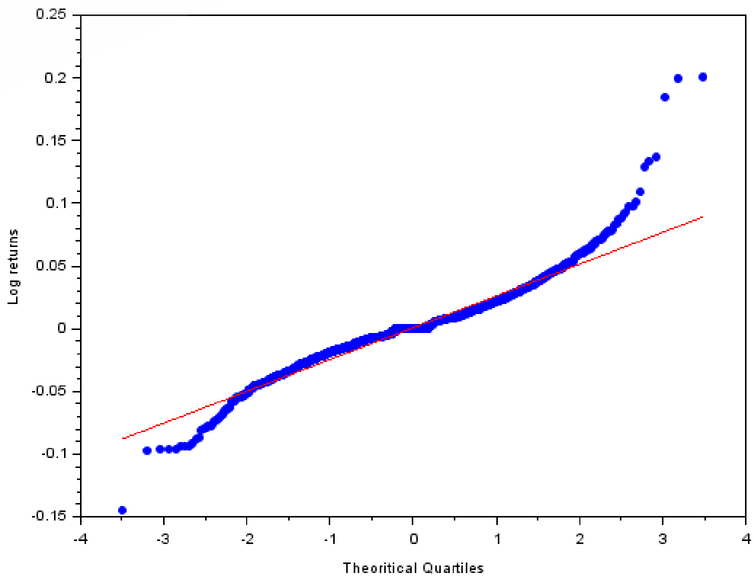

Figure 1.1: Normal and observed distribution

Figure 1.1 shows the comparison between normal distribution (the grey-colored linear line) and observed distribution (the black-colored curve). It clearly can be seen that the observed distribution has fat tails at both negative and positive ends of the return curve. In practice, the assumption of normality on the return which is not normally distributed is widely used. According to the previous studies, they assumed the return distribution to be normal (Saejiang *et al.*,2001) and (Hull, 2009). However, in this study the lack of normality of the return is accepted. We show the method that transform the return to be normality distributed by following an improved robust transformation proposed by Peter J. Huber in 1964. After that, we obtain the returns fluctuation by estimating their standard deviation over time horizon. This leads to volatility of stock returns .

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Volatility is a measure of the uncertainty of the return realized on an asset (Hull, 2009). In other words, it describes the returns fluctuation whether going up or down over the period. In the financial field, volatility is one of the key variables to make an appropriate decision. Therefore, Investors and financial analysts concern in capturing volatility. In fact, the volatility has taken place in different areas in financial theory and practice, such as risk management, portfolio selection and derivative pricing (Arowolo, 2013). In many cases, the volatility is shown by low fluctuation in some period, then following by high fluctuation, and vice versa. It indicates that volatility is not constant over time. Obtaining the volatility as accurate as possible is needed since return can be obtained from volatility and price can be computed based on the return. We can employ time series model to capture the volatility of returns asset.

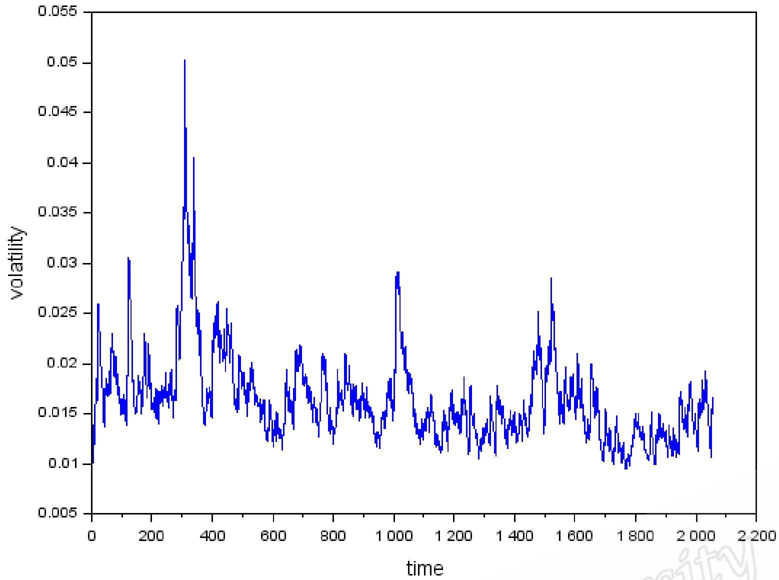

Figure 1.2: Volatility of stock returns of Bank Mandiri

The time series model that will be used must agree with heteroscedasticity property. Heteroscedasticity describes that the volatility of stock returns is not constant over time. It clearly can be seen, as an example, in Figure 1.2 that the volatility of Bank Mandiri changes over time. This leads to heteroscedasticity and we have to deal with this condition. One of heteroscedasticity models is Generalized Autoregressive Conditional Heteroscedasticity (GARCH) which was proposed by Bollerslev (1986). Furthermore, The GARCH gives volatility series which can be considered by investors to understand the behavior of returns fluctuation.

As in Figure 1.2, the volatility series is very fluctuating, we need to smooth the volatility series in order to simplify investigation of their change in many situations. Numerical method can be employed to do the job. This study uses natural cubic spline function which is a widely used technique for piecewise smooth curve fitting. This function is simply piecewise cubic polynomial which can be constructed so that the

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connections between adjacent cubic splines are visually smooth (Chapra and Canale, 2010). After the volatility series is smoothed by natural cubic spline, fitted volatility is obtained. The fitted volatility can be used in the process of assessing how well the GARCH can capture a known volatility.

The volatility of stock returns can be any possibilities in the market, thus enabling traders to design portfolios that increase in value when the volatility moves in a certain way. For this reason, it is important to have a good model for estimating volatility as accurate as possible. In this study we assess the volatility model which is simple and widely used GARCH(1,1) using Monte Carlo simulation.

1.2 Objectives and Scope of the Study

The objectives of the study are as follows:

1. To study the behavior of volatility of stock returns
2. To assess a volatility model using Monte Carlo simulation

The scope of the study is the analysis of assessing the performance of how well a volatility model using Monte Carlo simulation (see Figure 1.3). Data comprise of closing price on trading days of seven companies, which are Agro Lestari, Antam, Bank BNI, Bank BRI, Indofood, Indosat and Bank Mandiri starting from July 12, 2007 to September 29, 2015. This period was chosen since we would like to have the same period for all stocks and to see the volatility movement during the end of 2008 which correspond to financial crisis in Indonesia. These companies are of interest chosen from such difference sectors as agriculture, commodity, banking, food and telecommunication because they are among the largest companies in Indonesia. The data were obtained from Yahoofinance

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(2015)[September 29, 2015]. In this study, we consider the volatility model so-called GARCH(1,1) which will be explained in chapter 2.

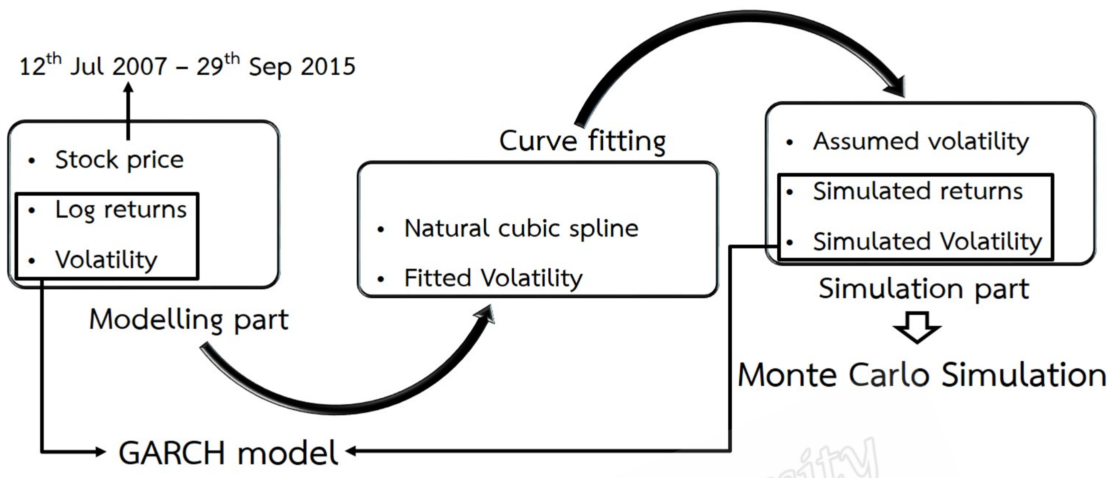

Figure 1.3: Diagram of the scope of the study

Firstly, we obtain the returns from the price and use Huber robust transformation to meet the returns with normal distribution. After that, the GARCH(1,1) model was used to fit the returns then the volatility was obtained. Moreover, we employ natural cubic spline to fit volatility in order to study the behavior of volatility over the period. Finally, Monte Carlo simulation was used to assess the GARCH(1,1).

Modelling part can be seen clearly in Figure 1.4. Stock price data comprise of 2056 observations that will be used in obtaining returns. This study define returns as continuously compounded returns. Before obtaining volatility, basic analysis of stock returns is presented. After that, GARCH model fit the data and the volatility will be obtained. The information from modelling part will be used in curve fitting.

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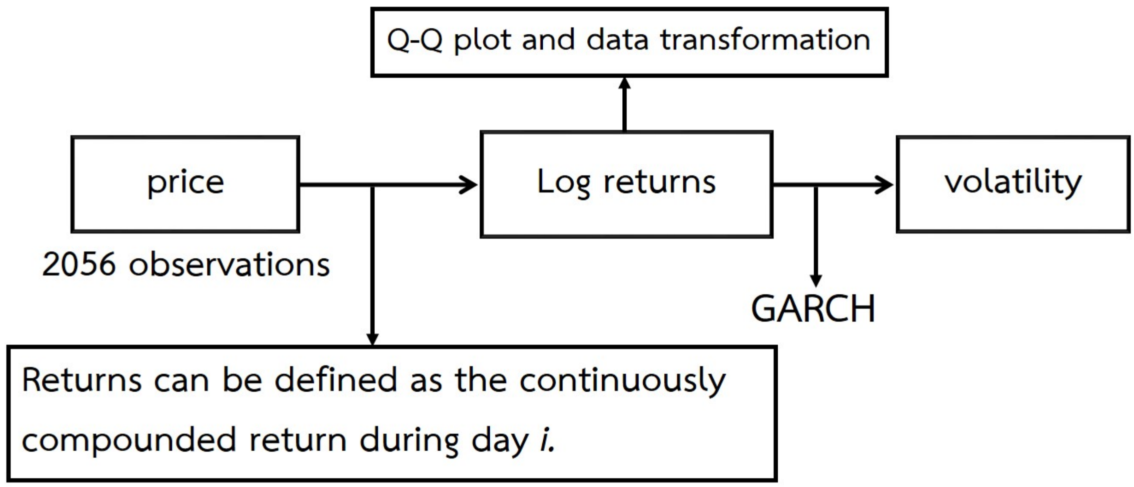

Figure 1.4: Diagram of modelling part

1.3 Terminologies

Dividend: a cash payment made to the owner of a stock (Hull, 2009)

Returns: the ratio of operating profit to shareholders (Parry, 2003)

Stock: an investment that represents part ownership of a company (Parry, 2003)

Volatility: a measure of the uncertainty of the return realized on an asset (Hull, 2009)

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1.4 Literature Reviews

Most studies in modelling the volatility of stock returns are using GARCH models which was proposed by Bollerslev (1986). We first investigate the stylize fact of stock returns. In fact, the returns in financial asset show leptocurtic (heavy tail) (Arowolo, 2013), non-normal distribution, positive skewed, stationary (Namugaya *et al.*, 2014) and volatility clustering (Kamau, 2015). A study by Ahmed and Suliman (2011) used GARCH models to fit the stock returns of Khartoum Stock Exchange. They showed that the volatility process is highly persistent (explosive process) and there is the positive correlation between the volatility and the expected stock returns.

Namugaya *et al.* (2014) applied the GARCH in modelling volatility of stock returns of Uganda Securities Exchange (USE). The study found that the GARCH(1,1) outperformed the other GARCH(p,q) models in modelling volatility of USE returns based on Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC). This result confirmed the study by Hansen and Lunde (2005) which argued that the GACRH(1,1) works well in modelling volatility of financial returns compared to more complicated models including EGARCH, GJR-GARCH etc.

Kamau *et al.*(2015) used the GARCH(1,1) to estimate volatility of stock returns in Kenyan stock market. The parameters of the model were estimated by Maximum Likelihood Estimation. Once the parameters have been determined, the volatility of stock returns will be obtained. They found that negative returns shocks have higher volatility than positive returns shocks.

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According to the studies by Ahmed and Suliman (2011), Namugaya *et al.* (2014) and Kamau *et.al* (2015), they assumed the returns are normally distributed in the process of estimating their volatility. Moreover, the volatility that have been obtained (see for example, Figure 1.2) did not show clearly its behavior over the period whether going up or down. For these reasons, we would like to address the gap between our study and preceding studies by presenting the method to transform the returns to be normally distributed and to smooth the volatility of stock returns using cubic spline function.

Moreover, we would also like to assess the performance of the model using Monte Carlo simulation. A study was done by Cartea and Karyampas (2012) in assessing volatility estimators using the Monte Carlo simulation. The method was able to test various volatility estimators by assuming price path under different assumption about the distribution of variable in question to be Gaussian. This study is similar to a study by Simionescu (2014) proposed steps in assessing process such as assuming the mean and the standard deviation of the parameters price then generating the price of normal distribution. The study showed that the Monte Carlo simulation can be a tool to assess the uncertainty forecasts.

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Chapter 2

Theory and Methods

This chapter describes mathematical and statistical methods which were used for analyzing of stocks returns and volatility in this study. These methods consist of obtaining the returns from stock price data, estimating volatility of stock returns and assessing volatility model using Monte Carlo simulation. The details will be explained as follows:

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2.1 Obtaining returns from stock price

In this section, the stock returns is either percentage returns or log returns. Let S_t be a stock price at the end of day t. Percentage returns u_t (often called returns) are defined as the percentage change in the market variable between the end of day $t-1$ and the end of day t (Hull, 2009). It can be written as:

$$u_t = \frac{S_t - S_{t-1}}{S_{t-1}}.$$

Returns can also be defined as the continuously compounded returns during day t (between the end of day $t-1$ and the end of day t) (Hull, 2009), as:

$$R_t = \ln \frac{S_t}{S_{t-1}}.$$

Commonly, continuously compounded returns R_t are called log returns. This study uses log returns R_t since it is convenient for multi period as well as approximately equal to percentage returns u_t over short period (Ruppert and Matteson, 2015). To show that $u_t \approx R_t$, the returns u_t can be written as

$$u_t + 1 = \frac{S_t}{S_{t-1}}.$$

Taking natural logarithm on both sides

$$\ln(u_t + 1) = \ln \frac{S_t}{S_{t-1}}. \tag{2.1}$$

The right hand side of (2.1), in fact is the log returns R_t. If the increment time is very small, then the percentage returns u_t is small (Alexander, 2008). The $\ln(u_t + 1)$ can be approximated using power series expansion as follow

$$\ln(u_t + 1) = u_t - \frac{u_t^2}{2} + \frac{u_t^3}{3} - \frac{u_t^4}{4} + \ldots.$$

Since u_t is very small, then u_t^n when $n \geq 2$ are so small that they can be neglected. Then we have

$$\ln(u_t + 1) \approx u_t. \tag{2.2}$$

In other word, from (2.1) and (2.2)

$$u_t \approx R_t = \ln \frac{S_t}{S_{t-1}}.$$

This proved that the log returns are approximately equal to percentage returns over short period. Further, basic analysis of stock returns will be described including Quantile-Quantile plot and data transformation.

2.1.1 Quantile-Quantile plot

The Quantile-Quantile (Q-Q) plot is a graphical technique for determining if two data come from population with a normal distribution. This technique is formed by

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plotting estimated quantiles from data set 1 on the horizontal axis and estimated quantiles from data set 2 on the vertical axis. From Figure 2.1a, it clearly can be seen that the tails of the log returns of agriculture are more dispersed than the theoretical quantiles. The low tail of the log returns occur at more negative value than the theoretical quantiles. Similarly, the high tail occur at the greater than the theoretical quantiles. However, the log returns in the middle seem to follow the theoretical quantiles. In this study we want the log returns to follow the normal distribution shown in Figure 2.1b. Thus, we need to transform both the low tail and the high tail to meet the middle section using Huber robust transformation.

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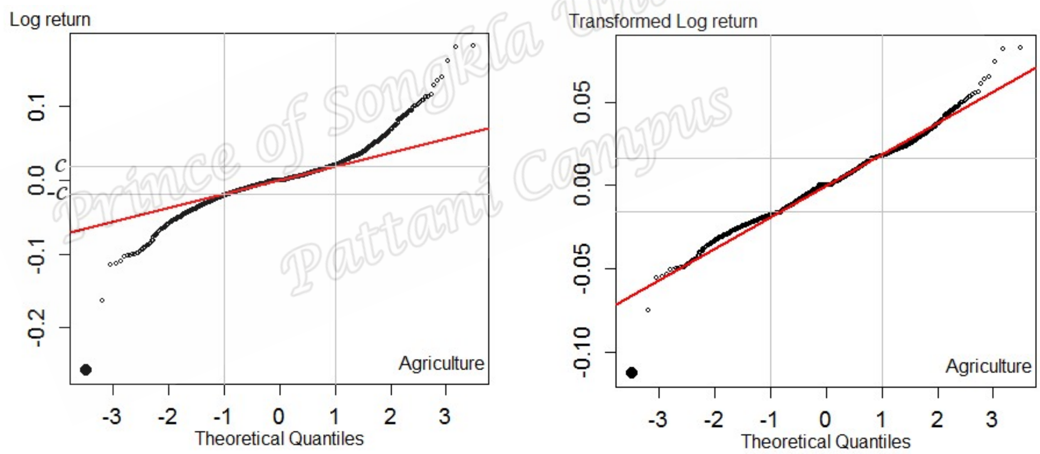

(a) (b)

Figure 2.1: The Q-Q plot of non-normal distribution (a) and approximately normal distribution (b)

2.1.2 Data Transformation

Log returns of stock price has fat tail which its Q-Q plot is similar to Figure 2.1a, deviating from linear line. Most of the time, the returns reflect piecewise linear behavior of three sections as parts of polygon (Figure. 2.2). Our desire is to have the returns follow one linear line, instead of three. To solve this problem, we use Huber robust transformation. In fact, we determine symmetrical constants c that indicate the turning point at the ends of the middle line $y = x$. The method depicted in Figure 2.2.

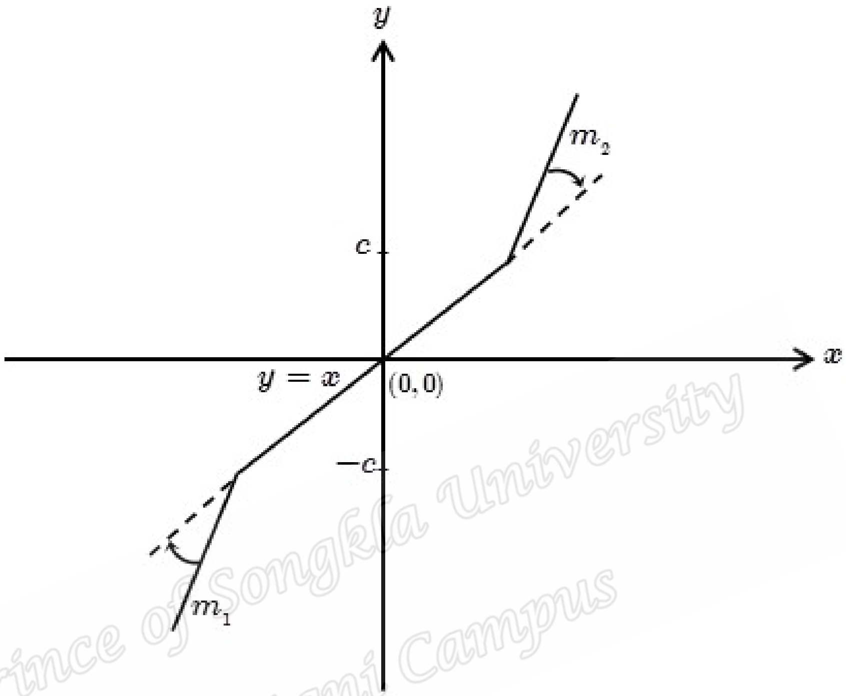

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Figure 2.2: Huber robust transformation using linear equation

From Figure 2.2 m_1 and m_2 are the slopes of the first and the third sections, respectively. It is straightforward to check that the equations of the first and the third section are $y = m_1x+c_1$ and $y = m_2x+c_2$, respectively. The application can be seen in Figure 2.1a where we determine the symmetrical constant c as the turning points at the ends of the diagonal of the rectangle. Huber (1964) suggested a method for transforming the data by shrinking their tails symmetrically. Using linear map with a constant a, we replace the observed value y greater than a specified constant c by $c+\left(\frac{y-c}{a}\right)$, and similarly replace the observed values y smaller than $-c$ by $-c+\left(\frac{y+c}{a}\right)$. After transforming the data, we now can obtain the information of stock returns fluctuation which is volatility series over time by fitting GARCH(1,1) to the transformed returns.

