Thai Natural Rubber Price Simulation and Stability Analysis based on Polynomial Fits

Chakkraphong Tomood

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science in Applied Mathematics
Prince of Songkla University
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I hereby certify that this work has not been accepted in substance for any degree, and is not being currently submitted in candidature for any degree.

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ชื่อวิทยานิพนธ์	การจำลองราคายางธรรมชาติของไทยและการวิเคราะห์เสถียรภาพของราคาที่กระชับด้วยพหุนาม
ผู้เขียน	นายจักรพงศ์ โต๊ะหมูด
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ปีการศึกษา	2559

บทคัดย่อ

ยางธรรมชาติเป็นสินค้าทางการเกษตรที่สำคัญของประเทศไทย สืบเนื่องจากรูปแบบการเคลื่อนไหวของราคายางคล้ายคลึงกับราคาหุ้น เราจึงสนใจศึกษารูปแบบดังกล่าว งานวิจัยส่วนใหญ่ศึกษารูปแบบการเคลื่อนไหวของราคาเป็นกระบวนการต่อเนื่องซึ่งจำลองด้วยการเคลื่อนที่แบบบราวน์ (Brownian motion) ซึ่งมีวิถีตัวอย่างต่อเนื่อง จากการสังเกตราคายางที่มีการเปลี่ยนแปลงทีละน้อยบ้างและมากบ้างนำไปสู่คำถามเรื่องการกระโดด (jump) ของราคา งานวิจัยนี้มีวัตถุประสงค์เพื่อจำลองราคายางแผ่นดิบของตลาดหาดใหญ่จากวันที่ 3 มกราคม พ.ศ. 2550 ถึงวันที่ 27 กุมภาพันธ์ พ.ศ. 2558 ด้วยตัวแบบจำลองสองแบบ แบบหนึ่งใช้เมื่อสมมติให้ราคามีความต่อเนื่อง อีกแบบใช้เมื่อสมมติให้ราคามีการกระโดด ผลการจำลองที่ดีกว่าสามารถวัดได้ด้วยค่าความคลาดเคลื่อนร้อยละสัมพัทธ์เฉลี่ย (Average Relative Percentage Error) ที่น้อยกว่า ซึ่งแสดงให้เห็นว่าตัวแบบที่มีการกระโดดให้ผลการกระชับที่ดีกว่าโดยประมาณ ยิ่งไปกว่านั้นงานวิจัยนี้ยังวิเคราะห์เสถียรภาพของราคายางแผ่นดิบในช่วงเวลาสั้น ๆ โดยการกระชับราคาด้วยพหุนามดีกรีไม่เกินสาม ผลที่ได้คือภาวะเสถียรของจุดสมดุลเปลี่ยนไปตามพหุนามที่ใช้และราคา อย่างไรก็ตามไม่ว่าจุดสมดุลจะเสถียรหรือไม่เกษตรกรสามารถตัดสินใจได้อย่างเหมาะสมตามพฤติกรรมความเสถียรและราคาปัจจุบัน

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ABSTRACT

Natural rubber is one of the most important agricultural product of Thailand. Since the movement of the natural rubber price is similar to the stock price, it is interesting to study its pattern. Most of the work considered prices as continuous processes which are modeled based on Brownian motion having a continuous sample paths. However, many small and large changes of the rubber price observed by eyes brought to the question of jumps. This research aims to simulate the Unsmoked Sheet Rubber (USS) price of Hat Yai market starting from January 3, 2007 to February 27, 2015 with two models. One is when the price is assumed to be continuous, the other is when it is assumed to have jumps. A better simulation result is measured by a smaller value of Average Relative Percentage Error (ARPE), showing that the model with jump provided an approximately better fit than the continuous model. Moreover, this research also studies stability analysis of the USS price in a short interval of time in which the price is fitted with the polynomials up to degree three. The results showed that the number of equilibrium points and their stability behaviors varied by the polynomial fitting and the price. However, no matter the equilibrium point is stable or not a farmer can make an appropriate decision according to the stability behaviors and the current price.

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This thesis would not have been successful without the help of mentioned people here, I would like to grab this opportunity to give a big thanks for them.