2.2 Estimating volatility of stock returns

It is well known that financial data contain non-constant variance over time so-called heteroscedasticity. Capturing the heteroscedasticity can be done by *Generalize Autoregressive Conditional Heteroscedasticity* (GARCH) model which was proposed by Bollerslev (1986). We first present the definition of general process of GARCH in order to meet the model for estimating the volatility.

Definition 1. Let $(w_t)_{t\in\mathbb{N}}$ be a sequence of independent and identically distributed (i.i.d) random variables such that $(w_t) \sim \mathcal{N}(0,1)$. The R_t is called the generalized autoregressive conditionally heteroscedasticity or GARCH(p,q) process (Posedel, 2005) if

$$R_t = \sigma_t w_t, \qquad t \in \mathbb{N},$$

where σ_t is a nonnegative process such that

$$\sigma_t^2 = \gamma V_L + \alpha_1 R_{t-1}^2 + ... + \alpha_q R_{t-q}^2 + ... + \beta_p \sigma_{t-p}^2, \qquad t \in \mathbb{N},$$

and

$$\gamma > 0, \quad \alpha_i \geq 0 \quad i = 1, ..., q \quad \beta_j \geq 0 \quad j = 1, ..., p,$$

where integers p and q are orders of σ_t^2 and R_t^2, respectively. The weights γ, α_i and β_j must sum to unity, that is

$$\gamma + \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j = 1.$$

In particular, GARCH(1,1) is the simplest and frequently useful model to estimate volatility (Arowolo, 2013) which is given by:

$$\sigma_t^2 = \gamma V_L + \alpha R_{t-1}^2 + \beta \sigma_{t-1}^2,$$

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where γ, α and β are the weight assigned to long-run average variance rate V_L, returns squared R_{t-1}^2 and variance σ_{t-1}^2, respectively. Now, we set $\omega = \gamma V_L$, the GARCH(1,1) model can also be written

$$\sigma_t^2 = \omega + \alpha R_{t-1}^2 + \beta \sigma_{t-1}^2, \tag{2.3}$$

where $\omega > 0, \alpha \geq 0$ and $\beta \geq 0$. In order to guarantee the variance to be positive, we set $\alpha + \beta < 1$. The formula (2.3) is used often for the purpose of estimating volatility. After that, the parameters α and β will be estimated by maximum likelihood method.

2.2.1 Estimating parameters of GARCH(1,1)

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Estimating parameters of the model can be done by maximum likelihood method which involves historical data of returns of the seven companies over 2007 to 2015. The method gives values of the parameters that maximize the likelihood function of the variable of interest (Hull, 2009). Now, we have the transformed returns, R_t, which is approximately normal with mean zero and variance σ_t^2 as required in definition 1. Initially, we determine the probability density function of $R_t, t = 1, 2, 3, ..., n$. Since for each t we have

$$f(r_t) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(\frac{-r_t^2}{2\sigma_t^2}\right),$$

then the likelihood function $L(r_t) = f(r_1, ..., r_n)$. For each t, R_t is independent so that

$$\begin{aligned} L(r_t) &= \prod_{t=1}^{n} f(r_t) \\ &= \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left(\frac{-r_t^2}{2\sigma_t^2}\right). \end{aligned} \tag{2.4}$$

By monotonicity of logarithm function, maximizing likelihood function can be done by maximizing its logarithm (Myung, 2003). Therefore, we now can maximize (2.4) by taking natural logarithm. Then we have,

$$
\begin{aligned}
l(r_t) = \ln L(r_t) &= \ln\left(\prod_{t=1}^{n}\frac{1}{\sqrt{2\pi\sigma_t^2}}\exp\left(\frac{-r_t^2}{2\sigma_t^2}\right)\right) \\
&= \sum_{t=1}^{n}\left(\frac{1}{2}\left(-\ln(2\pi)-\ln\left(\sigma_t^2\right)\right)+\frac{1}{2}\left(\frac{-r_t^2}{\sigma_t^2}\right)\right).
\end{aligned}
$$

Ignoring constant multiplicative factors of $l(r_t)$ gives

$$
\hat{l}(r_t) = \sum_{t=1}^{n}\left(-\ln\left(\sigma_t^2\right)-\frac{r_t^2}{\sigma_t^2}\right), \tag{2.5}
$$

where r_t and σ_t^2 are the log returns and the variance at day t, respectively. The parameters that maximize $l(r_t)$, also maximize $\hat{l}(r_t)$. Substituting formula (2.3) to (2.5) gives

$$
\hat{l}(\alpha,\beta;r_t) = \sum_{t=1}^{n}\left(-\ln\left(\omega+\alpha R_{t-1}^2+\beta\sigma_{t-1}^2\right)-\frac{r_t^2}{\omega+\alpha R_{t-1}^2+\beta\sigma_{t-1}^2}\right). \tag{2.6}
$$

We estimate the parameters α and β in the formula (2.6) using *damped* Newton's method which is given by

$$
\theta_n = \theta_{n-1} - d[H(\theta_{n-1})]^{-1}W(\theta_{n-1}), \qquad n \in \mathbb{N}, \tag{2.7}
$$

where θ_n is 2×1 matrix approximating the log likelihood function $\hat{l}$ containing estimates of α and β. The W_n and H_n are 2×1 matrix containing first derivative and 2×2 matrix of second derivative at iteration n, respectively, while d is a constant between 0 and 1. The H and W are given by

$$
H(\hat{l}) = \begin{bmatrix} \frac{\partial^2\hat{l}}{\partial\alpha^2} & \frac{\partial^2\hat{l}}{\partial\beta\partial\alpha} \\ \frac{\partial^2\hat{l}}{\partial\alpha\partial\beta} & \frac{\partial^2\hat{l}}{\partial\beta^2} \end{bmatrix}
$$

and

$$
W(\hat{l}) = \begin{bmatrix} \frac{\partial\hat{l}}{\partial\alpha} \\ \frac{\partial\hat{l}}{\partial\beta} \end{bmatrix},
$$

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respectively. Although the logarithm of the log likelihood can be calculated using formula (2.7), similar formulas are not available for its derivatives, so they are clculated numerically at each step of the iteration. The constant d(marquardt damping factor) is designed to decrease the changes at each iteration and thus prevent overshooting maximum values, which are constrained within the triangle $0 < \alpha < 1, 0 < \beta < 1, 0 < \alpha + \beta < 1$. After obtaining the parameters of model, we can fit the stock returns using GARCH(1,1) and the volatility will be given. As we have mentioned in chapter 1, the volatility is fluctuating a lot over the period. We would like to smooth the volatility series by using cubic spline interpolation in order to simplify investigation of their change in many situations.

2.2.2 Fitting volatility series using cubic spline

According to the preceding section (2.2), the GARCH(1,1) gives daily volatility series over the period. In order to study the behavior of volatility, we employ the natural cubic spline to fit volatility series obtaining from GARCH(1,1). It is because the natural cubic spline is the lowest degree splines that has such attractive properties as smoothness, continuity of the first and second derivative so that many financial institutions use the method for curve fitting (Alexander, 2008). Therefore, we can get the information on rate of change and cumulative change of volatility series over the period.

Let $(t_1, y_1), (t_2, y_2), ..., (t_n, y_n)$ be a series of knot points, where $t_1 < t_2 < ... < t_n$ and $s(t)$ be a cubic spline function which fits consecutive knot points. We proposed an natural cubic spline which easily to apply in the data. It was improved by McNeil et al.

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(2011). The cubic spline function is defined as:

$$s(t) = a + bt + \sum_{k=1}^{p} c_k \left(t - t_k\right)_+^3, \tag{2.8}$$

where t denotes time and $(t - t_k)_+$ is $t - t_k$ for $t > t_k$ and zero otherwise. Since this formula (2.8) is linear function of the coefficients a, b and c_k, it is fitted to data using linear regression. However, linearity in the future means that the quadratic and cubic coefficients are 0 for $t > t_p$ by setting $s''(t) = 0$. The condition can be seen as: Consider the formula (2.8) we get

$$s'(t) = b + 3\sum_{k=1}^{p} c_k (t - t_k)^2$$

$$s''(t) = 6\sum_{k=1}^{p} c_k (t - t_k) = 6\left(t\sum_{k=1}^{p} c_k - \sum_{k=1}^{p} c_k t_k\right).$$

To make $s''(t) = 0$, we set two conditions

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$$\sum_{k=1}^{p} c_k = 0, \sum_{k=1}^{p} c_k t_k = 0.$$

In order to have a simple and applicable natural cubic spline, we let $x = c_p, y = c_{p-1}, \alpha = t_p, \beta = t_{p-1}$,

$$\lambda = \sum_{k=1}^{p-2} c_k \tag{2.9}$$

and

$$\mu = \sum_{k=1}^{p-2} c_k t_k. \tag{2.10}$$

Then we have

$$x + y = -\lambda \tag{2.11}$$

and

$$\alpha x + \beta y = -\mu. \tag{2.12}$$

We obtain y and x by multiplying α through (2.11) and subtracting from (2.12). We get

$$\begin{aligned}(\alpha-\beta)y &= -\lambda\alpha+\mu\\ y=c_{p-1} &= \frac{\mu-\lambda\alpha}{\alpha-\beta}.\end{aligned} \tag{2.13}$$

Obtaining x can be done by substituting (2.13) to (2.11). We get

$$\begin{aligned}x+y &= -\lambda\\ x+\left(\frac{\mu-\lambda\alpha}{\alpha-\beta}\right) &= -\lambda\\ x &= \frac{-\lambda(\alpha-\beta)-\mu+\lambda\alpha}{\alpha-\beta}\\ x=c_p &= \frac{\lambda\beta-\mu}{\alpha-\beta}.\end{aligned} \tag{2.14}$$

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Now,the formula (2.8) can also be written as

$$\begin{aligned}s(t) &= a+bt+\sum_{k=1}^{p-2} c_k\,(t-t_k)_+^3 + c_{p-1}\,(t-t_{p-1})_+^3 + c_p\,(t-t_p)_+^3\\ &= a+bt+\sum_{k=1}^{p-2} c_k\,(t-t_k)_+^3 + \frac{\mu-\lambda\alpha}{\alpha-\beta}\,(t-t_{p-1})_+^3 + \frac{\lambda\beta-\mu}{\alpha-\beta}\,(t-t_p)_+^3.\end{aligned} \tag{2.15}$$

Substituting $\alpha = t_p, \beta = t_{p-1}$ and the two conditions in (2.9) and (2.10) to formula (2.15) gives

$$\begin{aligned}s(t) &= a+bt+\sum_{k=1}^{p-2} c_k\,(t-t_k)_+^3 + \left(\frac{\sum_{k=1}^{p-2} c_k t_k - t_p\sum_{k=1}^{p-2} c_k}{t_p-t_{p-1}}\right)(t-t_{p-1})_+^3\\ &\quad+\left(\frac{t_{p-1}\sum_{k=1}^{p-2} c_k - \sum_{k=1}^{p-2} c_k t_k = \mu}{t_p-t_{p-1}}\right)(t-t_p)_+^3.\\ &= a+bt+\sum_{k=1}^{p-2} c_k\left[(t-t_k)_+^3 + \left(\frac{t_k-t_p}{t_p-t_{p-1}}\right)(t-t_{p-1})_+^3 + \left(\frac{t_{p-1}-t_k}{t_p-t_{p-1}}\right)(t-t_p)_+^3\right]\end{aligned}$$

or

$$s(t) = a+bt+\sum_{k=1}^{p-2} c_k\left[(t-t_k)_+^3 - \left(\frac{t_p-t_k}{t_p-t_{p-1}}\right)(t-t_{p-1})_+^3 + \left(\frac{t_{p-1}-t_k}{t_p-t_{p-1}}\right)(t-t_p)_+^3\right]. \tag{2.16}$$

In practice, formula (2.16) was easily used for smoothing the volatility series. The parameters a, b, and $c_k, k = 1, ..., p-2$ in the formula (2.16) were estimated by Least square.

2.3 Assessing model Using Monte Carlo Simulation

The usual way, fitting a model involve the concept of taking a sample from a population where the sample distribution is known. In this case, the volatility of stock returns are unknown and different samples of data from the population provide different estimates of their values. In assessing the model, we reverse the process of fitting by assuming that the population parameters are known and use the Monte Carlo to generate repeated sample from distribution with known parameters. Thus, the objective in simulation is not to determine the volatility series, but rather to assess the model that estimates them.

The Monte Carlo simulation generates repeated samples from a distribution and these samples should be random but repeatable. Therefore, we should be able to generate exactly the same set of random numbers if we want to. A device for exactly reproducing a sample is to use a specific seed for starting the random numbers in a simulation. By changing the seed, different sets of random numbers can be generated and they can be reproduced exactly by using the same seed that was used to create them in the first place.

The idea to reproduce the repeatable random numbers are considering a probability space and a real valued random variable X on it, which records the outcome of random experiment. We can model repetitions of this experiment by introducing a sequence of

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random variables $X_1, X_2, ..., X_n$ which has the same probability information as X . We now propose a definition of a finite sequence of random variables which are identically distributed.

Definition 2. A sequence $X_1, X_2, ..., X_n$ of random variables is called identically distributed if

$$F_{X_1}(x) = F_{X_2}(x) = ... = F_{X_n}(x), \qquad \forall x,$$

where $F_{X_i}(x)$ is the distribution function of $X_i, i = 1, 2, ..., n$. (Briani, 2002)

If we assume that the random variables $X_1, X_2, ..., X_n$ are independent then we can consider the sequence as a model for repeated and independent runs of the experiment. We first propose the theorem to shows that with probability one, we can conclude the sample mean converge to the distribution mean as the sample size increases.

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Theorem 1 (Strong Law of Large Numbers)**.** *Let $X_1, X_2, ..., X_n$ be a sequence of independent, identically distributed, integrable random variables defined on the same probability space, such that for $i = 1, 2, ..., n$,*

$$\mu = \mathbb{E}\left[X_i\right],$$

then

$$\mathbb{P}\left(\lim_{n\to\infty} \frac{X_1 + ... + X_n}{n} = \mu\right) = 1.$$

Proof. (Briani, 2002)

The Strong Law of Large Numbers states that for almost every sample $\omega \in \Omega$,

$$\frac{X_1(\omega) + ... + X_n(\omega)}{n} \to \mu,$$

as $n \to \infty$. We want the error in estimating the sample mean in normally distributed. The following theorem guarantee that the sum of random variables have a distribution tend to normal distribution as the sample size increases.

Theorem 2 (Central Limit Theorem)**.** *Let* $X_1, X_2, ..., X_n$ *be a sequence of independent and identically distributed (i.i.d), real-valued random variables with, for* $i = 1, 2, ..., n$,

$$\mathbb{E}[X_i] = \mu, Var[X_i] = \sigma^3 > 0.$$

We set

$$S_n = X_1 + ... + X_n.$$

then for all $-\infty < a < b < +\infty$

$$\lim_{n\to\infty} \mathbb{P}\left(a \leq \frac{S_n - nm}{\sigma\sqrt{n}} \leq b\right) = \frac{1}{2\pi}\int_a^b \exp^{\frac{-x^2}{2}} dx$$

Proof. (Briani, 2002)

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Chapter 3

Results

This study concerns two objectives related to studying the behavior of volatility of stock returns over the period and assessing the performance of volatility model. Firstly, we show the behavior of stock returns of seven companies. After that, we present the volatility series obtaining from the GARCH(1,1). Finally, we show the result of assessing model using Monte Carlo simulation.

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3.1 Stock returns

We involve data from daily closing prices of the seven companies of Indonesia from 12 July 2007 to 29 September 2015.

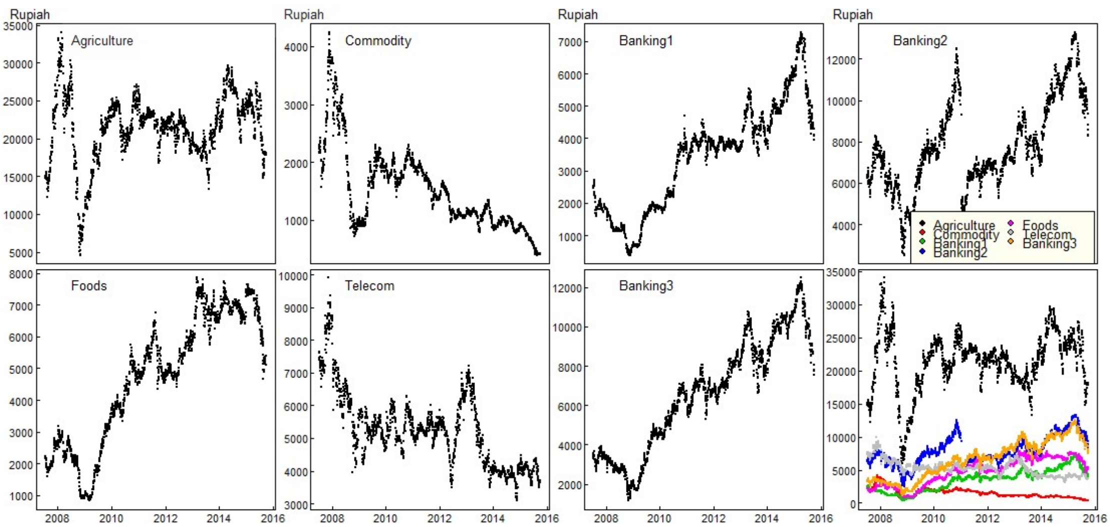

Figure 3.1: Stock prices of seven companies over the period

Looking at stock price graphed in Figure 3.1, we see that prices for stocks in bangking type 1, foods and bangking type 3 had quite similar patterns, all increasing after 2009. On the other side, commodity and telecommunication also show similar pattern, all decreasing substantially starting at the end of 2008 and remained low, whereas agriculture and bangking type 2 varied substantially. As the bottom right panel shows, there is also huge variation on the scales of the prices, with price for agriculture orders of magnitude greater than the others.

Furthermore, we calculated the returns of those stock price. In this study, returns were defined as log returns which has been mentioned in chapter 2. The log returns R_t is given by

$$R_t = \ln \frac{S_t}{S_{t-1}},$$

where S_t is stock price at then end of day t.

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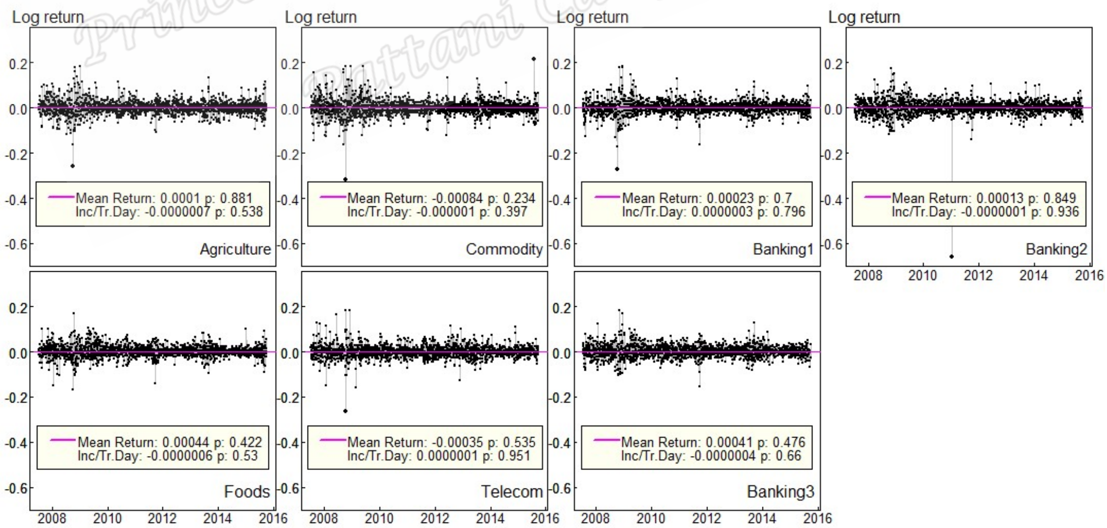

Figure 3.2: Log returns of seven companies over 2007 to 2015

Figure 3.2 shows the stock returns distribution over the period. Since all p-values of means returns and increasing per trading day for all stocks greater than 0.05 indicating they are not statistically significant which means that the mean returns and the increasing of returns are not zero. In this case, some smart investors can make money from these companies. In particular case, looking at the log returns of banking type 2, we see that the returns had the lowest value in a trading day during 2011, but subsequently remained relatively stable ranging from -0.1 up to 0.1. According to the information that has been issued by banking type 2 which is Bank BRI, the Bank made a decision to split the stock on 1 November 2011 for strategic purposes. This might be a reason of decreasing the returns at that time.

Furthermore, we investigate the returns distribution by Q-Q plot shown in Figure 3.3. The log returns are plotted on the y-axis and corresponding quantiles from theoretical quantiles on x-axis. It clearly can be seen from all panels that the stock returns are normal in the middle which are bounded between -1 and 1 of the theoretical quantile values, but have stretched tails on both sides.

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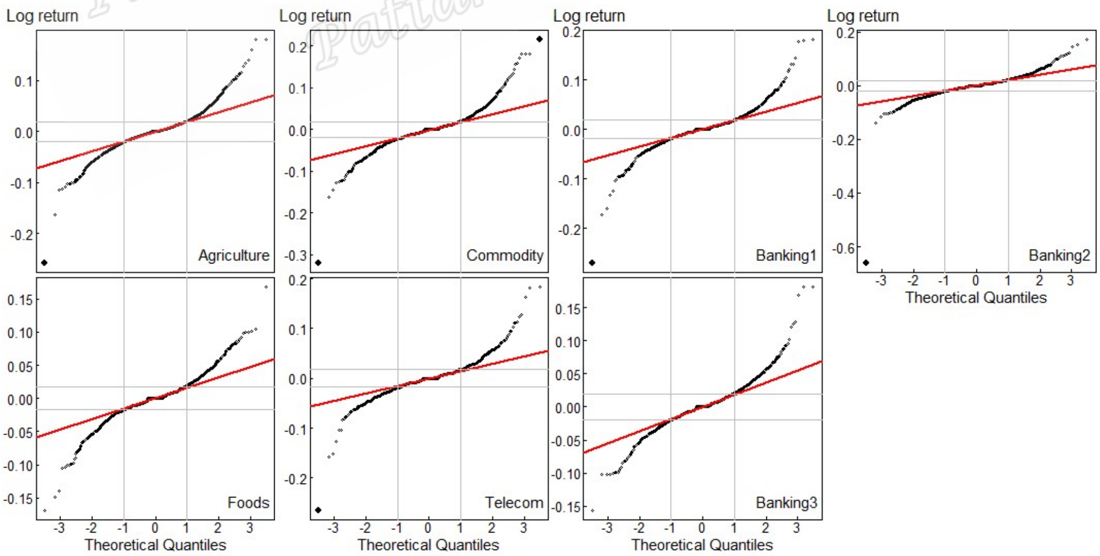

Figure 3.3: Q-Q plots of log returns

The low tail of the log returns occur at more negative value than the theoretical quantiles. Likewise, the high tail occur at the greater than the theoretical quantiles.

Points distant from the straight line indicated non-normality. This indicates that the returns distribution contain fat tail (heavy tail). We used Huber robust transformation to overcome this condition by replacing the observed value y greater than a specified constant c by $c+\left(\frac{y-c}{a}\right)$ and the observed values y smaller than $-c$ by $-c+\left(\frac{y+c}{a}\right)$, where the constant $a = 2.5$ and the constants c is 0.014 for food, 0.015 for telecom, and 0.016 for agriculture, commodity, banking type 1, banking type 2 and banking type 3.

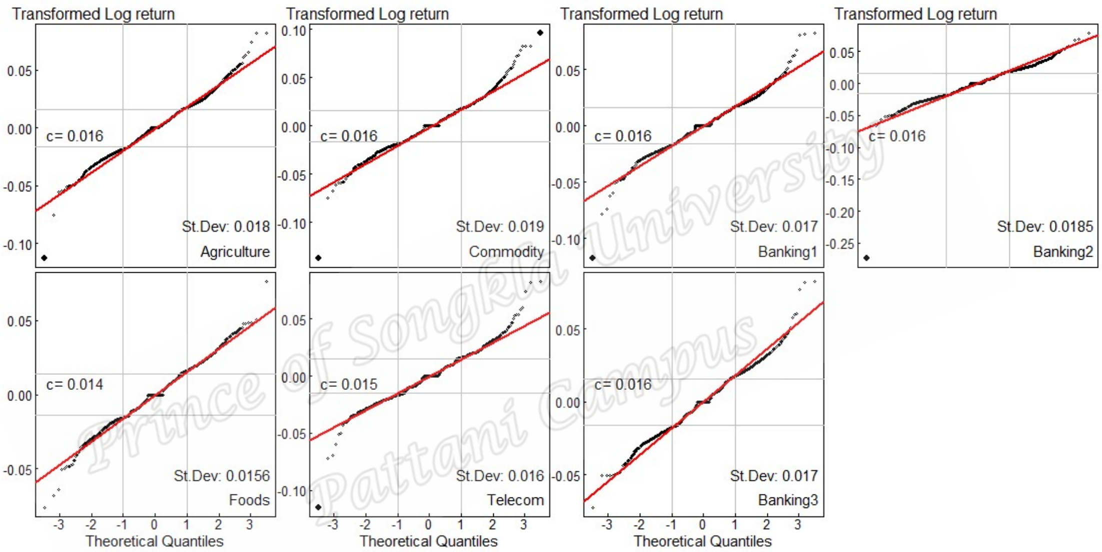

Figure 3.4: Q-Q plots of transformed log returns

Figure 3.4 shows the results of transforming log returns using the Huber robust transformation. The constants c vary for each stock between 0.014 to 0.016. It clearly can be seen that the returns series are following linear line even though there are minor stretch tail in some panels, indicating the returns is approximately normal. After transforming the data, we now can obtain the information of returns fluctuation over time by fitting GARCH(1,1) to the transformed returns.

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3.2 Volatility of stock returns

We can employ time series model to obtain the volatility of stock returns. Note that the time series model that will be used must agree with heteroscedasticity. In this study, we used GARCH(1,1) which has been described in chapter 2. The GARCH(1,1) is given by

$$\sigma_t^2 = \omega + \alpha R_{t-1}^2 + \beta \sigma_{t-1}^2. \tag{3.1}$$

Before fitting the GARCH(1,1) to the transformed returns, we estimated the parameters α and β using maximum likelihood method. This method was used to determine the values of parameters which maximize the log likelihood function. The transformed returns of seven companies comprising 2055 observations were involved in the log likelihood. Estimating the parameters was preceded by determining the log likelihood and total likelihood for any initial values.

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We set the initial values $\alpha = 0.124$ and $\beta = 0.824$ for agriculture, commodity, bangking type 1, foods and bangking type 3. The other two companies have different initial values, that is $\alpha = 0.224$ and $\beta = 0.674$ for bangking type 2, while $\alpha = 0.174$ and $\beta = 0.724$ for telecommunication. After that, we examined the following log likelihood function

$$\hat{l}(\alpha, \beta; r_t) = \sum_{t=1}^{n} \left(-\ln\left(\omega + \alpha R_{t-1}^2 + \beta \sigma_{t-1}^2\right) - \frac{r_t^2}{\omega + \alpha R_{t-1}^2 + \beta \sigma_{t-1}^2} \right). \tag{3.2}$$

Finding values of α and β that maximize that maximize the formula (3.2) can be achieved by using the damped Newton's method iterative procedure, in which initial values for these parameters are selected and successively updated using the formula

$$\theta_n = \theta_{n-1} - d[H(\theta_{n-1})]^{-1} W(\theta_{n-1}), \qquad n \in \mathbb{N},$$

where θ_{n-1} and W_{n-1} are 2×1 vectors containing estimates of α and β and their first derivative, respectively, at iteration $n-1$, H_{n-1} is the corresponding 2×2 matrix of second derivatives, and d is a constant between 0 and 1.

Table 3.1 shows the estimates parameters for all stocks. The estimated values of α range from 0.0999612 up to 0.2290749, β from 0.6897706 up to 0.8799266 and corresponding values of ω from 0.000004891346 up to 0.00002783173.

Table 3.1: Estimates of parameters of GARCH(1,1) of seven companies

Stock	ω	α	β	log likelihood
Agriculture	0.000009939594	0.1051743	0.8643040	14723.47
Commodity	0.000010334310	0.1120047	0.8592676	14300.04
Banking type 1	0.000008772274	0.1060997	0.8635160	15019.07
Banking type 2	0.000027831730	0.2290749	0.6897706	14662.61
Foods	0.000004891346	0.0999612	0.8799266	15362.81
Telecom	0.000019801130	0.1713836	0.7512006	15168.23
Banking type 3	0.000009021595	0.1069880	0.8616199	14961.33

According to the results in Table 3.1, the GARCH(1,1) that were used in this study are:

1. Agriculture: $\sigma_t^2 = 0.000009939594 + 0.1051743 R_{t-1}^2 + 0.8643040 \sigma_{t-1}^2$

2. Commodity: $\sigma_t^2 = 0.000010334310 + 0.1120047 R_{t-1}^2 + 0.8592676 \sigma_{t-1}^2$

3. Banking type 1: $\sigma_t^2 = 0.000008772274 + 0.1060997 R_{t-1}^2 + 0.8635160 \sigma_{t-1}^2$

4. Banking type 2: $\sigma_t^2 = 0.000027831730 + 0.2290749 R_{t-1}^2 + 0.6897706 \sigma_{t-1}^2$

5. Foods: $\sigma_t^2 = 0.000004891346 + 0.0999612R_{t-1}^2 + 0.8799266\sigma_{t-1}^2$

6. Telecommunication: $\sigma_t^2 = 0.000019801130 + 0.1713836R_{t-1}^2 + 0.7512006\sigma_{t-1}^2$

7. Banking type 3: $\sigma_t^2 = 0.000009021595 + 0.1069880R_{t-1}^2 + 0.8616199\sigma_{t-1}^2$

where R_t and σ_t are returns and variance at day t.

Fitting the GARCH(1,1) model to the transformed returns gives the volatility series plotted in Figure 3.5, where a and b corresponds to parameters α and β, respectively.

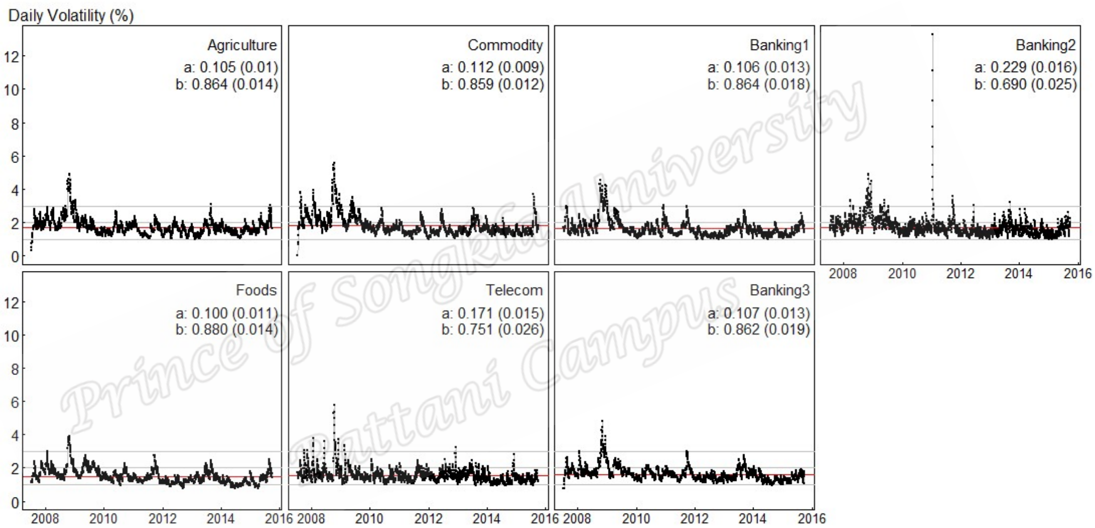

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Figure 3.5: Volatility series of seven stock returns

Figure 3.5 show evidence that the returns have higher volatility during the end of 2008, but subsequently remained relatively stable. The seven companies are big companies that can possibly reflect the economy of Indonesia. The increasing volatility at the end of 2008 indicates the economic crisis in Indonesia. We smoothed the volatility series using natural cubic spline.

To study the behavior of the volatility can be done by natural cubic spline. It is given by

$$s(t) = a+bt+\sum_{k=1}^{p-2} c_k \left[(t-t_k)_+^3 - \left(\frac{t_p - t_k}{t_p - t_{p-1}} \right) (t-t_{p-1})_+^3 + \left(\frac{t_{p-1} - t_k}{t_p - t_{p-1}} \right) (t-t_p)_+^3 \right]. \tag{3.3}$$

The equation (3.3) can be written as

$$s(t) = a + bt + \sum_{k=1}^{p-2} c_k s_k, \tag{3.4}$$

with $p = 8$, the estimates of parameters $a, b, c_1, ..., c_6$ of the seven stocks were obtained and shown in Table 3.2-3.8

Table 3.2: Estimates of parameters of natural cubic splines of agriculture

Parameters	Estimate	Std. Error
a	1.6209803118578	0.0426000214165
b	0.0039138539960	0.0002237637290
c_1	-0.0000000224625	0.0000000009926
c_2	0.0000000670617	0.0000000030491
c_3	-0.0000000807706	0.0000000042692
c_4	0.0000000625317	0.0000000045395
c_5	-0.0000000506278	0.0000000046061
c_6	0.0000000493617	0.0000000045649

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Table 3.3: Estimates of parameters of natural cubic splines of commodity

Parameters	Estimate	Std. Error
a	1.973047336312	0.049364458809
b	0.003003372918	0.000259295066
c_1	-0.000000017438	0.000000001150
c_2	0.000000048158	0.000000003533
c_3	-0.000000049142	0.000000004947
c_4	0.000000032144	0.000000005260
c_5	-0.000000034131	0.000000005338
c_6	0.000000044281	0.000000005290

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Table 3.4: Estimates of parameters of natural cubic splines of banking type 1

Parameters	Estimate	Std. Error
a	1.375588223397	0.043028055462
b	0.004256200227	0.000226012049
c_1	-0.000000022853	0.000000001003
c_2	0.000000069956	0.000000003080
c_3	-0.000000092804	0.000000004312
c_4	0.000000088962	0.000000004585
c_5	-0.000000084488	0.000000004652
c_6	0.000000076429	0.000000004611

Table 3.5: Estimates of parameters of natural cubic splines of banking type 2

Parameters	Estimate	Std. Error
a	1.548486974886	0.062455884875
b	0.003828803178	0.000328059969
c_1	-0.000000020715	0.000000001455
c_2	0.000000064622	0.000000004470
c_3	-0.000000089622	0.000000006259
c_4	0.000000090253	0.000000006655
c_5	-0.000000084456	0.000000006753
c_6	0.000000071287	0.000000006693

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Table 3.6: Estimates of parameters of natural cubic splines of foods

Parameters	Estimate	Std. Error
a	1.4829464380597	0.0367111794334
b	0.0027102694115	0.0001928316027
c_1	-0.0000000142093	0.0000000008554
c_2	0.0000000410967	0.0000000026276
c_3	-0.0000000504271	0.0000000036791
c_4	0.0000000495958	0.0000000039119
c_5	-0.0000000597616	0.0000000039694
c_6	0.0000000708997	0.0000000039338

Table 3.7: Estimates of parameters of natural cubic splines of telecommunication

Parameters	Estimate	Std. Error
a	1.7029743969015	0.0426938663851
b	0.0008916388805	0.0002242566654
c_1	-0.0000000068217	0.0000000009948
c_2	0.0000000199555	0.0000000030558
c_3	-0.0000000200805	0.0000000042786
c_4	0.0000000075404	0.0000000045495
c_5	-0.0000000056156	0.0000000046163
c_6	0.0000000172645	0.0000000045749

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Table 3.8: Estimates of parameters of natural cubic splines of banking type 3

Parameters	Estimate	Std. Error
a	1.5001309910969	0.0379616856029
b	0.0032211330833	0.0001994000953
c_1	-0.0000000171289	0.0000000008845
c_2	0.0000000518810	0.0000000027171
c_3	-0.0000000678287	0.0000000038044
c_4	0.0000000651640	0.0000000040452
c_5	-0.0000000654464	0.0000000041046
c_6	0.0000000632528	0.0000000040678

After estimating the parameters of natural cubic spline of volatility series for all seven companies, the fitted volatility were illustrated in Figure 3.6. We fitted the volatility series of seven stock returns using 8 knots natural cubic spline. It shows the volatility fitted by natural cubic spline which reflect the volatility signals. The lower right panel shows the volatility signals of stock returns for each of the seven stocks on the same axes. It can be seen that the seven volatility signals have the same trends, particularly during the end of 2008. In addition, regarding the natural cubic spline curves, food and telecom might simply reflect flat volatility over the period. These volatility signals can be used in the process of assessing the model. The volatility model is expected to capture the volatility as accurate as possible, so we assessed the GARCH(1,1) by employing Monte Carlo simulation.

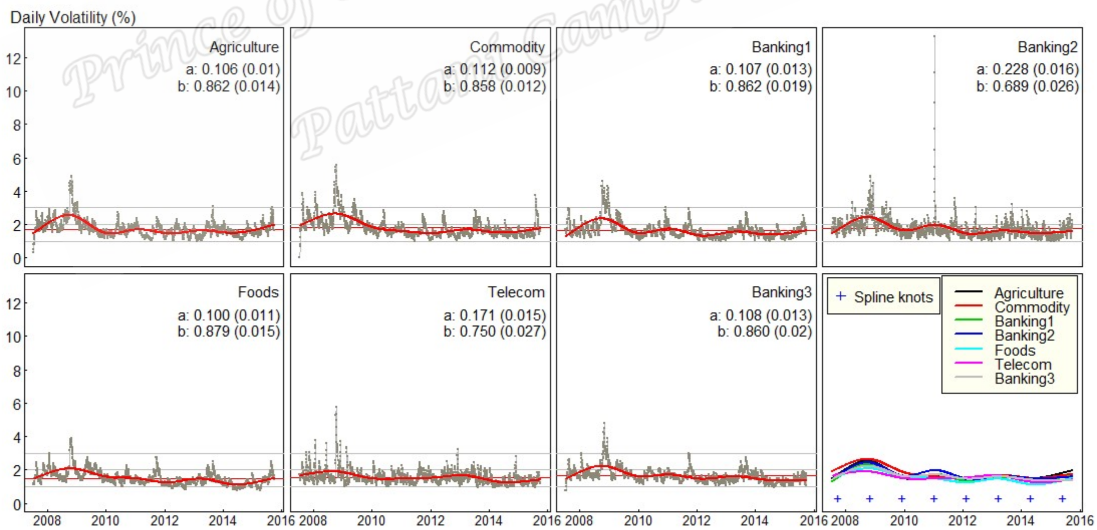

Figure 3.6: Fitted volatility series

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3.3 Assessing the Performance of GARCH(1,1)

Figure 3.7 shows the assumed volatility and fitted volatility of commodity stock as an example.

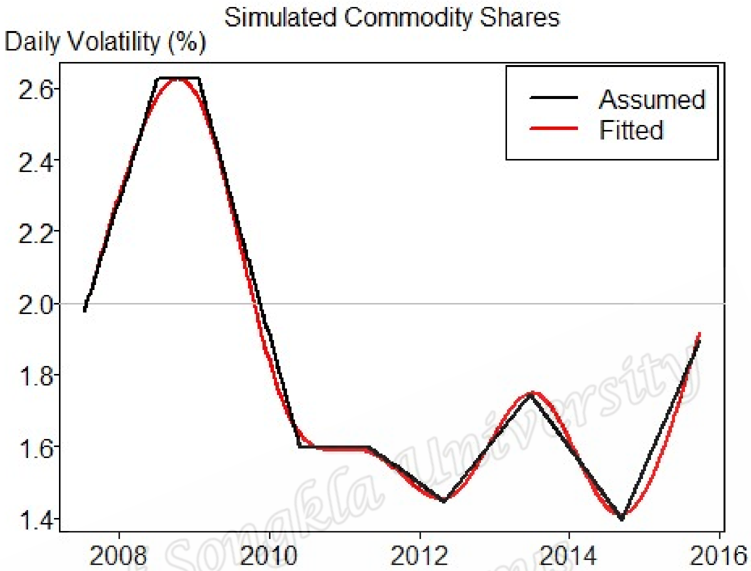

Figure 3.7: Assuming simple path of the volatility

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To see how well the GARCH(1,1) model can estimate volatility in a series of stock returns, we assume a specific simple shape which is a piecewise linear spline for the volatility that approximates what we found for commodity stock returns. We generate random numbers from known distribution which is the normal distribution. Multiplying these random numbers with values in assumed volatility give the returns which are graphed in Figure 3.8.