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Chakkraphong Tomood

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Chapter 1

Introduction and Literature Reviews

1.1 Background and Literature reviews

Natural rubber is one of the most important agricultural product of Thailand. In fact, Thailand is the world's largest natural rubber producer following by Indonesia and Vietnam (Workman, 2015) as in Figure 1.1. Rubber plantations are wildly planted in Thailand according to the promoting by the Thai government from 1961 forwards through special policies and programs. There are many rubber smallholders around the country, they collect the rubber latex and sell to the agents or companies for producing rubber products. In the south of Thailand, there is the central rubber market in Hat Yai city, Songkhla province where there is rubber pricing activity. Rubber pricing in the market depends on demand and supply of rubber production and also on economics situations. According to information from 1999-2014, Thailand exports rubber products increasing in each year especially for China which the rubber demand is continue increasing shown in Figure 1.2.

It is interesting to study the pattern of Thai rubber price. There are many models in finance that can describe the pattern of the asset prices. One famous model was suggested by two mathematicians: Fischer Sheffey Black and Myron Samuel Scholes in 1973 (Black and Scholes, 1973). The model is wildly used for describing the pattern of an asset price S_t which is assumed to be continuous as follows:

$$dS_t = \mu S_t dt + \sigma S_t dB_t, \tag{1.1}$$

where B_t is Brownian motion, and parameters μ and σ are called drift and volatility, respectively.

However, one of the shortcoming of the model (1.1) is that it does not consider the random jumps which can occur in the prices at any time. In 1976, a mathematician, Robert Cox Merton (Merton, 1976) improved the continuous model by considering the jump parts. In other word, it is assumed that the pattern of an asset prices S_t is not purely continuous:

$$dS_t = \mu S_t dt + \sigma S_t dB_t + (y_t - 1) S_t dN_t \tag{1.2}$$

where y_t is absolute price jump size and N_t is counting process which is Poisson process.

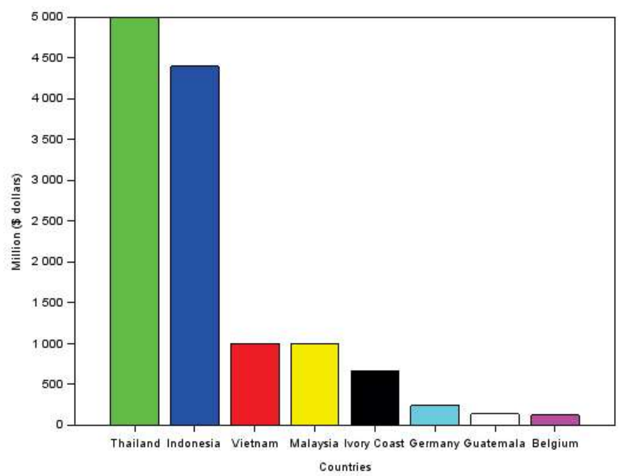


Figure 1.1 Eight countries that exported the highest dollar value worth of natural rubber during 2015.

There are many researches which studied the results between the continuous model (Black and Scholes, 1973; Klein, 1996; Khaled and Samia, 2010) and the model with jump (Merton, 1976; Kou, 2002; Maekawa et al., 2008; Gondal, 2011; Yan, 2011; El-Khatib and Al-Mdallal, 2012). For example, Maekawa and his research team (Maekawa et al., 2008) compared the results between the continuous model and the jump model (Kou, 1996) which both models applied to Japanese stock market. The result has shown that the model with jump outperforms the continuous model. Neupane and Calkins (Neupane and Calkins, 2013) studied the statistical models to capture price volatility of latex type Ribbed Smoked Rubber Sheet No.3 (RSS3) in Thailand for the period 2004-2011 which the daily price of latex type RSS3 was modeled by GARCH, GARCH-GJR and EGARCH models. The results showed that the price volatility of RSS3 was strongly persistent and the estimated results were statistically valid. The pricing model for such a jump diffusion model did not have a closed form formula since the market is incomplete (El-Khatib and Al-Mdallal, 2012). This study is considered the historical data of Unsmoked Sheet Rubber (USS) price in Hat Yai market (the Thai rubber association, 2015) starting from Jan 3, 2007 to Feb 27, 2015 (see Figure 1.3). During this period, there is 1584 daily prices which are observed in the official trade day which is around 7 years. The maximum price and minimum price in this period are 186 and 30 bahts/kg, respectively. In general, the log return R is calculated by the difference of the log of today price, S_t, and the log of yesterday price, S_{t-1}, of formula (1.3)