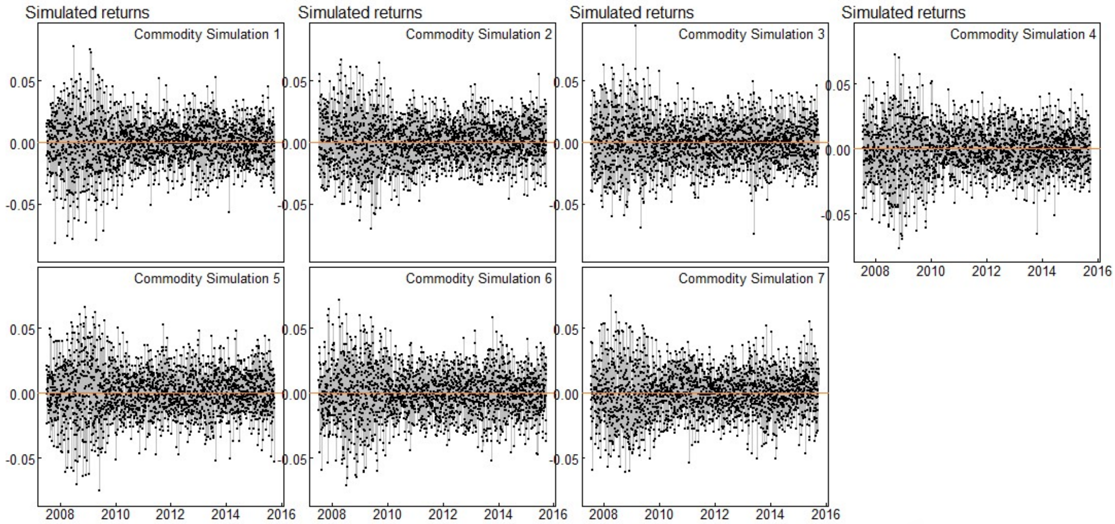

Figure 3.8: Simulated returns

Before employing GARCH(1,1) to fit simulated returns, we assess the normalility of the data using Q-Q plot as shown in Figure 3.9.

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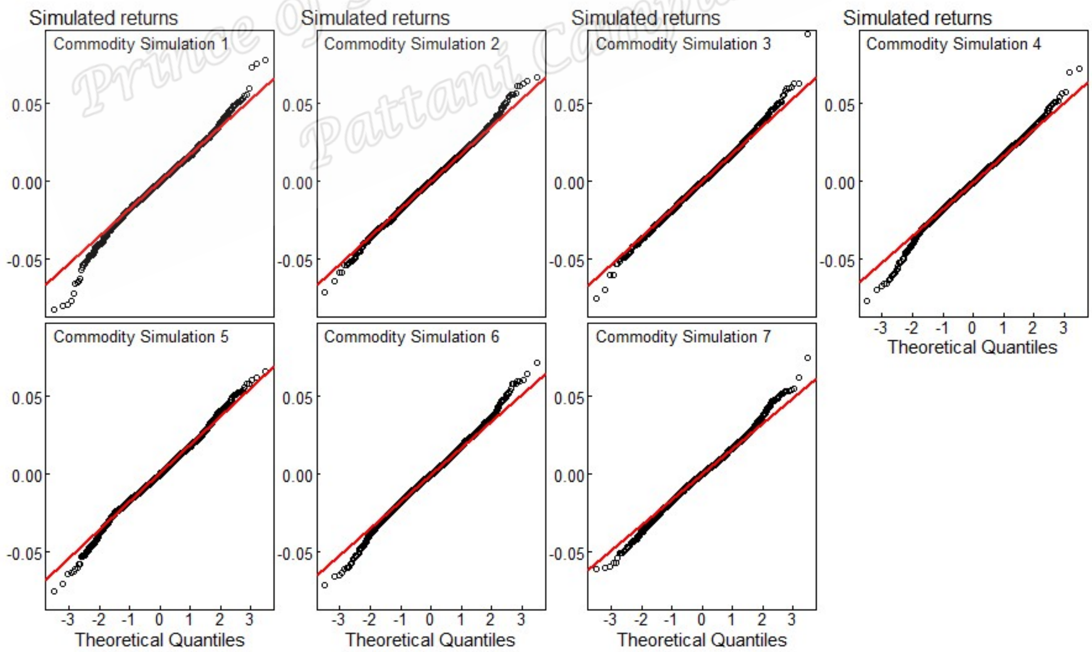

Figure 3.9: Q-Q plot of simulated returns

From Figure 3.9, it can be seen that the simulated returns are following linear line indicating normality. However, there are stretch tail in some panels even though the random numbers come from normal distribution. This is due to the fact that the variance is not constant. For each series of simulated commodity share returns, we can compute their prices (assuming that the closing price on 12 July 2007 is the same as was observed, i.e. 2225.94 rupiah) by exponentiating accumulated returns. The results are graphed in Figure 3.10.

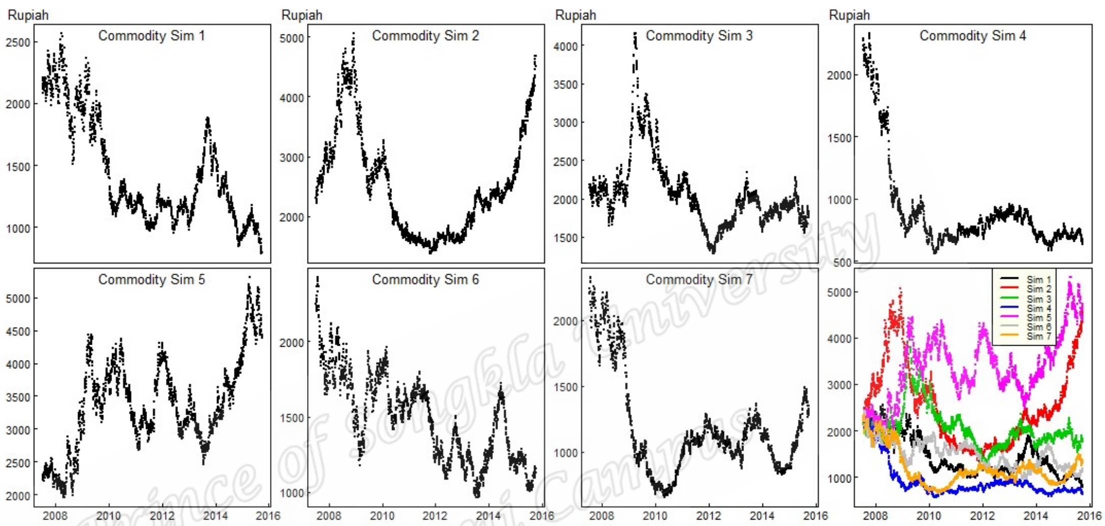

Figure 3.10: Simulated price of commodity stock

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We fitted the simulated returns series using GARCH(1,1) to obtain their volatility. However, we need to estimate the parameters of the model as we did in modelling part. Table 3.9 shows the estimates parameters of GARCH(1,1) of seven simulation. Estimated values of α range from 0.0160812 up to 0.04883133, β from 0.9277765 up to 0.9818305 and corresponding values of ω range from 0.0000006619928 up to 0.000008368814.

Table 3.9: Estimates of parameters of GARCH(1,1) of seven simulations

Simulation	ω	α	β	log likelihood
1	0.0000008910728	0.01608120	0.9812698	14470.01
2	0.0000010097810	0.01805914	0.9792222	14325.59
3	0.0000010768590	0.01966104	0.9772828	14416.06
4	0.0000010824210	0.02188261	0.9749664	14455.89
5	0.0000006619928	0.01624257	0.9818305	14495.20
6	0.0000021556710	0.02676629	0.9668474	14450.22
7	0.0000083688140	0.04883133	0.9277765	13766.77

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Therefore the GARCH(1,1) models that were used in simulation are

1. Simulation1: $\sigma_t^2 = 0.0000008910728 + 0.0160812R_{t-1}^2 + 0.9812698\sigma_{t-1}^2$
2. Simulation2: $\sigma_t^2 = 0.0000010097810 + 0.01805914R_{t-1}^2 + 0.9792222\sigma_{t-1}^2$
3. Simulation3: $\sigma_t^2 = 0.0000010768590 + 0.01966104R_{t-1}^2 + 0.9772828\sigma_{t-1}^2$
4. Simulation4: $\sigma_t^2 = 0.0000010824210 + 0.02188261R_{t-1}^2 + 0.9749664\sigma_{t-1}^2$
5. Simulation5: $\sigma_t^2 = 0.0000006619928 + 0.01624257R_{t-1}^2 + 0.9818305\sigma_{t-1}^2$
6. Simulation6: $\sigma_t^2 = 0.0000021556710 + 0.02676629R_{t-1}^2 + 0.9668474\sigma_{t-1}^2$
7. Simulation7: $\sigma_t^2 = 0.0000083688140 + 0.04883133R_{t-1}^2 + 0.9277765\sigma_{t-1}^2$.

Fitting those GARCH(1,1) models to the simulated returns gives simulated volatility series. Again, we used natural cubic spline to smooth the simulated volatility series. The estimates of parameters $a, b, c_1, ..., c_6$ for all stocks were shown in Table 3.10-3.16.

Table 3.10: Estimates of parameters of natural cubic spline of simulation 1

Parameters	Estimate	Std. Error
a	1.7173056646673	0.0090350925913
b	0.0030350833787	0.0000474583332
c_1	-0.0000000139274	0.0000000002105
c_2	0.0000000350123	0.0000000006467
c_3	-0.0000000291169	0.0000000009055
c_4	0.0000000142523	0.0000000009628
c_5	-0.0000000192543	0.0000000009769
c_6	0.0000000273078	0.0000000009682

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Table 3.11: Estimates of parameters of natural cubic spline of simulation 2

Parameters	Estimate	Std. Error
a	1.6178967524672	0.0100216178610
b	0.0047113086280	0.0000526402220
c_1	-0.0000000207472	0.0000000002335
c_2	0.0000000532822	0.0000000007173
c_3	-0.0000000471546	0.0000000010043
c_4	0.0000000223495	0.0000000010679
c_5	-0.0000000189427	0.0000000010836
c_6	0.0000000247701	0.0000000010739

Table 3.12: Estimates of parameters of natural cubic spline of simulation 3

Parameters	Estimate	Std. Error
a	1.8776711908311	0.0117636948886
b	0.0032011203738	0.0000617907726
c_1	-0.0000000166211	0.0000000002741
c_2	0.0000000444864	0.0000000008420
c_3	-0.0000000421554	0.0000000011789
c_4	0.0000000219930	0.0000000012535
c_5	-0.0000000188195	0.0000000012720
c_6	0.00000002461900	0.0000000012606

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Table 3.13: Estimates of parameters of natural cubic spline of simulation 4

Parameters	Estimate	Std. Error
a	1.6298662895912	0.0101829825377
b	0.0041184678650	0.0000534878169
c_1	-0.0000000187335	0.0000000002373
c_2	0.0000000478790	0.0000000007288
c_3	-0.0000000415913	0.0000000010205
c_4	0.0000000200516	0.0000000010851
c_5	-0.0000000200225	0.0000000011010
c_6	0.0000000254282	0.0000000010912

Table 3.14: Estimates of parameters of natural cubic spline of simulation 5

Parameters	Estimate	Std. Error
a	1.9696456012943	0.0114062611768
b	0.0025103849915	0.0000599132923
c_1	-0.0000000121842	0.0000000002658
c_2	0.0000000297152	0.0000000008164
c_3	-0.0000000223117	0.0000000011431
c_4	0.0000000081687	0.0000000012155
c_5	-0.0000000142714	0.0000000012333
c_6	0.0000000251387	0.0000000012223

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Table 3.15: Estimates of parameters of natural cubic spline of simulation 6

Parameters	Estimate	Std. Error
a	1.6136714052574	0.0121617658701
b	0.0039023614873	0.0000638817069
c_1	-0.0000000190756	0.0000000002834
c_2	0.0000000515146	0.0000000008705
c_3	-0.0000000510035	0.0000000012188
c_4	0.0000000316573	0.0000000012960
c_5	-0.0000000301402	0.0000000013150
c_6	0.0000000332725	0.0000000013032

Table 3.16: Estimates of parameters of natural cubic spline of simulation 7

Parameters	Estimate	Std. Error
a	1.7352852464381	0.0204433406164
b	0.0033863884618	0.0001073820617
c_1	-0.0000000168293	0.0000000004763
c_2	0.0000000458898	0.0000000014632
c_3	-0.0000000458825	0.0000000020488
c_4	0.0000000260182	0.0000000021784
c_5	-0.0000000183871	0.0000000022104
c_6	0.0000000184621	0.0000000021906

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The result of fitting natural cubic spline to simulated volatility series are depicted in Figure 3.11.

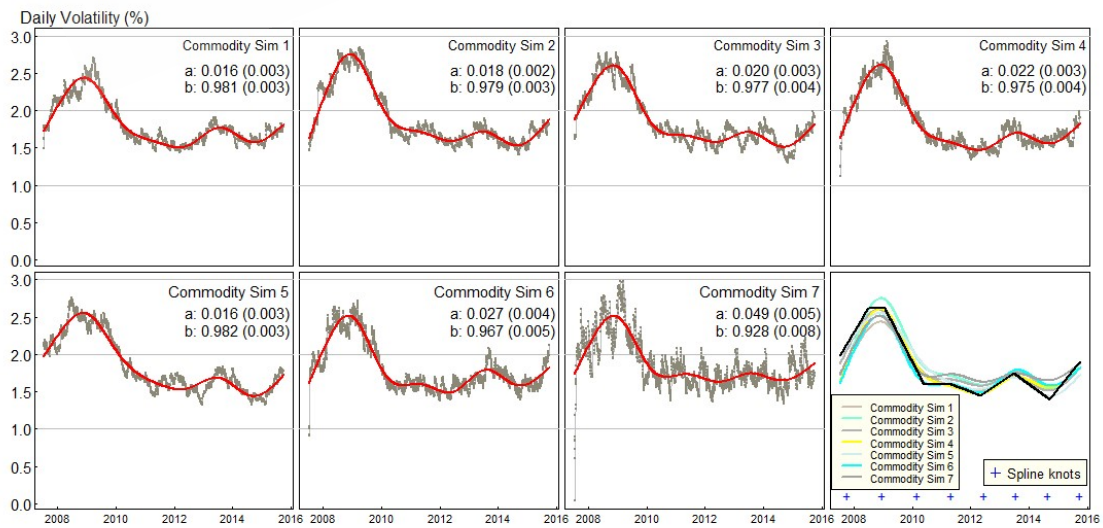

Figure 3.11: Assumed volatility and seven simulated volatility

Figure 3.11 shows 8 knots natural cubic spline functions fitted to the estimated daily volatility for the seven simulation with the estimated parameters $a = \alpha$ and $b = \beta$. The lower right panel shows the volatility signals together with the known volatility realization which has been assumed before. Clearly, the GARCH(1,1) has captured the population volatility quite well which was shown by volatility signals of seven simulation close to assumed volatility.

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Chapter 4

Conclusions and Discussions

In this chapter, conclusions and discussion are presented from our study. The objectives of the study were studying the behavior of volatility of stock returns and assessing the volatility model using Monte Carlo simulation. The data in this study comprise of closing price on trading days of seven companies, which are AALI (Agro Lestari), ANTM (Antam), BBNI (Bank BNI), BBRI (Bank BRI), INDF (Indofood), ISAT (indosat) and BMRI (Bank Mandiri), starting from 12 July 2007 to 29 September 2015, yielding 2056 observations on each series.

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In this study we used 5 sectors of stock price in Indonesia stock exchange, which are agriculture, commodity, banking, foods and telecommunication because these sectors can represent the 9 sectors which are agriculture, mining, industrial, commodity, consumer goods, property, banking, telecommunication and foods that have been traded in Indonesia market. The result of the study could be used as a procedure to obtain a good volatility model.

Result for the returns distribution shown in the first section of the results in chapter 3, suggested that the data need to be transformed. We used Huber robust transformation to do the job. After that, we employed GARCH(1,1) to estimate the daily volatility of stock returns over the period. In order to study the behavior of the volatility, we used natural cubic spline to smooth them. The volatility series for all stock had higher volatility during the end of 2008, but subsequently remained relatively stable. The higher volatility might be affected economic crisis in Indonesia. In addition, foods and telecom might simply reflect flat volatility over the period. In this study, we have considered the importance of having a good volatility model, so we assessed the model using Monte Carlo simulation. We assumed the simple path of the volatility that approximate the true volatility obtaining from GARCH(1,1). We used specified seed to have simulated returns random and repeatable. Multiplying these random numbers by corresponding values in the assumed volatility gives simulated returns. Furthermore, we apply GARCH(1,1) again to estimate the volatility of simulated returns. In each case the GARCH(1,1) was able to recapture the shape of the volatility series in population. Therefore, the GARCH(1,1) is able to capture the volatility quite well.

The following points are possible limitations of the study. Involving the representative companies from 5 sectors instead of 9 sectors might not accurately reflect the economy in Indonesia. Also determining constant c in Huber transformation was subjective decision by initially looking at the graph then fix the constants c for each stock.This procedure might not be easy to implement for other set of data. Determining location and number of knots of natural cubic spline has not been done properly in practice. This become a crucial problem of having an appropriate curve fitting. One could possibly consult the expertise in economy whether the knots using was leading to an appropriate fitting.

It would be better if employing the representative companies from all sectors so that the result might be more appropriate to reflect the economy in Indonesia. In addition, the longer period can be involved that cover extreme events to see the volatility move-

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ment during the period. Finding the applicable procedure in determining constant c will give a good contribution in Huber transformation. Various volatility models might be involved to present the comparison of the performance of the models so that the best model will be obtained to capture the volatility as accurate as possible.

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Appendix

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July 27 - 29, 2016

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an Oral Presenter

Supa Hannongbua

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Dean of Faculty of Science

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Modelling the Volatility and Assessing the Performance of the Model

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ABSTRACT

This paper presented GARCH(1,1) model for estimating volatility of daily returns of stock prices of Indonesia over the period from July 2007 to September 2015. Parameters of the model were estimated by Maximum Likelihood Estimation. We fitted volatility series using natural cubic spline to study the behavior of the volatility over the period. In order to obtained a good model, we assessed the performance of how good the GARCH(1,1) capturing volatility using Monte Carlo simulation. Our finding shows that the GARCH(1,1) is able to capture the volatility quite well.

Keywords: Volatility, GARCH(1,1), Natural cubic spline, Monte Carlo simulation.

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1. INTRODUCTION

In the financial field, volatility is one of the key variables to make an appropriate decision. According to [8] the volatility can be defined as a degree of fluctuation in asset price which can be going up or down. In fact, the volatility has taken place in different areas in financial theory and practice, such as risk management, portfolio selection and derivative pricing [2]. In many cases, the volatility is shown by low fluctuation in some period, then following by high fluctuation, and vice versa. It indicates that volatility is not constant over time. Estimating the volatility as accurate as possible is needed since return can be obtained from volatility and price can be computed based on the return. We can employ time series model to capture the volatility of returns asset.

The time series model that will be used must agree with heteroscedasticity property. Heteroscedasticity describes the volatility changes over time horizon. One of heteroscedasticity models is Generalized Autoregressive Conditional Heteroscedasticity (GARCH) which was proposed by Bollerslev [3]. Estimating volatility using the GARCH has been frequently studied by many researchers. Kamau *et al.* [8] used GARCH(1,1) to estimate the volatility of stock return in Kenyan stock markets. Their finding is that the returns stylized facts including volatility clustering, non-normal distribution and mean. Volatility clustering is the situation that high fluctuations in the returns of an asset are often followed by other high flactuations, likewise low flactuations are followed by other low fluctuations.

Their finding is similar to a study by Namugaya *et al.* [11] which showed that Uganda Securities Exchange (USE) returns have non-normal distribution, positively skewed and stationary. In fact, those returns attributes usually appear in financial time series data. It is well known that the volatility series give important information of the data. Thus, we need to investigate the volatility behavior through the period.

In order to simplify investigation of the volatility behavior, we need to smooth the volatility series. Numerical method can be employed to do the job. This leads to natural cubic spline function which is a widely used technique for piecewise smooth curve fitting. This function is simply piecewise cubic polynomial which can be constructed so that the connections between adjacent cubic splines are visually smooth [5]. Further, we note that volatility signal is obtained due to the natural cubic spline fitting of the volatility series. The signal can be used in the process of assessing how well the GARCH(1,1) model can capture a known volatility.

To assess how GARCH(1,1) can estimate volatility of stock returns ,we address the Monte Carlo simulation which is frequently used in evaluating financial models. Prior study was done by Cartea and karyampas [4] in assessing volatility estimators using the Monte Carlo simulation. The method was able to test various volatility estimators by assuming price path under different assumption about the distribution of interest variable to be Gaussian. The data of Gaussian distribution can be generated by assuming mean and variance.

The rest of the paper is organized as follows. Section 2 describes the research methodology. Section 3 reveals the result and discussion. Finally, we present conclusions.

2. METHODOLOGY

This section describes mathematical and statistical methods which were used for analyzing of volatility of stock returns in this paper. These methods comprise of obtaining the returns from stock price data, transforming the returns distribution, using GARCH(1,1) to estimate the volatility of the returns, smoothing volatility series using natural cubic spline and assessing volatility model using Monte Carlo simulation. The details will be explained as follows.

2.1 Obtaining return from stock price

We involve data from daily closing prices of the seven companies of Indonesia from July 2007 to September 2015. We can obtain returns series from stock prices data by differencing log of the price from one day to the next. Returns can be defined as the continuously compounded return during day t (between the end of day $t-1$ and the end of day t) [7], as:

$$R_t = \ln \frac{S_t}{S_{t-1}},$$

where S_t is the price at day t. Commonly, continuously compounded return, R_t, is called log return.

Figure 1 shows the stock returns distribution. It clearly can be seen from the *p*-values that the means returns for all stocks are not statistically significant which means that all means returns are not zero. In this case, some smart investors can make money from these companies. Furthermore, we investigate the returns distribution by Quantile-Quantile theory shown in figure 2

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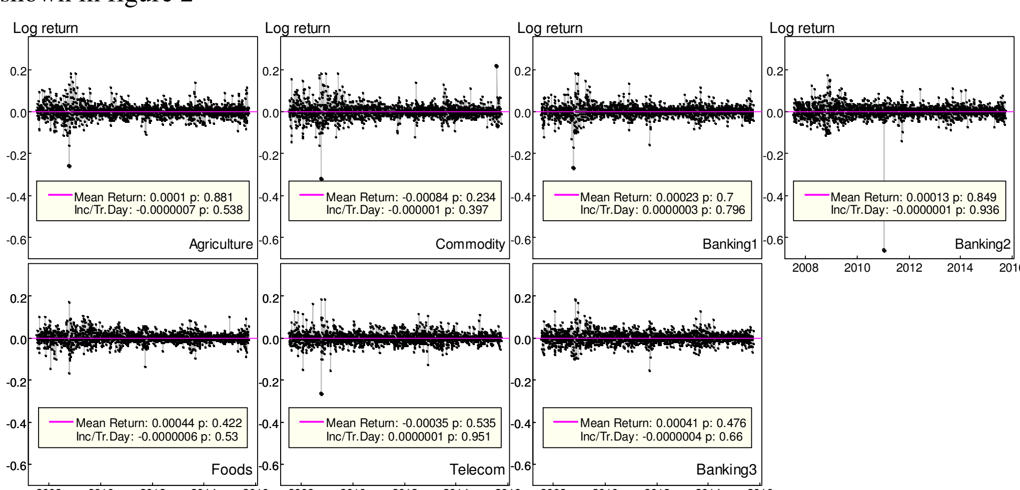

Figure 1. Stock returns distribution over 12 July 2007 to 29 September 2015

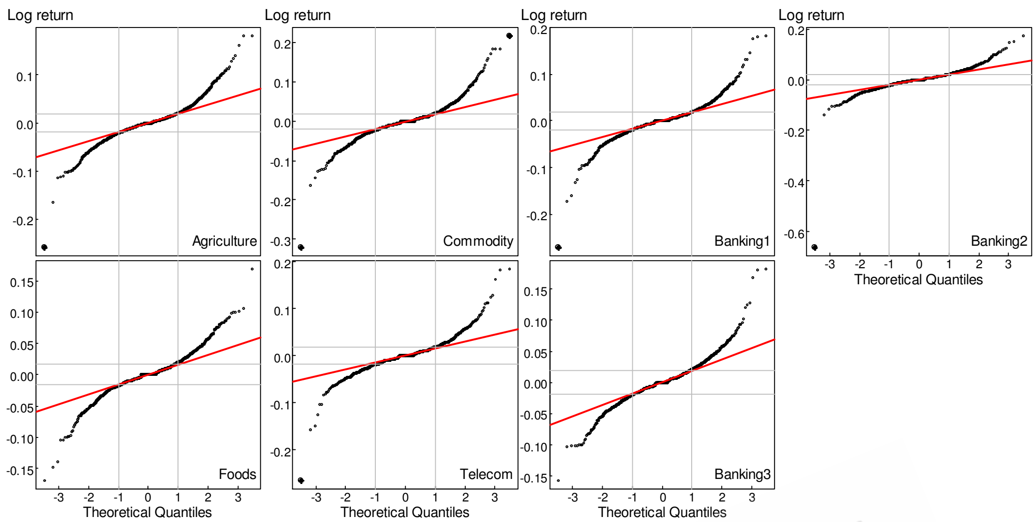

Figure 2. Quantile-Quantile plots of stock returns

In figure 2 the data are plotted on the y-axis and corresponding quantiles from a standardize normal distribution on x-axis. It clearly can be seen from all panels that the stock returns are normal in the middle, but have stretched tails on both sides. Points distant from a fitted line indicated non-normality. In other words, the returns distribution contain fat tail (heavy tail). We use Huber robust transformation to overcome this condition so that the transformed returns would be approximately normal.

2.2 Transforming stock returns using Huber robust transformation

Most of the time, the returns of financial data reflect piecewise linear behavior of three sections as parts of polygon (figure. 3). Our desire is to have the returns follow one linear model, instead of three. To solve this problem, we use the Huber robust transformation. In fact, we determine symmetrical constants c which are the turning point at the ends of $y = x$. Huber [6] suggested a method for transforming the data by shrinking their tails symmetrically. It involves replacing observed value y greater than a specified constant c by $c + \left(\frac{y-c}{a}\right)$, and similarly replacing values smaller than $-c$ by $-c + \left(\frac{y+c}{a}\right)$. The method depicted in the following figure 3.

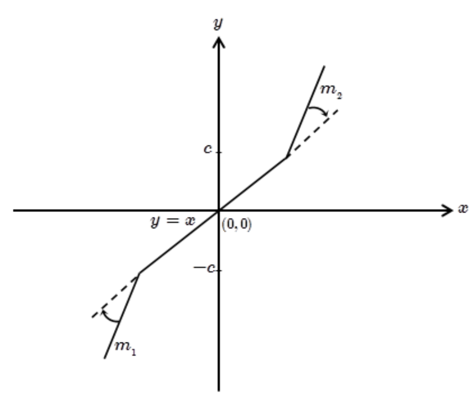

Figure 3. Huber robust transformation using linear equation

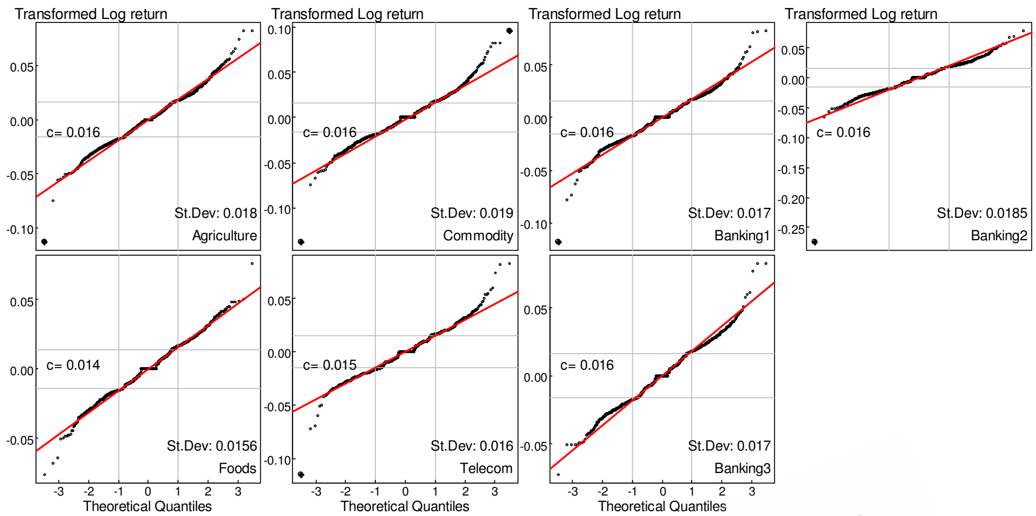

Figure 4. Quantile-Quantile plots of transformed returns

Figure. 4 shows the stock returns series after transforming using the Huber robust transformation with different constants c. It clearly can be seen that the transformed returns are approximately normal. After transforming the data, we can obtain the information of return fluctuation (volatility series) over time by fitting GARCH(1,1) to the transformed return.

2.3 Obtaining volatility series using GARCH(1,1)

Financial data contain non-constant variance over time. It is well known as heteroscedasticity. Capturing heteroscedasticity can be done by GARCH model. We involve the definition of general process of GARCH which is GARCH (*p,q*).

Definition 1.

Let $(w_t)_{t>0}$ be a sequence of independent and identically distributed (i.i.d) random variables such that $w_t \sim N(0,1)$. The R_t is called the generalized autoregressive conditionally heteroscedasticity or GARCH (*p,q*) process [12] if

$$R_t = \sigma_t w_t, \qquad t \in N,$$

where σ_t is a nonnegative process such that,

$$\sigma_t^2 = \gamma V_L + \alpha_1 R_{t-1}^2 + \ldots + \alpha_q R_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_p \sigma_{t-p}^2, \qquad t \in N,$$

and

$$\gamma > 0,\ \ \alpha_i \geq 0\ \ i = 1,\ldots,q\ \ \ \beta_i \geq 0\ \ i = 1,\ldots,p,$$

where integers p and q are orders of σ_t^2 and R_t^2, respectively. In particular, GARCH(1,1) is the simplest and frequently useful model [2] which is given by:

$$\sigma_t^2 = \gamma V_L + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2,$$

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where γ, α and β are the weight assigned to long-run average variance rate V_L, returns squared R_{t-1}^2, and variance σ_{t-1}^2, respectively. The weights γ, α and β must sum to unity, that is

$$\gamma+\alpha+\beta=1.$$

Now, we set $\omega=\gamma V_L$, the GARCH(1,1) model can also be written

$$\sigma_t^2=\omega+\alpha u_{t-1}^2+\beta\sigma_{t-1}^2, \tag{1}$$

where $\omega>0$, $\alpha\geq 0$ and $\beta\geq 0$. In order to guarantee the variance to be positive, we set $\alpha+\beta<1$. The formula (1) is often used for the purpose of estimating the volatility. After that, we estimate the parameters α and β by maximum likelihood method.

2.4. Using Maximum Likelihood Method to estimate parameters of GARCH(1,1)

The method gives values of the parameters that maximize the likelihood function of the variable of interest [7]. Now, we have the transformed returns R_t which is approximately normal with mean zero and variance σ_t^2 as required in definition 1. Initially, we determine the probability density function of R_t, $t=1,2,3,\ldots,n$. Since for each t we have

$$f\left(r_t\right)=\frac{1}{\sqrt{2\pi\sigma_t^2}}\exp\left(\frac{-r_t^2}{2\sigma_t^2}\right),$$

then the likelihood function $L\left(r_t\right)=f\left(r_1,\ldots,r_n\right)$. For each t, R_t is independence so that

$$\begin{aligned}L\left(r_t\right)&=\prod_{i=1}^{n}f\left(r_t\right)\\&=\prod_{t=1}^{n}\left[\frac{1}{\sqrt{2\pi\sigma_t^2}}\exp\left(\frac{-r_t^2}{2\sigma_t^2}\right)\right],\end{aligned} \tag{2}$$

by monotonicity of logarithm function, maximizing likelihood function can be done by maximizing its logarithm [10]. Therefore, we now can maximize (2) by taking natural logarithm. Then we have,

$$\begin{aligned}l\left(r_r\right)=\ln L\left(r_r\right)&=\ln\left(\prod_{t=1}^{n}\frac{1}{\sqrt{2\pi\sigma_t^2}}\exp\left(\frac{-r_t^2}{2\sigma_t^2}\right)\right)\\&=\sum_{t=1}^{n}\left(\frac{1}{2}\left(-\ln\left(2\pi\right)-\ln\left(\sigma_t^2\right)\right)+\frac{1}{2}\left(\frac{-r_t^2}{\sigma_t^2}\right)\right).\end{aligned}$$

Ignoring constant multiplicative factors of $l\left(r_t\right)$ gives

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$$\hat{l}\left(r_t\right) = \sum_{t=1}^{n}\left[-\ln\left(\sigma_t^2\right) - \frac{r_t^2}{\sigma_t^2}\right], \tag{3}$$

where r_t and σ_t^2 are the returns and the variance at day t, respectively. The parameters that maximize $l\left(r_t\right)$, also maximize $\hat{l}\left(r_t\right)$. Furthermore, we solve formula (3) numerically by damped Newton's method. In summary, fitting the GARCH(1,1) gives volatility series of the seven stock returns plotted in figure. 5 which describe the return fluctuation over the period.

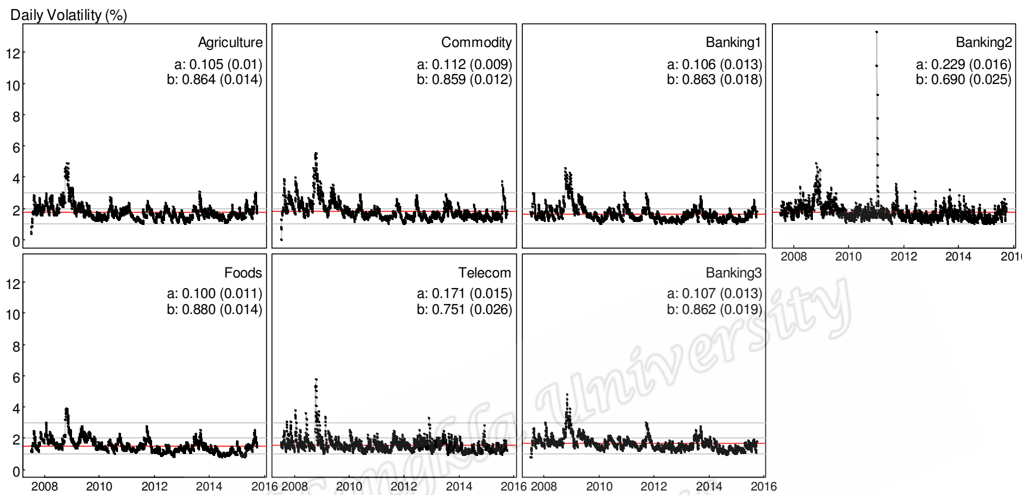

Figure 5. Volatility series of seven stock returns

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It is clear from figure. 5 that all stock returns have higher volatility during the end of 2008, but subsequently remained relatively stable. The seven companies are big companies that can possibly reflect the economy of Indonesia. The increasing volatility at the end of 2008 corresponds to the economic crisis in Indonesia at that time. As in figure 5, the volatility series is very fluctuating, we need to smooth the volatility series in order to simplify investigation of their change in many situations. The volatility series will be smoothed using natural cubic spline.

2.5. Fitting volatility series using cubic spline function

According to the preceding section (2.4), the GARCH(1,1) gives daily volatility series over the period. In order to study the behavior of volatility, we employ the natural cubic spline to fit volatility series obtaining from GARCH(1,1). It is because the natural cubic spline has such attractive properties as smoothness, continuity of the first and second derivative so that many financial institutions use the method for curve fitting [1]. Therefore, we can get the information on rate of change and cumulative change of volatility series over the period.

Let $\left(t_1, y_1\right), \left(t_2, y_2\right), ..., \left(t_n, y_n\right)$ where $t_1 < t_2 < ... < t_n$ and $s\left(t\right)$ be a series of knot points and cubic spline function which fits consecutive knot points, respectively. We employed a natural cubic spline which easily to apply in the data. It was improved by McNeil *et al.* [9]. The cubic spline function is defined as:

$$s(t) = a + bt + \sum_{k=1}^{p} c_k \left(t - t_k\right)_+^3, \tag{4}$$

where t denotes time, $t_1 < t_2 < ... < t_p$ are specified knots and $\left(t - t_k\right)_+$ is $t - t_k$ for $t > t_k$ and 0 otherwise. Since the formula (4) is linear function of the coefficients a, b and c_k, it is fitted to the data using linear regression. However, linearity in the future means that the quadratic and cubic coefficients are 0 for $t > t_p$ by setting $s''(\mathrm{t}) = 0$. Therefore the formula (4) can also be written as

$$s(t) = a + bt + \sum_{k=1}^{p-2} c_k \left[\left(t - t_k\right)_+^3 - \left(\frac{t_p - t_k}{t_p - t_{p-1}}\right)\left(t - t_{p-1}\right)_+^3 + \left(\frac{t_{p-1} - t_k}{t_p - t_{p-1}}\right)\left(t - t_p\right)_+^3 \right].$$

In summary, we fitted the volatility series of seven stock returns using eight-knot natural cubic spline, the results are graphed in figure. 6. It shows the volatility fitted by natural cubic spline which reflect the volatility signals. The lower right panel shows the volatility signals of stock returns for each of the seven stocks on the same axes. It can be seen that the seven volatility signals have the same trends, particularly during end of 2008. In addition, foods and telecom might simply reflect flat volatility over the period. These volatility signals can be used in assessing the model.

The volatility model is expected to capture the volatility as accurate as possible, so we need to assess the GARCH(1,1) using Monte Carlo simulation.

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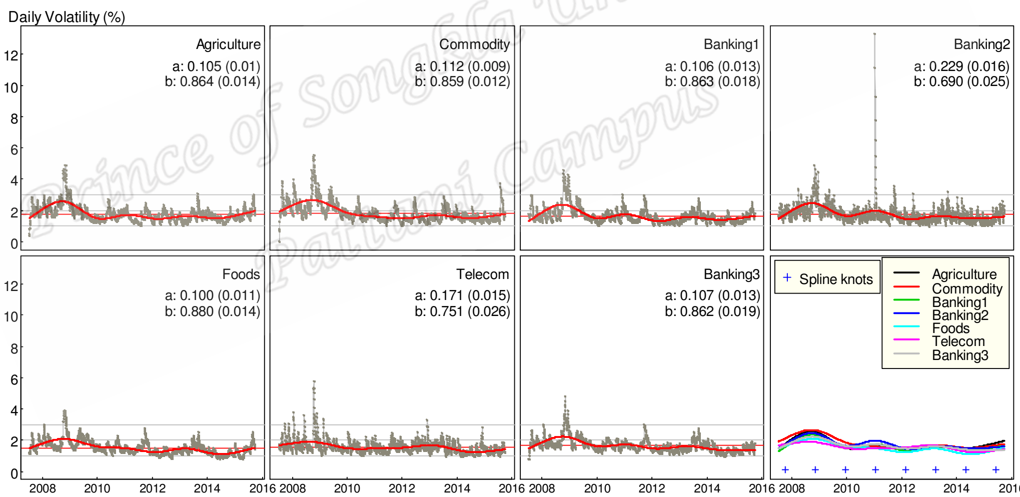

Figure 6. Fitted volatility series

2.6.Assessing model using Monte Carlo simulation

The usual way, fitting a model involve the concept of taking a sample from a population where the sample distribution is known. In this case, the volatility of stock returns are unknown and different samples of data from the population provide different estimates of their values. In assessing the model, we reverse the process of fitting by assuming that the population parameters are known and use the Monte Carlo to generate repeated sample from distribution with known parameters. Thus, the objective in simulation is not to determine the volatility series, but rather to assess the model that estimating them.

The Monte Carlo simulation generates repeated samples from a distribution and these samples should be random but repeatable. Therefore, we should be able to generate exactly the same set of random numbers if we want to. A device for exactly reproducing a sample is to use a specific seed for starting the random numbers in a simulation. By changing the seed, different sets of random numbers can be generated and they can be reproduced exactly by using the same seed that was used to create them in the first place. As an example, the following figure 7 shows the assumed volatility and fitted volatility of commodity stock.

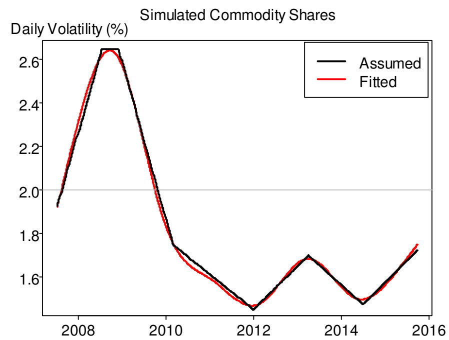

Figure 7. Assuming volatility path

To see how well the GARCH(1,1) model can estimate volatility in a series of stock returns, we assume a specific simple shape which is a piecewise linear spline for the volatility that approximates what we found for commodity stock returns.