$$R = log\left(\frac{S_t}{S_{t-1}}\right); t = 2, 3, ..., n. \tag{1.3}$$

The small and large changes of the log return observed by eyes (see Figure 1.4), brought to the question of jumps. So that, it is interesting to simulate and fit the price with both the continuous model and the model with jump. Moreover, the stability of the rubber price during short interval of time will be considered. The price model is fitted by polynomials up to degree three while the stability is studied by cobweb analysis. Even though, the linear cobweb model is recommended to be used for the assessment of impact of policy decision (Anokye and Oduro, 2012) but in the real economic situations sometimes the nonlinear approach is needed to make a better prediction (Anokye et al., 2014).

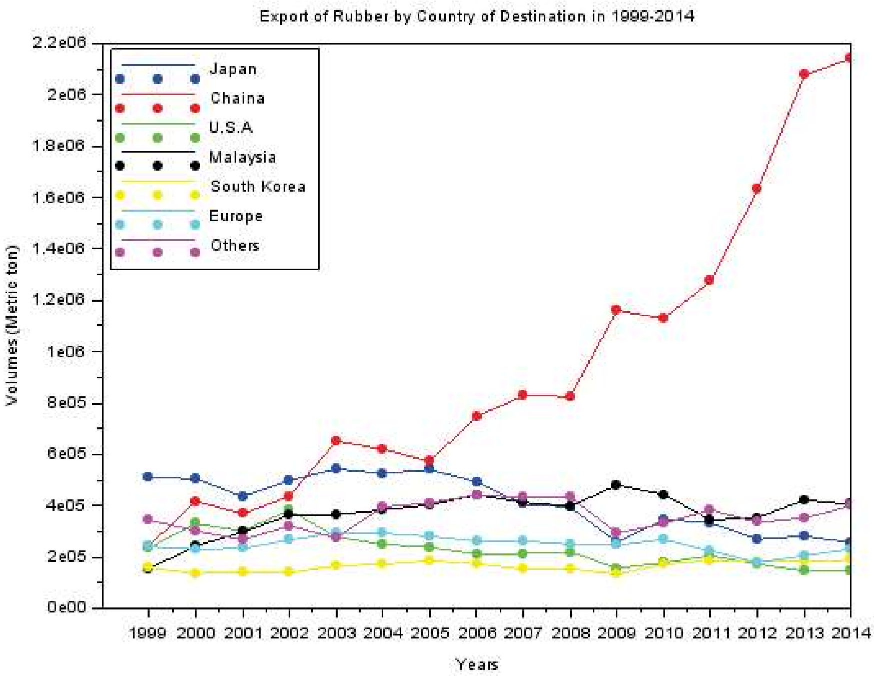


Figure 1.2 Export of rubber by country of destination in 1999-2014.

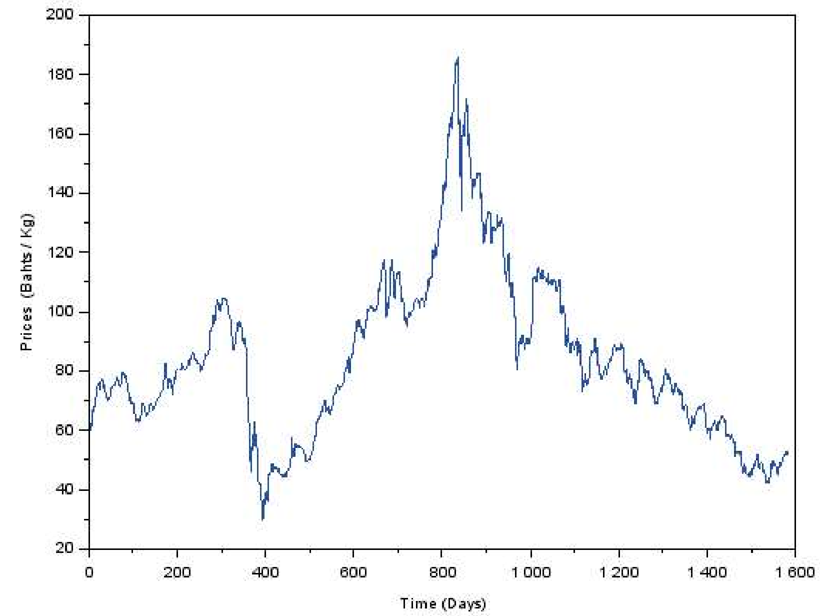


Figure 1.3 The historical data of USS price starts from Jan 3, 2007 to Feb 27, 2015.