Fitting the GARCH(1,1) model gives the volatility series plotted on figure.8 for the seven simulations. The estimated values of alpha range from 0.016 up to 0.045, and corresponding values of beta range from 0.930 up to 0.983. Figure. 8 shows 8 knots natural cubic spline functions fitted to the estimated daily volatility for the seven simulations. The lower right panel shows the volatility signals together with the known volatility realization which has been assumed before. Clearly, the GARCH(1,1) has captured the population volatility quite well which was shown by volatility signals of seven simulations close to assumed volatility.

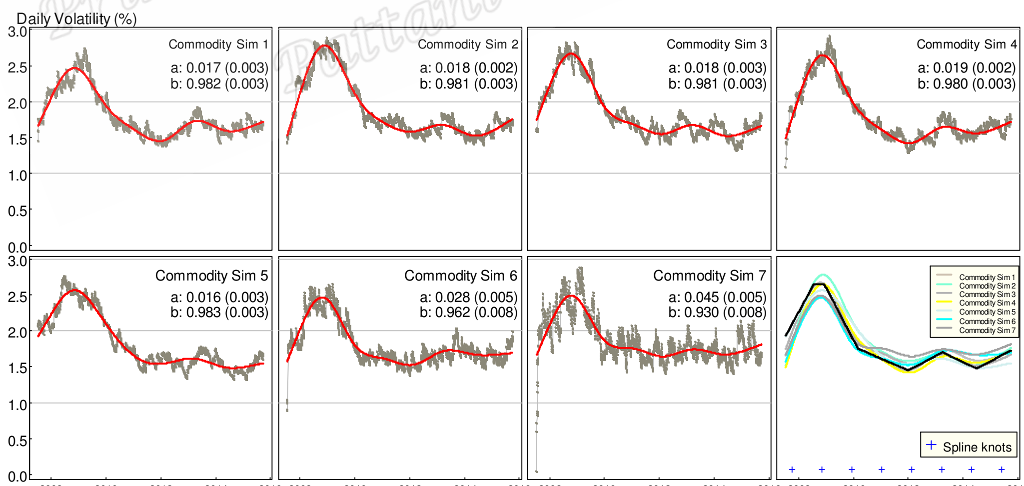

Figure 8. Estimated volatility of seven simulations

3. RESULT AND DISCUSSION

We generated seven realizations and estimated the daily volatility series for each using a GARCH(1,1). In each case the GARCH(1,1) was able to recapture the shape of the volatility series in population. We saw the Monte Carlo simulation assumed the simple path of the known volatility obtaining from GARCH(1,1). Further, we can compute the returns based on the assumed volatility. Therefore, the Monte Carlo simulation can be used to assess the

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accuracy of a specified model for determining unknown population parameters based on a sample.

Further work is needed to gain a better model to capture the volatility which gives the volatility signals of simulation very close to the assumed volatility. In addition, the model have to be more accurate and simple in process.

4. CONCLUSIONS

In this study, we have considered the importance of having a good volatility model. The Monte Carlo simulation was used to assess the GARCH(1,1) model. Our finding showed that the GARCH(1,1) is able to capture the volatility quite well.

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Program Code of the Study

```
setwd("E:/subhan")
options(scipen=8)                                # display numbers with 8 decimal places
shareTypes <-
c("Agriculture","Commodity","Banking1","Banking2","Foods","Telecom","Banking3")
read.table("agro.txt",h=T,as.is=T) -> k1         #agriculture
read.table("antam.txt",h=T,as.is=T) -> k2        #commodity
read.table("bni.txt",h=T,as.is=T) -> k3          #bangking type 1
read.table("bri.txt",h=T,as.is=T) -> k4          #bangking type 2
read.table("indofood.txt",h=T,as.is=T) -> k5     #foods
read.table("indosat.txt",h=T,as.is=T) -> k6      #telecommunication
read.table("mandiri.txt",h=T,as.is=T) -> k7      #bangking type 3
k1$date <- as.Date(k1$date)
k2$date <- as.Date(k2$date)
k3$date <- as.Date(k3$date)
k4$date <- as.Date(k4$date)
k5$date <- as.Date(k5$date)
k6$date <- as.Date(k6$date)
k7$date <- as.Date(k7$date)
date1 <- c(k1$date[1],k2$date[1],k3$date[1],k4$date[1],k5$date[1],k6$date[1],k7$date[1])
cbind(shareTypes,as.character(date1))
```

- Start all at July 12 2007

```
k1 <- subset(k1,date>"2007-07-11")[,c(1,2)]
k2 <- subset(k2,date>"2007-07-11")[,c(1,2)]
k3 <- subset(k3,date>"2007-07-11")[,c(1,2)]
k4 <- subset(k4,date>"2007-07-11")[,c(1,2)]
k5 <- subset(k5,date>"2007-07-11")[,c(1,2)]
k6 <- subset(k6,date>"2007-07-11")[,c(1,2)]
k7 <- subset(k7,date>"2007-07-11")[,c(1,2)]
```

- Check that dates are consistent

```
merge(k1,k2,by.x="date",by.y="date") -> k12
names(k12)[2:3] <- shareTypes[1:2]
str(k12)
merge(k12,k3,by.x="date",by.y="date") -> k1..3
names(k1..3)[4] <- shareTypes[3]
str(k1..3)
```

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```
merge(k1..3,k4,by.x="date",by.y="date") -> k1..4
names(k1..4)[5] <- shareTypes[4]
str(k1..4)
merge(k1..4,k5,by.x="date",by.y="date") -> k1..5
names(k1..5)[6] <- shareTypes[5]
str(k1..5)
merge(k1..5,k6,by.x="date",by.y="date") -> k1..6
names(k1..6)[7] <- shareTypes[6]
str(k1..6)
merge(k1..6,k7,by.x="date",by.y="date") -> kT
names(kT)[8] <- shareTypes[7]
str(kT)
```

- Check that dates are dates

```
kT$date <- as.Date(kT$date)
summary(kT)
rm(k1,k2,k3,k4,k5,k6,k7,k12,k1..3,k1..4,k1..5,k1..6)        # tidy up
```

- Figure 1.3: Stock price of seven companies over the period

```
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,0,2.5,1),mar=c(0.4,2.8,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
for (j in c(1:4)) {
 plot(kT$date,kT[,(j+1)],pch=20,cex=0.6,xaxt="n",xlab="",ylab="")
 mtext(side=3,adj=-0.13,line=0.2,"Rupiah",cex=0.8)
 legend("topleft",inset=c(0.1,0),leg=shareTypes[j],bty="n",cex=1.2,x.intersp=0)
}
legend("bottomright",inset=c(0.01,0.003),leg=shareTypes,y.intersp=0.6,
         cex=1.2,pch=21,pt.bg=c(1:4,6,8,"orange"),bg="ivory",ncol=2)
for (j in c(5:7)) {
plot(kT$date,kT[,(j+1)],pch=20,cex=0.6,xlab="",ylab="")
legend("topleft",inset=c(0.1,0),leg=shareTypes[j],bty="n",cex=1.2,x.intersp=0)
}
ymax <- max(kT[,c(2:8)])
ymin <- min(kT[,c(2:8)])
plot(kT$date,kT[,2],ylim=c(ymin,ymax),pch=20,cex=0.6,xlab="",ylab="")
points(kT$date,kT[,3],pch=20,cex=0.6,col=2)
points(kT$date,kT[,4],pch=20,cex=0.6,col=3)
points(kT$date,kT[,5],pch=20,cex=0.6,col=4)
points(kT$date,kT[,6],pch=20,cex=0.6,col=6)
```

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```
points(kT$date,kT[,7],pch=20,cex=0.6,col=8)
points(kT$date,kT[,8],pch=20,cex=0.6,col="orange")
```

- Log price

```
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,0,2.5,1),mar=c(0.4,2.8,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
for (j in c(1:4)) {
 plot(kT$date,log(kT[,(j+1)]),pch=20,cex=0.6,xaxt="n",yaxt="n",xlab="",ylab="")
 mtext(side=3,adj=-0.13,line=0.2,"Rupiah",cex=0.8)
 legend("topleft",inset=c(0.12,0),leg=shareTypes[j],bty="n",cex=1.2,x.intersp=0)
 laby <- c(12,24,50,100,240,500,1200,2400,5000,12000,24000,50000,120000,240000)
 aty <- log(laby)
 axis(side=2,at=aty,lab=laby)
}
legend("bottomright",inset=c(0.01,0),leg=shareTypes,y.intersp=0.8,
          cex=1.2,pch=21,pt.bg=c(1:4,6,8,"orange"),bg="ivory")
for (j in c(5:7)) {
 plot(kT$date,log(kT[,(j+1)]),pch=20,cex=0.6,xlab="",yaxt="n",ylab="")
 legend("topleft",inset=c(0.12,0),leg=shareTypes[j],bty="n",cex=1.2,x.intersp=0)
 axis(side=2,at=aty,lab=laby)
}
ymax <- max(log(kT[,c(2:8)]))
ymin <- min(log(kT[,c(2:8)]))
plot(kT$date,log(kT[,2]),ylim=c(ymin,ymax),pch=20,cex=0.6,yaxt="n",xlab="",ylab="")
points(kT$date,log(kT[,3]),pch=20,cex=0.6,col=2)
points(kT$date,log(kT[,4]),pch=20,cex=0.6,col=3)
points(kT$date,log(kT[,5]),pch=20,cex=0.6,col=4)
points(kT$date,log(kT[,6]),pch=20,cex=0.6,col=6)
points(kT$date,log(kT[,7]),pch=20,cex=0.6,col=8)
points(kT$date,log(kT[,8]),pch=20,cex=0.6,col="orange")
axis(side=2,at=aty,lab=laby)

n <- nrow(kT)                  # number of trading days
kT$tDay <- c(0:(n-1))          # trading days after Day 1 (2007-07-12)
kT$agro.r <- NA
kT$comm.r <- NA
kT$bank1.r <- NA               # initialize returns from one trading day to next
kT$bank2.r <- NA
kT$food.r <- NA
```

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```
kT$telec.r <- NA
kT$bank3.r <- NA
```

- Compute the returns of stock price

```
kT[2:n,10:16] <- log(kT[2:n,2:8])-log(kT[c(1:(n-1)),2:8])
ymin <- min(kT[-1,10:16])
ymax <- max(kT[-1,10:16])+0.1
```

- Plot the returns and fit linear model for all share groups

```
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,0.5,2.5,1),mar=c(0.4,2,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
for (j in c(1:4)) {
   if(j<4) plot(kT$date,kT[,9+j],type="l",col=8,ylim=c(ymin,ymax),
         xlab="",ylab="",xaxt="n",yaxt="n")
   if(j==4) plot(kT$date,kT[,9+j],type="l",col=8,ylim=c(ymin,ymax),
       xlab="",ylab="",yaxt="n",cex.axis=1.2)  #no xaxt to show the period in x axis
 legend("bottomright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1.4,x.intersp=0)
 z <- kT[,9+j]                                    #returns
 siz <- ifelse(abs(z)>0.2,1.2,0.6)                #indicated outliers
 points(kT$date,kT[,9+j],pch=20,cex=siz)
 mtext(side=3,adj=-0.1,line=0.2,"Log return")   # legend on top left
 lm(kT[,9+j]~1) -> mod1                          #linear model (least square) in means returns
 summary(mod1) -> rez1
 abline(h=mean(kT[-1,9+j]),col=6)
 lm(data=kT,kT[,9+j]~tDay) -> mod2               #linear model for increasing tDay
 kT[,j+16] <- exp(log(kT[1,j+1])+c(0,cumsum(mod2$fit)))   # what is that?
 summary(mod2) -> rez2
 axis(side=2,cex.axis=1.2)
 lg1 <- paste("Mean Return: ",round(mod1$coef[1],5)," p: ",round(rez1$coef[1,4],3),sep="")
 lg2 <-  paste("Inc/Tr.Day: ",round(mod2$coef[2],7)," p: ",round(rez2$coef[2,4],3),sep="")
 lg <- c(lg1,lg2)
 legend("bottom",inset=c(0.01,0.17),leg=lg,lwd=2,col=c(6,"ivory"),
         x.intersp=0.2,y.intersp=0.8,bg="ivory",cex=1.3)
}
for (j in c(5:7)) {
 plot(kT$date,kT[,9+j],type="l",col=8,ylim=c(ymin,ymax),
         xlab="",ylab="",cex.axis=1.2)
 legend("bottomright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1.5,x.intersp=0)
 z <- kT[,9+j]
```

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```
 siz <- ifelse(abs(z)>0.2,1.2,0.6)
 points(kT$date,kT[,9+j],pch=20,cex=siz)
 lm(kT[,9+j]~1) -> mod1
 summary(mod1) -> rez1
 abline(h=mean(kT[-1,9+j]),col=6)
 lm(data=kT,kT[,9+j]~tDay) -> mod2
 kT[,j+16] <- exp(log(kT[1,j+1])+c(0,cumsum(mod2$fit)))
 summary(mod2) -> rez2
 axis(side=2,cex.axis=1.2)
 lg1 <- paste("Mean Return: ",round(mod1$coef[1],5)," p: ",round(rez1$coef[1,4],3),sep="")
 lg2 <-  paste("Inc/Tr.Day: ",round(mod2$coef[2],7)," p: ",round(rez2$coef[2,4],3),sep="")
 lg <- c(lg1,lg2)
 legend("bottom",inset=c(0.01,0.17),leg=lg,lwd=2,col=c(6,"ivory"),
           x.intersp=0.2,y.intersp=0.8,bg="ivory",cex=1.3)}
```

➢ Assess normality assumption for returns (figure 3.3: Q-Q plots of log returns)

```
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,0,2,1),mar=c(0.4,2.7,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
fff <- 0.01*c(1.6,1.6,1.6,1.6,1.4,1.5,1.6)
for (j in c(1:7)) {
 z <- kT[-1,j+9]
 ptp <- ifelse(abs(z)>0.2,20,1)
 siz <- ifelse(abs(z)>0.2,2,0.6)
 qqnorm(z,main="",ylab="",xlab="",xaxt="n",cex.axis=1.2,pch=ptp,cex=siz)
 legend("bottomright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1.4,x.intersp=0)
 if (j>3) {
  axis(side=1,cex.axis=1.2)
  axis(side=1,at=0,lab="Theoretical Quantiles",padj=1.4,cex.axis=1.4,tcl=0)
 }
 qqline(z,col=2,lwd=2)
 abline(v=c(-1,1),col=8)             #v: vertical & h: horizontal
 abline(h=fff[j]*c(-1.2,1.2),col=8)
 if (j<5) mtext(side=3,"Log return",adj=-0.18,line=0.5,cex.axis=1.4,tcl=0)
}
```

➢ Q-Q plot of agriculture returns (figure 2.1a)

```
windows(5,5)
fff <- 0.01*c(1.6,1.6,1.6,1.6,1.4,1.5,1.6)
for (j in c(1:1)) {
```

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```
 z <- kT[-1,j+9]
 ptp <- ifelse(abs(z)>0.2,20,1)
 siz <- ifelse(abs(z)>0.2,2,0.6)
 qqnorm(z,main="",ylab="",xlab="",xaxt="n",cex.axis=1.2,pch=ptp,cex=siz)
 legend("bottomright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1,x.intersp=0)
axis(side=1,at=0,lab="Theoretical Quantiles",padj=1.4,cex.axis=1,tcl=0)
axis(side=1,cex.axis=1.2)
qqline(z,col=2,lwd=2)}
 if (j>3) {
  axis(side=1,cex.axis=1.2)
  axis(side=1,at=0,lab="Theoretical Quantiles",padj=1.4,cex.axis=1.4,tcl=0)
 }
 qqline(z,col=2,lwd=2)
 abline(v=c(-1,1),col=8)             #v: vertical & h: horizontal
 abline(h=fff[j]*c(-1.2,1.2),col=8)
 if (j<5) mtext(side=3,"Log return",adj=-0.18,line=0.2,cex.axis=1.4,tcl=0)
```

- Figure 3.4: Q-Q plots of transformed log returns

```
stDevs <- NULL
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,0,2,1),mar=c(0.4,2.7,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
cut <- fff
mm <- 0.4                          #slope
for (j in c(1:7)) {
 z <- kT[-1,j+9]
 f <- cut[j]
 kT[,j+23] <- NA
 zt <- ifelse(z< -f,-f+mm*(z+f),ifelse(z>f,f+mm*(z-f),z))
 kT[-1,j+23] <- zt
 names(kT)[j+23] <- paste(shareTypes[j],".tr",sep="")
 ptp <- ifelse(abs(z)>0.2,20,1)
 siz <- ifelse(abs(z)>0.2,2,0.6)
 qqnorm(zt,main="",ylab="",xlab="",xaxt="n",cex.axis=1.2,pch=ptp,cex=siz)
 abline(v=c(-1,1),col=8)                  #vertical line
 abline(h=fff[j]*c(-1,1),col=8)           #horizontal line
 legend("bottomright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1.4,x.intersp=0)
 c <- paste("c= ",round(cut[j],4),sep="")     #show constant c
 legend("topleft",inset=c(-0.01,0.4),leg=c,bty="n",x.intersp=0.,cex=1.4)
 if (j>3) {
```

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```
  axis(side=1,cex.axis=1.2)
  axis(side=1,at=0,lab="Theoretical Quantiles",padj=1.4,cex.axis=1.4,tcl=0)
 }
 qqline(zt,col=2,lwd=2)
 if (j<5) mtext(side=3,"Transformed Log return",adj=-0.3,line=0.2,cex.axis=1.4,tcl=0)
 sigma <- paste("St.Dev: ",round(sd(zt),4),sep="")
 legend("bottomright",inset=c(-0.01,0.1),leg=sigma,bty="n",x.intersp=0.,cex=1.4)
 stDevs <- c(stDevs,sd(zt))
}
```

- Transformed agriculture returns (figure 2.1b)

```
stDevs <- NULL
windows(5,5)
cut <- fff
mm <- 0.4
for (j in c(1:1)) {
 z <- kT[-1,j+9]
 f <- cut[j]
 kT[,j+23] <- NA
 zt <- ifelse(z< -f,-f+mm*(z+f),ifelse(z>f,f+mm*(z-f),z))
 kT[-1,j+23] <- zt
 names(kT)[j+23] <- paste(shareTypes[j],".tr",sep="")
 ptp <- ifelse(abs(z)>0.2,20,1)
 siz <- ifelse(abs(z)>0.2,2,0.6)
 qqnorm(zt,main="",ylab="",xlab="",xaxt="n",cex.axis=1.2,pch=ptp,cex=siz)
 abline(v=c(-1,1),col=8)
 abline(h=fff[j]*c(-1,1),col=8)
 legend("bottomright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1,x.intersp=0)
 axis(side=1,cex.axis=1.2)
  axis(side=1,at=0,lab="Theoretical Quantiles",padj=1.4,cex.axis=1,tcl=0)
 qqline(zt,col=2,lwd=2)
 if (j<5) mtext(side=3,"Transformed Log return",adj=-0.2,line=0.2,cex.axis=1.4,tcl=0)}
```

- Plot tail-shrunk returns and fit linear model for all share groups

```
ymin <- -0.3
ymax <- 0.2
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,0,2.5,1),mar=c(0.4,2,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
aj <- 2.5
```

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```
for (j in c(1:4)) {
 rj <- kT[,9+j]
 cj <- fff[j]
 trj <- ifelse(rj>cj,cj+(rj-cj)/aj,ifelse(rj<(-cj),-cj+(rj+cj)/aj,rj))
 plot(kT$date,trj,type="l",col=8,ylim=c(ymin,ymax),
          xlab="",ylab="",xaxt="n",yaxt="n")
 legend("bottomright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1.4,x.intersp=0)
 z <- trj
 siz <- ifelse(abs(z)>0.1,1.2,0.6)              #size of outlier point
 points(kT$date,trj,pch=20,cex=siz)
 lm(trj~1) -> mod1
 summary(mod1) -> rez1
 abline(h=mean(trj[-1]),col=6)
 mtext(side=3,adj=-0.15,line=0.2,"Transformed Log return")
 lm(data=kT,trj~tDay) -> mod2
 fv <- mod2$fit
 fv.str <- ifelse(fv>cj,cj+(fv-cj)*aj,ifelse(fv<(-cj),-cj+(fv+cj)*aj,fv))
 kT[,j+30] <- exp(log(kT[1,j+1])+c(0,cumsum(fv.str)))
 summary(mod2) -> rez2
 lg1 <- paste("Mean: ",round(mod1$coef[1],5)," p-value: ",round(rez1$coef[1,4],3),sep="")
 lg2 <- paste("Inc/Tr.Day: ",round(mod2$coef[2],7)," p: ",round(rez2$coef[2,4],3),sep="")
 lg <- c(lg1,lg2)
 legend("topright",inset=c(0.01,0.01),leg=lg,lwd=2,col=c(6,"ivory"),
          x.intersp=0.2,y.intersp=0.8,bg="ivory",cex=1.1)
 axis(side=2,cex.axis=1.2)
}
for (j in c(5:7)) {
 rj <- kT[,9+j]
 cj <- fff[j]
 trj <- ifelse(rj>cj,cj+(rj-cj)/aj,ifelse(rj<(-cj),-cj+(rj+cj)/aj,rj))
 plot(kT$date,trj,type="l",col=8,ylim=c(ymin,ymax),
          xlab="",ylab="",cex.axis=1.2)
 legend("bottomright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1.4,x.intersp=0)
 z <- trj
 siz <- ifelse(abs(z)>0.1,1.2,0.6)
 points(kT$date,trj,pch=20,cex=siz,)
 lm(trj~1) -> mod1
 summary(mod1) -> rez1
 abline(h=mean(trj[-1]),col=6)
```

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```
 lm(data=kT,trj~tDay) -> mod2
 fv <- mod2$fit
 fv.str <- ifelse(fv>cj,cj+(fv-cj)*aj,ifelse(fv<(-cj),-cj+(fv+cj)*aj,fv))
 kT[,j+30] <- exp(log(kT[1,j+1])+c(0,cumsum(fv.str)))
 summary(mod2) -> rez2
 lg1 <- paste("Mean: ",round(mod1$coef[1],5)," p-value: ",round(rez1$coef[1,4],3),sep="")
 lg2 <-  paste("Inc/Tr.Day: ",round(mod2$coef[2],7)," p: ",round(rez2$coef[2,4],3),sep="")
 lg <- c(lg1,lg2)
 legend("topright",inset=c(0.01,0.01),leg=lg,lwd=2,col=c(6,"ivory"),
          x.intersp=0.2,y.intersp=0.8,bg="ivory",cex=1.1)
 axis(side=2,cex.axis=1.2)
}
```

- Put models on plots using deflation factors to match means

```
ymin <- min(kT[-1,10:16])
ymax <- max(kT[-1,10:16])
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,0,2.5,1),mar=c(0.4,2.8,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
DF1 <- c(1:7)*0
lWid1 <- c(1,1,1,1,1,1,1)
lWid <- c(1,1,1,1,1,1,1)
for (j in c(1:4)) {
 DF1[j] <- mean(kT[,(j+1)])/mean(kT[,(j+30)])
 DF[j] <- mean(kT[,(j+1)])/mean(kT[,(j+16)])
 plot(kT$date,log(kT[,(j+1)]),pch=20,cex=0.6,xaxt="n",yaxt="n",xlab="",ylab="")
 points(kT$date,log(DF1[j]*kT[,j+30]),type="l",col=2,lwd=lWid1[j])
 points(kT$date,log(DF[j]*kT[,j+16]),type="l",col=4,lwd=lWid[j])
 mtext(side=3,adj=-0.13,line=0.2,"Rupiah",cex=0.8)
 legend("topleft",inset=c(0.1,0),leg=shareTypes[j],bty="n",cex=1.2,x.intersp=0)
 laby <- c(12,24,50,100,240,500,1200,2400,5000,12000,24000,50000,120000,240000)
 aty <- log(laby)
 axis(side=2,at=aty,lab=laby)
}
legend("bottomright",bty="n",leg=c("Raw Returns","Transformed"),lwd=2,col=c(4,2),cex=1.2)
for (j in c(5:7)) {
 DF1[j] <- mean(kT[,(j+1)])/mean(kT[,(j+30)])
 DF[j] <- mean(kT[,(j+1)])/mean(kT[,(j+16)])
 plot(kT$date,log(kT[,(j+1)]),pch=20,cex=0.6,yaxt="n",xlab="",ylab="")
 points(kT$date,log(DF1[j]*kT[,j+30]),type="l",col=2,lwd=lWid1[j])
```

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```
 points(kT$date,log(DF[j]*kT[,j+16]),type="l",col=4,lwd=lWid[j])
 points(kT$date,log(DF1[j]*kT[,j+30]),type="l",col=2,lwd=2)
 legend("topleft",inset=c(0.1,0),leg=shareTypes[j],bty="n",cex=1.2,x.intersp=0)
 axis(side=2,at=aty,lab=laby)
}
ymax <- max(log(kT[,c(2:8)]))+0.7
ymin <- min(log(kT[,c(2:8)]))

plot(kT$date,log(DF1[1]*kT[,31]),ylim=c(ymin,ymax),pch=20,cex=0.6,yaxt="n",xlab="",ylab
="")
legend("topleft",inset=c(0,0),leg=shareTypes,y.intersp=0.8,x.intersp=0.4,
          cex=1.2,pch=21,pt.bg=c(1:4,6,8,"orange"),bty="n",ncol=3)
points(kT$date,log(DF1[2]*kT[,32]),pch=20,cex=0.6,col=2)
points(kT$date,log(DF1[3]*kT[,33]),pch=20,cex=0.6,col=3)
points(kT$date,log(DF1[4]*kT[,34]),pch=20,cex=0.6,col=4)
points(kT$date,log(DF1[5]*kT[,35]),pch=20,cex=0.6,col=6)
points(kT$date,log(DF1[6]*kT[,36]),pch=20,cex=0.6,col=8)
points(kT$date,log(DF1[7]*kT[,37]),pch=20,cex=0.6,col="orange")
axis(side=2,at=aty,lab=laby)
```

➢ Fit GARCH(1,1) to transformed returns

```
ff <- function(a,b,u) {                        #log likelihood function
 eps <- 0.000001                               # to avoid zero returns
 VL <- var(u)
 w <- VL*(1-a-b)
 n <- length(u)
 v <- 0*u                                      # initialize v
 v[1] <- 0
 v[2] <- max(u[1]^2,eps)
 lik <- -log(v[2])-u[2]^2/v[2]                 #log likelihood function
 for (i in 3:n) {
  v[i] <- max(w+a*u[i-1]^2+b*v[i-1],eps)#GARCH(1,1)
  lik <- lik-log(v[i])-u[i]^2/v[i]
 }
 lik
}
alpha <- NULL
beta <- NULL
seA <- NULL
```

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```
seB <- NULL
```

➢ Setting the initial values of alpha and beta

```
for (jj in c(1:7)) {
 z <- kT[-1,jj+23]                 # transformed returns
 lik <- matrix(NA,20,20)           # Find (a,b) cell where likelihood has maximum value
 a.Lmax <- 0
 b.Lmax <- 0
 Lmax <- -9999999
 for (j in 1:20) {
  b <- 0.05*j-0.026
  for (i in 1:(21-j)) {
   a <- 0.05*i-0.026
   lik[i,j] <- ff(a,b,z)
   if (lik[i,j]>Lmax) {
    a.Lmax <- a
    b.Lmax <- b
    Lmax <- lik[i,j]
   }
  }
 }
 VL <- var(z)            # long-term variance
 a <- a.Lmax             # initial parameter estimates
 b <- b.Lmax
```

➢ *damped* Newton's method

```
 H0 <- matrix(0,2,2)     # Hessian matrix which corresponds second derivative of log
likelihood function
 w0 <- matrix(0,2,1)     # column vector of first derivatives of log likelihood function
 ab0 <- w0
 ab0[1,1] <- a                     # initial values of alpha & beta
 ab0[2,1] <- b
 d <- 0.0001                              # dx & dy in numerical derivatives
 epsilon <- 0.000001                      # change in log-lik for convergence
 diff <- 1                                # initial change
 dd <- 0.05                               # Marquardt damping factor (0,1)
 nit <- 200                               # maximum number of iterations
 rez0 <- NULL                      # array containing results at each iteration
 it <- 0
```

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```
 while ( (abs(diff)>epsilon) && ((it <- it+1) < nit) ) {
  a <- ab0[1,1]
  b <- ab0[2,1]
  F <- ff(a,b,z)
  w0[1,1] <- (ff(a+d,b,z)-ff(a-d,b,z))/(2*d)                    # numerical derivatives( central difference 1st derivative w.r.t a)
  w0[2,1] <- (ff(a,b+d,z)-ff(a,b-d,z))/(2*d)          # w.r.t b
  H0[1,1] <- (ff(a+d,b,z)-2*ff(a,b,z)+ff(a-d,b,z))/d^2       #central difference 2nd derivative w.r.t a)
  H0[2,2] <- (ff(a,b+d,z)-2*ff(a,b,z)+ff(a,b-d,z))/d^2  # w.r.t. b
  H0[2,1] <- (ff(a+d,b+d,z)-ff(a+d,b-d,z)-ff(a-d,b+d,z)+ff(a-d,b-d,z))/(4*d^2) #w.r.t a & b
  H0[1,2] <- H0[2,1]                  #w.r.t alpha & beta
  rez0 <- rbind(rez0,c(F,w0[1,1],w0[2,1],H0[1,1],H0[2,2],H0[1,2],a,b))
  ab1 <- ab0 - dd*solve(H0) %*% w0
  ab0 <- ab1                                        # update estimates
  diff <- ff(ab0[1,1],ab0[2,1],z)-F
 }
 SE <- sqrt(-diag(solve(H0)))              # standard errors of a and b
 CIfor.a <- a+SE[1]*1.96*c(-1,1)# Is it confident interval?
 CIfor.b <- b+SE[2]*1.96*c(-1,1)
 alpha <- c(alpha,a)
 beta <- c(beta,b)
 seA <- c(seA,SE[1])      #standard error alpha
 seB <- c(seB,SE[2])
}
```

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➢ Plot alpha & beta

```
windows(4,4)
par(oma=c(0,0,0,0),mar=c(2.5,2.5,2.5,1),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
plot(beta,alpha,ylim=c(0,0.3),xlim=c(0.6,1),pch=20,ylab="")
polygon(c(0,1,0,0),c(0,0,1,1))
mtext(side=3,adj=-0.14,line=0.1,"alpha")
abline(mod$coef,col=2)
summary(mod) -> rez
round(rez$r.sq,2) -> rsq
leg2 <- paste("r-squared = ",rsq,sep="")
legend("topright",leg=c("fitted model"),lwd=1,col=2,bty="n")
legend("topright",inset=c(0,0.1),leg=leg2,bty="n")
```

➢ Compute and plot volatility series

```
vol <- NULL
for (j in c(1:7)) {
 z <- kT[-1,j+23]                       # transformed returns
 a <- alpha[j]
 b <- beta[j]
 w <- var(z)*(1-a-b)
 vt <- NA+z                             # trading day variances
 vt[2] <- z[1]^2
 for (i in 3:n) {
  vt[i] <- w+a*z[i-1]^2+b*vt[i-1]
 }
 vol <- cbind(vol,100*sqrt(vt)[-1])     # trading day volatilities
}
ymin <- min(vol)
ymax <- max(vol)
kT$day <- as.integer(kT$date-kT$date[1])
yy <- as.data.frame(vol)
names(yy) <- shareTypes
x <- as.integer(kT$date[-1])
```

➢ Figure 3.5: Volatility series of seven stock returns

```
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,2.5,2.5,1),mar=c(0.5,0.5,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
for (j in c(1:7)) {
 if (j<4) plot(kT$day[-1],vol[,j],type="l",col=8,ylim=c(ymin,ymax),
          xlab="",ylab="",xaxt="n",yaxt="n")
 if (j>3) plot(kT$date[-1],vol[,j],type="l",col=8,ylim=c(ymin,ymax),
          xlab="",ylab="",yaxt="n", cex.axis=1.2)
 mean1 <- mean(vol[,j])
 abline(h=mean1,col=2)
 abline(h=c(1:3),col=8)
 aa <- round(alpha[j],3)
 #aa<-c(expression(A),aa)
 aa <- ifelse(nchar(aa)==3,paste(aa,"00",sep=""),ifelse(nchar(aa)==4,paste(aa,"0",sep=""),aa))
#nchar
 la <- paste("a: ",aa," (",round(seA[j],3),")",sep="")   #standard error alpha
 bb <- round(beta[j],3)
 #bb<-c(expression(B),bb) #to show beta
```

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```
 bb <-
ifelse(nchar(bb)==3,paste(bb,"00",sep=""),ifelse(nchar(bb)==4,paste(bb,"0",sep=""),bb))
 lb <- paste("b: ",bb," (",round(seB[j],3),")",sep="")          # standard error beta
 legend("topright",inset=c(0,0.12),leg="",title=la,bg="white",
          x.intersp=0,bty="n",cex=1.4)
 legend("topright",inset=c(0,0.19),leg="",title=lb,bg="white",
          x.intersp=0,bty="n",cex=1.4)
 legend("topright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1.4,x.intersp=0)
 if (j<4) points(kT$day[-1],vol[,j],pch=20,cex=0.6)
 if (j>3) points(x,vol[,j],pch=20,cex=0.6)
 if (j==1) mtext(side=3,adj=-0.1,line=0.2,"Daily Volatility (%)")
 if (j %in% c(1,5)) axis(side=2,cex.axis=1.4)
}
```

➢ Natural cubic spline

```
kT$day <- as.integer(kT$date-kT$date[1])
x<-c(1:(n-1))
kn <- as.integer (2000/36*c(1,6,11,16,21,26,31,36))
p <- length(kn)             # number of spline knots
yy <- as.data.frame(vol)
names(yy) <- shareTypes
yy$x <- x
d1 <- kn[p]-kn[p-1]
for (j in c(1:(p-2))) {
 sj <- ifelse(x>kn[j],(x-kn[j])^3,0)
 sj <- sj-((kn[p]-kn[j])/d1)*ifelse(x>kn[p-1],(x-kn[p-1])^3,0)
 sj <- sj+((kn[p-1]-kn[j])/d1)*ifelse(x>kn[p],(x-kn[p])^3,0)
 yy[,(j+8)] <- sj
 names(yy)[j+8] <- paste("s",j,sep="")
}
fits <- NULL                        #starting fits
resids <- NULL
```

➢ Figure 3.6: Fitted volatility series

```
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,2.5,2.5,1),mar=c(0.5,0.5,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
for (j in c(1:7)) {
 if (j<5) plot(kT$date[-1],vol[,j],type="l",col="grey40",ylim=c(ymin,ymax),
          xlab="",ylab="",xaxt="n",yaxt="n")
```

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```
 if (j>4) plot(kT$date[-1],vol[,j],type="l",col="grey40",ylim=c(ymin,ymax),
          xlab="",ylab="",yaxt="n",cex.axis=1.2)
 abline(h=c(1:3),col=8)
 mean1 <- mean(vol[,j])
 abline(h=mean1,col=2)
 aa <- round(alpha[j],3)
 aa <- ifelse(nchar(aa)==3,paste(aa,"00",sep=""),ifelse(nchar(aa)==4,paste(aa,"0",sep=""),aa))
 la <- paste("a: ",aa," (",round(seA[j],3),")",sep="")
 bb <- round(beta[j],3)
 bb <-
ifelse(nchar(bb)==3,paste(bb,"00",sep=""),ifelse(nchar(bb)==4,paste(bb,"0",sep=""),bb))
 lb <- paste("b: ",bb," (",round(seB[j],3),")",sep="")
 legend("topright",inset=c(0,0.12),leg="",title=la,bg="white",
          x.intersp=0,bty="n",cex=1.4)
 legend("topright",inset=c(0,0.19),leg="",title=lb,bg="white",
          x.intersp=0,bty="n",cex=1.4)
 legend("topright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1.4,x.intersp=0)
 if (j<5) points(kT$date[-1],vol[,j],pch=20,cex=0.7, col="cornsilk4")
 if (j>4) points(kT$date[-1],vol[,j],pch=20,cex=0.7, col="cornsilk4")
 if (j==1) mtext(side=3,adj=-0.1,line=0.2,"Daily Volatility (%)")
 if (j %in% c(1,5)) axis(side=2,cex.axis=1.4)
 mod1 <- lm(data=yy,yy[,j]~x+s1+s2+s3+s4+s5+s6)                # parameter estimator
 if (j<5) points(kT$date[-1],mod1$fit,type="l",col=2,lwd=2)     # plot spline function
 if (j>4) points(kT$date[-1],mod1$fit,type="l",col=2,lwd=2)
 fits <- cbind(fits,mod1$fit)
 resids <- cbind(resids,mod1$resid)
}
plot(kT$date[-
1],fits[,1],type="l",lwd=2,col=1,ylim=c(ymin,ymax),ylab="",yaxt="n",xlab="",cex.axis=1.2)
clr <- c(1:6,8)
for (j in c(1:7)) {
 points(kT$date[-1],fits[,j],type="l",lwd=2,col=clr[j])
}
text(kT$date[kn],ymin,adj=c(0.5,0),"+",col="blue",cex=1.4)
legend("topright",inset=c(0.02,0.01),leg=shareTypes,lwd=2,col=clr,cex=1.4,bg="ivory",y.inter
sp=0.8)
legend("topleft",inset=c(0.02,0.02),leg="Spline
knots",pch=3,pt.cex=1.2,col="blue",cex=1.4,bg="ivory")
```

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- Refit the model incorporating fitted volatility for banking shares

```
for (j in c(1:7)) {
 kT[,j+38] <- kT[,j+23]
 kT[-1,j+38] <- kT[-1,j+38]/(fits[,j]/mean(fits[,j]))
}
names(kT)[39:45] <- paste(shareTypes,".trVs",sep="")
ymin <- -0.3
ymax <- 0.2
```

- Plot tail-shrunk and volatility-scaled returns and fit linear model for all share groups

```
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,2,2.5,1),mar=c(0.4,3,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
for (j in c(1:7)) {
 if (j<4) plot(kT$date,kT[,j+38],type="l",col=8,ylim=c(ymin,ymax),
          xlab="",ylab="",xaxt="n",yaxt="n")
 if (j>3) plot(kT$date,kT[,j+38],type="l",col=8,ylim=c(ymin,ymax),
          xlab="",ylab="",cex.axis=1.2)
 legend("bottomright",inset=c(-0.01,0),leg=shareTypes[j],bty="n",cex=1.4,x.intersp=0)
 z <- kT[,j+38]
 siz <- ifelse(abs(z)>0.1,1.2,0.6)
 points(kT$date,z,pch=20,cex=siz)
 lm(z~1) -> mod1
 summary(mod1) -> rez1
 abline(h=mean(z[-1]),col=6)
 if (j==1) mtext(side=3,adj=1,line=0.2,"Transformed & Volatility-scaled Return  ")
 lm(data=kT,z~tDay) -> mod2
 fv <- mod2$fit
 fvs <- fv*(fits[,j]/mean(fits[,j]))
 fv.str <- ifelse(fvs>cj,cj+(fvs-cj)*aj,ifelse(fvs<(-cj),-cj+(fvs+cj)*aj,fvs))
 kT[,j+45] <- exp(log(kT[1,j+1])+c(0,cumsum(fv.str)))
 summary(mod2) -> rez2
 lg1 <- paste("Mean: ",round(mod1$coef[1],5)," p-value: ",round(rez1$coef[1,4],3),sep="")
 lg2 <-  paste("Inc/Tr.Day: ",round(mod2$coef[2],7)," p: ",round(rez2$coef[2,4],3),sep="")
 lg <- c(lg1,lg2)
 legend("topright",inset=c(0.01,0.01),leg=lg,lwd=2,col=c(6,"ivory"),
          x.intersp=0.2,y.intersp=0.8,bg="ivory",cex=1.2)
 axis(side=2,cex.axis=1.2)
}
```

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- Put new models on plots using deflation factors to match means