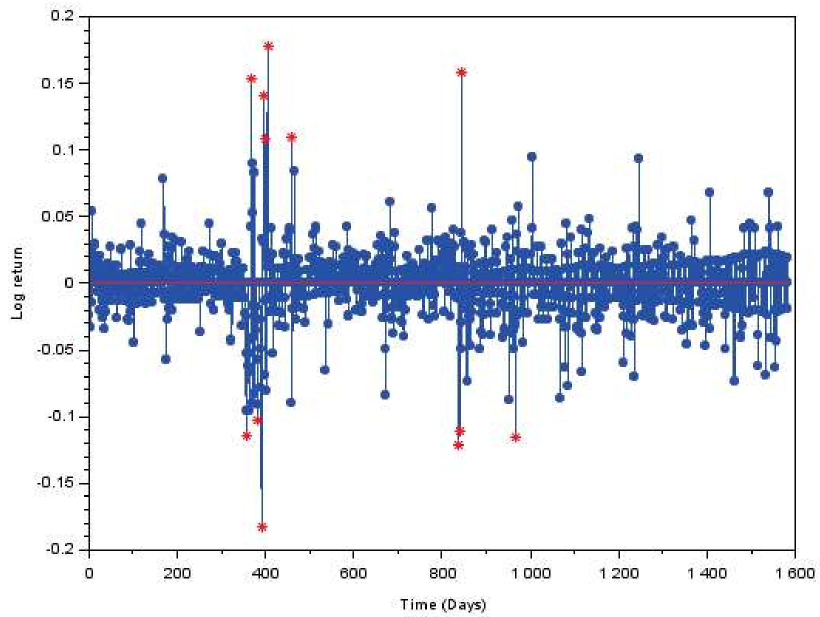


Figure 1.4 The log return of USS price.

1.2 Objectives of Research

The aims of this study are as follows:

1. To simulate and fit the Unsmoked Sheet Rubber (USS) price with the continuous model and the model with jump.
2. To compare the results obtained by both models.
3. To study stability analysis of the USS price in a short period of time by applying cobweb analysis in which case the USS price is fitted by polynomials up to degree three.

1.3 Expected Advantages

This study contains two expected advantages.

1. An investor can decide to choose an appropriate model for rubber pricing.
2. A farmer can behave appropriately corresponding to the stability analysis.

1.4 Scope of the research

The scopes of this study are as follows:

1. Use 1584 observation prices of the USS price starting from 3 January 2007 to 27 February 2015 obtained by the Thai Rubber Association.
2. Use the continuous model and the model with jump to fit the USS price.

3. Apply cobweb analysis to study the behavior of the USS price in term of stable and unstable price in each short interval of time.

Chapter 2

Theories and Methods

This chapter introduces the models for simulating the USS price as the stochastic differential equations and the methods involving in the simulation. Also, this chapter introduces all theorems and methods for studying the stability of the USS price.

2.1 Simulation and fitting of the data

In this section, let us describe the two models that will be used to fit the data which is mentioned in Chapter 1.

2.1.1 The continuous model and the model with jump

Fischer Sheffey Black and Myron Samuel Scholes (Black and Scholes, 1973) suggested the models for describing the pattern of an asset price assumes to has a constant drift and a constant volatility driven by a Brownian motion which is called the diffusion part as the Stochastic Differential Equation (SDE):

$$dS_t = \mu S_t dt + \sigma S_t dB_t. \tag{2.1}$$

Eq.(2.1) assumes that has a continuous solution which is the model for simulating the asset price.

However, the pattern of the asset price in the real situation is not actually continuous. This means that Eq.(2.1) ignores the random jumps which can occur at any time. In the case, the pattern of an asset price is not purely continuous, Merton (Merton, 1976) improved the pattern of the asset price in Eq.(2.1) by adding an independently and identically jump part which has Poisson distribution. The probability that the asset price jumps in a small interval Δt can be used a Poisson process dN_t with $E\big(dN_t\big) = Var\big(dN_t\big) = \lambda\Delta$ as follows.