```
ymin <- min(kT[-1,10:16])
ymax <- max(kT[-1,10:16])
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,0,2.5,1),mar=c(0.4,2.8,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
DF1 <- c(1:7)*0
DF2 <- DF1
lWid1 <- c(1,1,1,1,1,1,1)
lWid2 <- c(1,1,1,1,1,1,1)
for (j in c(1:4)) {
 DF1[j] <- mean(kT[,(j+1)])/mean(kT[,(j+30)])
 DF2[j] <- mean(kT[,(j+1)])/mean(kT[,(j+45)])
 plot(kT$date,log(kT[,(j+1)]),pch=20,cex=0.6,xaxt="n",yaxt="n",xlab="",ylab="")
 points(kT$date,log(DF1[j]*kT[,j+30]),type="l",col=4,lwd=lWid1[j])
 points(kT$date,log(DF2[j]*kT[,j+45]),type="l",col=2,lwd=lWid2[j])
 mtext(side=3,adj=-0.13,line=0.2,"Rupiah",cex=0.8)
 legend("topleft",inset=c(0.1,0),leg=shareTypes[j],bty="n",cex=1.2,x.intersp=0)
 laby <- c(12,24,50,100,240,500,1200,2400,5000,12000,24000,50000,120000,240000)
 aty <- log(laby)
 axis(side=2,at=aty,lab=laby)
}
legend("bottomright",bty="n",leg=c("Transformed Returns","Transformed & Scaled"),
          x.intersp=0.2,lwd=2,col=c(4,2),cex=1.2)
for (j in c(5:7)) {
 DF1[j] <- mean(kT[,(j+1)])/mean(kT[,(j+30)])
 DF2[j] <- mean(kT[,(j+1)])/mean(kT[,(j+45)])
 plot(kT$date,log(kT[,(j+1)]),pch=20,cex=0.6,yaxt="n",xlab="",ylab="")
 points(kT$date,log(DF1[j]*kT[,j+30]),type="l",col=4,lwd=lWid1[j])
 points(kT$date,log(DF2[j]*kT[,j+45]),type="l",col=2,lwd=lWid[j])
 legend("topleft",inset=c(0.1,0),leg=shareTypes[j],bty="n",cex=1.2,x.intersp=0)
 axis(side=2,at=aty,lab=laby)
}
ymax <- max(log(kT[,c(2:8)]))+0.7
ymin <- min(log(kT[,c(2:8)]))
plot(kT$date,log(DF2[1]*kT[,46]),ylim=c(ymin,ymax),pch=20,cex=0.6,yaxt="n",xlab="",ylab
="")
legend("topleft",inset=c(0,0),leg=shareTypes,y.intersp=0.8,x.intersp=0.4,
          cex=1.2,pch=21,pt.bg=c(1:4,6,8,"orange"),bty="n",ncol=3)
points(kT$date,log(DF2[2]*kT[,47]),pch=20,cex=0.6,col=2)
```

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```
points(kT$date,log(DF2[3]*kT[,48]),pch=20,cex=0.6,col=3)
points(kT$date,log(DF2[4]*kT[,49]),pch=20,cex=0.6,col=4)
points(kT$date,log(DF2[5]*kT[,50]),pch=20,cex=0.6,col=6)
points(kT$date,log(DF2[6]*kT[,51]),pch=20,cex=0.6,col=8)
points(kT$date,log(DF2[7]*kT[,52]),pch=20,cex=0.6,col="orange")
axis(side=2,at=aty,lab=laby)
```

- Assessing the performance of GARCH(1,1) Using Monte Carlo Simulation

Generate simulated samples assuming that the mean (compounded) return in the population is zero and the return distribution is normal.

```
samp <- 2                 # sample stock type: commodity as an example
n <- nrow(kT)             # length of series
S0 <- kT[1,samp+1]        # initial price of stock
nSim <- 7                 # number of simulated series
seedu <- 325649
set.seed(seedu)

# Assumed daily volatility of commodity
ft <- 1.975+0.65*c(1:230)/230              #1
ft <- c(ft,2.625+0*c(1:130)/130)           #2
ft <- c(ft,2.625-1.03*c(1:330)/330)        #3
ft <- c(ft,1.595-0*c(1:225)/225)           #4
ft <- c(ft,1.595-0.15*c(1:250)/250)        #5
ft <- c(ft,1.445+0.3*c(1:300)/300)         #6
ft <- c(ft,1.745-0.35*c(1:320)/320)        #7
ft <- c(ft,1.395+0.5*c(1:270)/270)         #8
```

- Plot the assumption

```
windows(5,4)
par(mar=c(2.4,2.6,2.4,1),mgp=c(1.1,0.2,0),oma=c(0,0,0,0),las=1,tcl=-0.2)
plot(kT$date[-1],fits[,samp],type="l",col="red",lwd=2,ylab="",xlab="")
abline(h=c(1:3),col=8)
points(kT$date[-1],ft,type="l",lwd=2) #kT$date[-1]
mtext(side=3,adj=-0.12,line=0.2,"Daily Volatility (%)")
mtext(side=3,adj=0.5,line=1,"Simulated Commodity Shares")
lg <- c("Assumed","Fitted")
legend("topright",inset=c(0.01,0.01),leg=lg,lwd=2,col=c(1,"red"))
```

- Simulated returns

```
rt <- NULL
```

```
for (j in c(1:nSim)) {
 rt <- cbind(rt,c(0,ft*rnorm(n-1)/100))              # simulated returns
}
ymin <- min(rt)
ymax <- max(rt)
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,0.5,2.5,1),mar=c(0.4,2,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
for (j in c(1:3)) {
 plot(kT$date[-1],rt[-1,j],type="l",col=8,ylim=c(-0.09,0.09),xaxt="n",
          xlab="",ylab="",cex.axis=1.2)
 points(kT$date[-1],rt[-1,j],pch=20,cex=0.6)
 abline(h=0,col="chocolate1")
 mtext(side=3,line=0.2,adj=-0.1,"Simulated returns")
 tit <- paste("Commodity Simulation",j,sep=" ")
 legend("topright",bty="n",inset=c(0,0),leg="",title=tit,cex=1.2)
}
for (j in c(4:7)) {
 plot(kT$date[-1],rt[-1,j],type="l",col=8,ylim=c(-0.08,0.09),
          xlab="",ylab="",cex.axis=1.2)
 if (j==4) mtext(side=3,line=0.2,adj=-0.1,"Simulated returns")
 points(kT$date[-1],rt[-1,j],pch=20,cex=0.6)
 abline(h=0,col="chocolate1")
 tit <- paste("Commodity Simulation",j,sep=" ")
 legend("topright",bty="n",inset=c(0,0),leg="",title=tit,cex=1.2)
}
```

➢ Q-Q plots of simulated returns

```
windows(10,6)
par(mfrow=c(2,4),oma=c(2.5,0.5,2.5,1),mar=c(0.4,2.7,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
for (j in c(1:7)) {
 z <- rt[-1,j]
 qqnorm(z,main="",ylab="",xlab="",xaxt="n",cex.axis=1.2,ylim=c(-0.08,0.09))
 if (j>3) {
  axis(side=1,cex.axis=1.2)
  axis(side=1,at=0,lab="Theoretical Quantiles",padj=1.4,cex.axis=1.4,tcl=0)
 }
 qqline(z,col=2,lwd=2)
 if (j<5) mtext(side=3,"Simulated returns",adj=-0.2,line=0.2,cex.axis=1.4,tcl=0)
 tit <- paste("Commodity Simulation",j,sep=" ")
```

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```
 legend("topleft",bty="n",inset=c(0.02,0),leg="",title=tit,cex=1.2)
}
```

- Plot corresponding prices

```
P0 <- kT[1,shareTypes[samp]]
rt <- as.data.frame(rt)
for (j in c(1:nSim)) {
  rt[-1,j+7] <- P0*exp(cumsum(rt[-1,j]))
}
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,0,2.5,1),mar=c(0.4,2.8,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
for (j in c(1:4)) {
 plot(kT$date[-1],rt[-1,(j+7)],pch=20,cex=0.6,xaxt="n",xlab="",ylab="")
 mtext(side=3,adj=-0.13,line=0.2,"Rupiah",cex=0.8)
 tit <- paste("Commodity Sim",j,sep=" ")
 legend("top",bty="n",inset=c(0.03,0),leg="",title=tit,cex=1.2)
}
for (j in c(5:7)) {
 plot(kT$date[-1],rt[-1,(j+7)],pch=20,cex=0.6,xlab="",ylab="")
 tit <- paste("Commodity Sim",j,sep=" ")
 legend("top",bty="n",inset=c(0.03,0),leg="",title=tit,cex=1.2)
}
ymax <- max(rt[-1,c(8:14)])
ymin <- min(rt[-1,c(8:14)])
plot(kT$date[-1],rt[-1,8],ylim=c(ymin,ymax),pch=20,cex=0.6,xlab="",ylab="")
points(kT$date[-1],rt[-1,9],pch=20,cex=0.6,col=2)
points(kT$date[-1],rt[-1,10],pch=20,cex=0.6,col=3)
points(kT$date[-1],rt[-1,11],pch=20,cex=0.6,col=4)
points(kT$date[-1],rt[-1,12],pch=20,cex=0.6,col=6)
points(kT$date[-1],rt[-1,13],pch=20,cex=0.6,col=8)
points(kT$date[-1],rt[-1,14],pch=20,cex=0.6,col="orange")
legend("topright",inset=c(0.15,0),leg=paste("Sim",c(1:7),sep=" "),y.intersp=0.8,
          cex=0.9,lwd=2,col=c(1:4,6,8,"orange"),bg="ivory")
```

- Plot all simulations in the same axes

```
xmin <- min(kT$date)
xmax <- max(kT$date)
ymax <- max(rt[-1,c(8:14)])
ymin <- min(rt[-1,c(8:14)])
```

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```
ymin1 <- log(ymin)
ymax1 <- log(ymax)
windows(9,5)
par(mar=c(2.5,2.8,2,1),mgp=c(1.1,0.2,0),oma=c(0,0,0,0),las=1,tcl=0.2)
clr <- c(1:7)                    #c(1:4,6,8,"orange")
plot(kT$date,log(rt[,8]),pch=20,col=clr[1],cex=0.4,xlim=c(xmin,xmax),
        ylim=c(ymin1,ymax1),xlab="",ylab="",yaxt="n")
mtext(side=3,adj=-0.04,line=0.2,"Rupiah")
for (j in c(2:7)) {
 points(kT$date,log(rt[,(j+7)]),pch=20,col=clr[j],cex=0.4)
}
ylab <- c(30,100,300,1000,3000,10000,30000)
yat <- log(ylab)
axis(side=2,at=yat,lab=ylab)
legend("topleft",inset=c(0.01,0.005),leg=paste("Sim",c(1:7),sep=" "),y.intersp=0.8,
        x.intersp=0.4,pch=21,pt.bg=c(1:4,6,8,"orange"),bg="ivory",ncol=3)
```

➢ Fit GARCH(1,1) to simulated returns

```
ff <- function(a,b,u) {
 eps <- 0.000001
 VL <- var(u)
 w <- VL*(1-a-b)
 n <- length(u)
 v <- 0*u                         # initialize v
 v[1] <- 0
 v[2] <- max(u[1]^2,eps)
 lik <- -log(v[2])-u[2]^2/v[2]
 for (i in 3:n) {
  v[i] <- max(w+a*u[i-1]^2+b*v[i-1],eps)
  lik <- lik-log(v[i])-u[i]^2/v[i]
 }
 lik
}
alpha.f <- NULL
beta.f <- NULL
seA.f <- NULL
seB.f <- NULL
nit.f <- NULL
```

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- Setting initial values of parameters alpha and beta

```
for (jj in c(1:7)) {
 z <- rt[-1,jj]                    # simulated returns
lik <- matrix(NA,20,20) # Find (a,b) cell where likelihood has maximum value
 a.Lmax <- 0
 b.Lmax <- 0
 Lmax <- -9999999
 for (j in 1:20) {
  b <- 0.05*j-0.026
  for (i in 1:(21-j)) {
   a <- 0.05*i-0.026
   lik[i,j] <- ff(a,b,z)
   if (lik[i,j]>Lmax) {
    a.Lmax <- a
    b.Lmax <- b
    Lmax <- lik[i,j]
   }
  }
 }
 VL <- var(z)          # long-term variance
 a <- a.Lmax           # initial parameter estimates
 b <- b.Lmax


 # damped Newton's method
 H0 <- matrix(0,2,2)               # Hessian matrix
 w0 <- matrix(0,2,1)               # column vector of derivatives
 ab0 <- w0
 ab0[1,1] <- a                     # intial values of alpha & beta
 ab0[2,1] <- b
 d <- 0.0001                       # dx & dy in numerical derivatives
 epsilon <- 0.00001                # change in log-lik for convergence
 diff <- 1                         # initial change
 dd <- 0.05                        # Marquardt damping factor
 nit <- 200
 rez0 <- NULL          # array containing results at each iteration
 it <- 0
 while ( (abs(diff)>epsilon) && ((it <- it+1) < nit) ) {
  a <- ab0[1,1]
```

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```
  b <- ab0[2,1]
  F <- ff(a,b,z)
  w0[1,1] <- (ff(a+d,b,z)-ff(a-d,b,z))/(2*d)          # numerical derivatives
  w0[2,1] <- (ff(a,b+d,z)-ff(a,b-d,z))/(2*d)
  H0[1,1] <- (ff(a+d,b,z)-2*ff(a,b,z)+ff(a-d,b,z))/d^2
  H0[2,2] <- (ff(a,b+d,z)-2*ff(a,b,z)+ff(a,b-d,z))/d^2
  H0[2,1] <- (ff(a+d,b+d,z)-ff(a+d,b-d,z)-ff(a-d,b+d,z)+ff(a-d,b-d,z))/(4*d^2)
  H0[1,2] <- H0[2,1]
  rez0 <- rbind(rez0,c(F,w0[1,1],w0[2,1],H0[1,1],H0[2,2],H0[1,2],a,b))
  ab1 <- ab0 - dd*solve(H0) %*% w0
  ab0 <- ab1                                           # update estimates
  diff <- ff(ab0[1,1],ab0[2,1],z)-F
 }
 SE <- sqrt(-diag(solve(H0)))                # standard errors of alpha and beta
 CIfor.a <- a+SE[1]*1.96*c(-1,1)
 CIfor.b <- b+SE[2]*1.96*c(-1,1)
 alpha.f <- c(alpha.f,a)
 beta.f <- c(beta.f,b)
 seA.f <- c(seA.f,SE[1])
 seB.f <- c(seB.f,SE[2])
 nit.f <- c(nit.f,it)
}
alpha.f; beta.f; seA.f; seB.f; nit.f

# compute and plot simulated volatility series
vol <- NULL
for (j in c(1:7)) {
 z <- rt[-1,j]                               # simulated returns
 a <- alpha.f[j]
 b <- beta.f[j]
 w <- var(z)*(1-a-b)
 vt <- NA+z                                  # trading day variances
 vt[2] <- z[1]^2
 for (i in 3:n) {
  vt[i] <- w+a*z[i-1]^2+b*vt[i-1]
 }
 vol <- cbind(vol,100*sqrt(vt)[-1])          # trading day volatilities
}
ymin <- min(vol)
```

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```
ymax <- max(vol)
```

- Natural cubic spline

```
kT$day <- as.integer(kT$date-kT$date[1])
x <-c(1:(n-1))
kn <- as.integer(2000/36*c(1,6,11,16,21,26,31,36))
p <- length(kn)            # number of spline knots
yy <- as.data.frame(vol)
names(yy) <- shareTypes
yy$x <- x
d1 <- kn[p]-kn[p-1]
for (j in c(1:(p-2))) {
 sj <- ifelse(x>kn[j],(x-kn[j])^3,0)
 sj <- sj-((kn[p]-kn[j])/d1)*ifelse(x>kn[p-1],(x-kn[p-1])^3,0)
 sj <- sj+((kn[p-1]-kn[j])/d1)*ifelse(x>kn[p],(x-kn[p])^3,0)
 yy[,(j+8)] <- sj
 names(yy)[j+8] <- paste("s",j,sep="")
}
fits.f <- NULL
resids.f <- NULL
sims <- paste(shareTypes[samp],"Sim",c(1:7),sep=" ")
```

- Volatility of simulated returns

```
windows(12,6)
par(mfrow=c(2,4),oma=c(2.5,2.5,2.5,1),mar=c(0.5,0.5,0,0),mgp=c(1.1,0.2,0),las=1,tcl=0.2)
#kT$day[-1]
for (j in c(1:4)) {
 plot(kT$date[-1],vol[,j],type="l",col=8,ylim=c(ymin,ymax),
          xlab="",ylab="",xaxt="n",yaxt="n")
 abline(h=c(1:3),col=8)
 aa <- round(alpha.f[j],3)
 aa <- ifelse(nchar(aa)==3,paste(aa,"00",sep=""),ifelse(nchar(aa)==4,paste(aa,"0",sep=""),aa))
 la <- paste("a: ",aa," (",round(seA.f[j],3),")",sep="")
 bb <- round(beta.f[j],3)
 bb <-
ifelse(nchar(bb)==3,paste(bb,"00",sep=""),ifelse(nchar(bb)==4,paste(bb,"0",sep=""),bb))
 lb <- paste("b: ",bb," (",round(seB.f[j],3),")",sep="")
 legend("topright",inset=c(0,0.12),leg="",title=la,bg="white",
          x.intersp=0,bty="n",cex=1.4)
```

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```
 legend("topright",inset=c(0,0.19),leg="",title=lb,bg="white",
          x.intersp=0,bty="n",cex=1.4)
 legend("topright",inset=c(-0.01,0),leg=sims[j],bty="n",cex=1.2,x.intersp=0)
 points(kT$date[-1],vol[,j],pch=20,cex=0.6,col="cornsilk4")
 if (j==1) mtext(side=3,adj=-0.11,line=0.2,"Daily Volatility (%)")
 if (j %in% c(1,5)) axis(side=2,cex.axis=1.4)
 mod1 <- lm(data=yy,yy[,j]~x+s1+s2+s3+s4+s5+s6)
 points(kT$date[-1],mod1$fit,type="l",col=2,lwd=2)                    # fit spline function
 fits.f <- cbind(fits.f,mod1$fit)
 resids.f <- cbind(resids.f,mod1$resid)
}
for (j in c(5:7)) {
 plot(kT$date[-1],vol[,j],type="l",col=8,ylim=c(ymin,ymax),
          xlab="",ylab="",yaxt="n",cex=1.4)
 abline(h=c(1:3),col=8)
 aa <- round(alpha.f[j],3)
 aa <- ifelse(nchar(aa)==3,paste(aa,"00",sep=""),ifelse(nchar(aa)==4,paste(aa,"0",sep=""),aa))
 la <- paste("a: ",aa," (",round(seA.f[j],3),")",sep="")
 bb <- round(beta.f[j],3)
 bb <-
ifelse(nchar(bb)==3,paste(bb,"00",sep=""),ifelse(nchar(bb)==4,paste(bb,"0",sep=""),bb))
 lb <- paste("b: ",bb," (",round(seB.f[j],3),")",sep="")
 legend("topright",inset=c(0,0.12),leg="",title=la,bg="white",
          x.intersp=0,bty="n",cex=1.4)
 legend("topright",inset=c(0,0.19),leg="",title=lb,bg="white",
          x.intersp=0,bty="n",cex=1.4)
 legend("topright",inset=c(-0.01,0),leg=sims[j],bty="n",cex=1.4,x.intersp=0)
 points(kT$date[-1],vol[,j],pch=20,cex=0.6,col="cornsilk4")
 mod1 <- lm(data=yy,yy[,j]~x+s1+s2+s3+s4+s5+s6)      #estimates parameters
 points(kT$date[-1],mod1$fit,type="l",col=2,lwd=2)
 if (j==5) axis(side=2,cex.axis=1.4)
 fits.f <- cbind(fits.f,mod1$fit)
 resids.f <- cbind(resids.f,mod1$resid)
}
```

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➢ Plot simulated volatility in the same axes

```
plot(kT$date[-
1],fits.f[,1],type="l",lwd=2,col=8,ylim=c(ymin,ymax),ylab="",yaxt="n",xlab="",cex.axis=1)
clr <-c("antiquewhite3","aquamarine","gray70","yellow","lightcyan2","cyan","gray64","gray60")
```

```
for (j in c(1:7)) {
 points(kT$date[-1],fits.f[,j],type="l",lwd=2,col=clr[j])
}
```

- Assumed volatility

```
ft <- 1.975+0.65*c(1:230)/230              #1
ft <- c(ft,2.625+0*c(1:130)/130)           #2
ft <- c(ft,2.625-1.03*c(1:330)/330)        #3
ft <- c(ft,1.595-0*c(1:225)/225)           #4
ft <- c(ft,1.595-0.15*c(1:250)/250)        #5
ft <- c(ft,1.445+0.3*c(1:300)/300)         #6
ft <- c(ft,1.745-0.35*c(1:320)/320)        #7
ft <- c(ft,1.395+0.5*c(1:270)/270)         #8
points(kT$date[-1],ft,type="l",lwd=2,col=1)
text(kT$date[kn],ymin,adj=c(0.5,0),"+",col="blue",cex=1.2)
legend("bottomleft",inset=c(0.004,0.083),leg=sims,lwd=2,col=clr,cex=1,bg="ivory",y.intersp=
0.9)
legend("bottomright",inset=c(0.01,0.1),leg="Spline
knots",pch=3,pt.cex=1.4,col=4,cex=1.2,bg="ivory",y.intersp=0.7)
```

Prince of Songkla University
Pattani Campus

Vitae

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Student ID: 5720320705

Educational Attainment:

Degree	Name of Institution	Year of Graduation
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Scholarship Awards during Enrolment:

1. Science and Technology (SAT)-ASEAN scholarship for international students from the Faculty of Science and Technology, Prince of Songkla University, Thailand.
2. Research scholarship from the Graduate School, Prince of Songkla University, Thailand.

Proceeding:

Awalludin, S. A & Saelim, R. 2016. Modelling the Volatility and Assessing the Performance of the Model. The 20th International Annual Symposium on Computational Science and Engineering. 27-29 July 2016. Bangkok, Thailand.