■ The probability of the asset price has once jump in a small interval Δt:

$$P\big(dN_t = 1\big) = \lambda\Delta t.$$

■ The probability of the asset price has more than once jump in a small interval Δt:

$$P\big(dN_t \geq 2\big) = 0.$$

■ The probability of the asset price has no jump in a small interval Δt:

$$P\big(dN_t = 0\big) = 1 - \lambda\Delta t,$$

where the parameter $\lambda \in \mathbb{R}^+$ is the intensity of the process N_t which is independent of time t .

Suppose that in a small interval Δt the asset price jumps from S_t to $y_t S_t$. The y_t is called an absolute price jump size. Then, the relative price jump size is

$$\frac{dS_t}{S_t} = \frac{y_t S_t - 1}{S_t} = y_t - 1,$$

where y_t is assumed to be non-negative random variable drawn from log-normal distribution which is $\ln\big(y_t\big)$ having independent and identical normal distribution with mean 0 and variance δ^2. Merton (Merton, 1976) introduced a new equation for describing the pattern of an asset price with jump as follows:

$$dS_t = \mu S_t dt + \sigma S_t dB_t + (y_t - 1) S_t dN_t. \tag{2.2}$$

To solve Eq.(2.2) in order to obtain a model of asset price, we apply the following proposition (Cont and Tankov, 2004).

Proposition 2.1. *(Ito's formula for jump diffusion process) Let X be a diffusion process with jumps, defined as the sum of a drift term, a Brownian stochastic integral and a compound Poisson process*

$$X_t = X_0 + \int_0^t a_s ds + \int_0^t b_s dB_s + \sum_{i=1}^{N_t} \Delta X_i,$$

where a_t and b_t are continuous non anticipating processes with

$$E\left[\int_0^T b_t dt\right] < \infty.$$

Then, for any $C^{1,2}$ function, $f : [0,T] \times \mathbb{R} \to \mathbb{R}$, the process $Y_t = f(t, X_t)$ can be represented as

$$\begin{aligned} f\Big(t, X_t\Big) - f\Big(0, X_0\Big) &= \int_0^t \left[\frac{\partial f}{\partial s}(s, X_s) + \frac{\partial f}{\partial X}(s, X_s) a_s\right] ds \\ &+ \int_0^t \left[\frac{\partial^2 f}{\partial X^2}(s, X_s) b_s^2\right] ds \\ &+ \int_0^t \left[\frac{\partial f}{\partial X}(s, X_s) b_s\right] dB_s \\ &+ \sum_{\{i \geq 1, T_i \geq t\}} \left[f\big(X_{T_i^-} + \Delta X_i\big) - f\big(X_{T_i^-}\big)\right]. \qquad (2.3) \end{aligned}$$

Now, Eq.(2.2) will be solved by applying Eq.(2.3) with $f(t, X_t) = \ln(S_t)$. When the parameters are constant and ω is fixed, then

$$a_t = \mu S_t, \qquad b_t = \sigma S_t, \qquad \Delta X_i = (y_t - 1) S_t,$$

$$\frac{\partial f}{\partial t}(t, S_t) = 0, \qquad \frac{\partial f}{\partial S_t} = \frac{1}{S_t}, \qquad \frac{\partial^2 f}{\partial S_t^2} = -\frac{1}{S_t^2}.$$

All calculations are substituted into Eq.(2.3):

$$
\begin{aligned}
\ln\big(S_t\big) - \ln\big(S_0\big) &= \int_0^t \Big[0 + \frac{1}{S_s}\mu S_s\Big] ds + \frac{1}{2}\int_0^t \sigma^2 S_s^2 \Big(-\frac{1}{S_s^2}\Big) ds \\
&\quad + \int_0^t \Big[\sigma S_s\Big(\frac{1}{S_s}\Big)\Big] dB_s + \sum_{i=1}^{N_t}\Big[\ln\big(S_{t_-} + (y_t - 1)S_{t_-}\big)\Big] \\
\ln\left(\frac{S_t}{S_0}\right) &= \int_0^t \mu ds - \frac{1}{2}\int_0^t \sigma^2 ds + \int_0^t \sigma dB_s \\
&\quad + \sum_{i=1}^{N_t}\Big[\ln\Big(\frac{S_{t^-} + (y_t - 1)S_{t^-}}{S_{t^-}}\Big)\Big] \\
\ln\left(\frac{S_t}{S_0}\right) &= \mu(t-0) - \frac{1}{2}\sigma^2(t-0) + \sigma(B_t - B_0) + \sum_{i=1}^{N_t}\big[\ln(y_t)\big] \\
\ln\left(\frac{S_t}{S_0}\right) &= \Big(\mu - \frac{1}{2}\sigma^2\Big)t + \sigma B_t + \sum_{i=1}^{N_t}\big[\ln(y_t)\big]. \qquad (2.4)
\end{aligned}
$$

Take an exponential function both sides of Eq.(2.4), so

$$
\begin{aligned}
\exp\left(\ln\left(\frac{S_t}{S_0}\right)\right) &= \exp\left(\Big(\mu - \frac{1}{2}\sigma^2\Big)t + \sigma B_t + \sum_{i=1}^{N_t}\big[\ln(y_t)\big]\right) \\
S_t &= S_0 \exp\left(\Big(\mu - \frac{1}{2}\sigma^2\Big)t + \sigma B_t + \sum_{i=1}^{N_t}\big[\ln(y_t)\big]\right) \\
S_t &= S_0 \exp\left(\Big(\mu - \frac{1}{2}\sigma^2\Big)t + \sigma B_t + \sum_{i=1}^{N_t} Y_i\right), \qquad (2.5)
\end{aligned}
$$

where μ is again the drift, σ is again the volatility, λ is the intensity of the jump process, y_t is the absolute value of random jumps size and N_t is the counting process which is a Poisson process with intensity λ. The $\sum_{i=1}^{N_t} Y_i$ represents the jump part of the process. Note that for $\lambda = 0$ it means that there is no any jumps occur in the process. Then the solution in Eq.(2.5) is reduced to be the solution of Eq.(2.1) as

$$
S_t = S_0 \exp\left(\Big(\mu - \frac{1}{2}\sigma^2\Big)t + \sigma B_t\right). \qquad (2.6)
$$

Eq.(2.6) is known as the Geometric Brownian Motion (GBM) and wildly used in financial mathematics as the behavior of underlying asset price of the Black-Scholes model.

2.1.2 The simulations

This section explains how to simulate the USS price with Eq.(2.5) and Eq.(2.6) so that it fits the data. Suppose that a fixed set of dates t_i is given as follows:

$$0 = t_0 < t_1 = t_0 + \Delta t < ... < t_i = t_{i-1} + \Delta t = T$$

with time step Δt. Let us first consider the continuous model with constants μ and σ . For the today price at $t = t_i$, the discretization version of (2.6) is

$$S_{t_i} = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t_i + \sigma B_{t_i}\right)$$

and for the price of the next day at $t = t_i$ is

$$S_{t_i+\Delta t} = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)(t_i + \Delta t) + \sigma B_{t_i+\Delta t}\right).$$

Then the return can be obtained by

$$\frac{S_{t_i+\Delta t}}{S_{t_i}} = \frac{S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)(t_i + \Delta t) + \sigma B_{t_i+\Delta t}\right)}{S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t_i + \sigma B_{t_i}\right)}.$$

With some mathematical manipulations and property of exponential functions, the above formula becomes

$$\begin{aligned}
\frac{S_{t_i+\Delta t}}{S_{t_i}} &= \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t\right) \exp\left(\sigma B_{t_i+\Delta t} - \sigma B_{t_i}\right) \\
\frac{S_{t_i+\Delta t}}{S_{t_i}} &= \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta B_{t_i+\Delta t}\right) \\
S_{t_i+\Delta t} &= S_{t_i} \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta B_{t_i+\Delta t}\right), \qquad (2.7)
\end{aligned}$$

where $\Delta B_{t_i+\Delta t} = B_{t_i+\Delta t} - B_{t_i}$. By the properties of Brownian motion (independent and identically increment), $\Delta B_{t_i+\Delta t} = B_{t_i+\Delta t} - B_{t_i} \overset{d}{=} B_{t_i+\Delta t-t_i} = B_{\Delta t} \overset{d}{=} \sqrt{\Delta t} B_1$

which this notation $\stackrel{d}{=}$ means that both sides has the same distribution and B_1 is standard normal distribution $\mathcal{N}(0,1)$.

Next, the model with jump (2.5) will be discretized and manipulated as similarly as the continuous case. Then, the today price S_{t_i}:

$$S_{t_i} = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t_i + \sigma B_{t_i} + \sum_{i=1}^{N_{t_i}} Y_i\right)$$

and the next day price $S_{t_i+\Delta t}$:

$$S_{t_i+\Delta t} = S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)(t_i + \Delta t) + \sigma B_{t_i+\Delta t} + \sum_{i=1}^{N_{t_i+\Delta t}} Y_i\right),$$

are obtained. Then also the return can be obtained by

$$\frac{S_{t_i+\Delta t}}{S_{t_i}} = \frac{S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)(t_i + \Delta t) + \sigma B_{t_i+\Delta t} + \sum_{i=1}^{N_{t_i+\Delta t}} Y_i\right)}{S_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t_i + \sigma B_{t_i} + \sum_{i=1}^{N_{t_i}} Y_i\right)}.$$

Again, with some mathematical manipulations and property of exponential functions, the above formula becomes

$$\frac{S_{t_i+\Delta t}}{S_{t_i}} = \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta B_{t_i+\Delta t}\right) \exp\left(\sum_{i=1}^{N_{t_i+\Delta t}} Y_i - \sum_{i=1}^{N_{t_i}} Y_i\right).$$

As $\sum_{i=1}^{N_{t_i+\Delta t}} Y_i - \sum_{i=1}^{N_{t_i}} Y_i = \sum_{i=N_{t_i+1}}^{N_{t_i+\Delta t}} Y_i$, then we get

$$S_{t_i+\Delta t} = S_{t_i} \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta B_{t_i+\Delta t} + \sum_{i=N_{t_i+1}}^{N_{t_i+\Delta t}} Y_i\right) \tag{2.8}$$

where $N_{t_i} \leq N_{t_i+\Delta t}$ and $N_{t_i+\Delta t}$ has Poisson distribution with parameter $\lambda\Delta t$.

2.1.3 The Parameters of two models

Throughout this study the parameters of both models will be investigated by Monte Carlo method. The Monte Carlo method is a simple and effective tool to simulate the asset prices that do not have closed form formulas and the method has been used by many researches (Prahl et al., 1989; Boyle et al., 1997; El-Khatib and Al-Mdallal, 2012). The formulas for parameters μ and σ of the continuous case (2.7) are obtained by the expectation and the variance of the log return

$$R = \ln\left(\frac{S_{t_i+\Delta t}}{S_{t_i}}\right) = \left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\Delta B_{t_i+\Delta t}\right),$$

respectively. Indeed,

$$\begin{aligned}
E[R] &= E\left[\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\Delta B_{t_i+\Delta t}\right] \\
&= E\left[\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t\right] + E\left[\sigma\Delta B_{t_i+\Delta t}\right] \\
&= \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma E\left[\Delta B_{t_i+\Delta t}\right]; B_{t_i+\Delta t} \overset{d}{=} \sqrt{\Delta t}B_1 \\
&= \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + 0 \\
&= \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t.
\end{aligned}$$

If there are n observations of the log return $(R_1, ..., R_n)$ by the law of large number the expectation of the log return can be obtained by

$$E[R] \approx \frac{1}{n}\sum_{i=1}^{n} R_i.$$

Therefore,

$$\begin{aligned}
\frac{1}{n}\sum_{i=1}^{n} R_i &\approx \left(\mu - \frac{1}{2}\sigma^2\right)\Delta t \\
\left(\mu - \frac{1}{2}\sigma^2\right) &\approx \frac{\sum_{i=1}^{n} R_i}{\Delta t} \\
\mu &\approx \frac{\sum_{i=1}^{n} R_i}{\Delta t} + \frac{1}{2}\sigma^2. \qquad (2.9)
\end{aligned}$$

Now, the formula for σ is obtained by computing the variance of the log return R. In fact,

$$\begin{aligned}
Var[R] &= Var\left[\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta B_{t_i+\Delta t}\right] \\
&= Var\left[\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t\right] + Var\left[\sigma \Delta B_{t_i+\Delta t}\right] \\
&= 0 + \sigma^2 Var\left[\Delta B_{t_i+\Delta t}\right]; \qquad B_{t_i+\Delta t} \stackrel{d}{=} \sqrt{\Delta t} B_1 \\
&= \sigma^2 Var\left[\sqrt{\Delta t} B_1\right] \\
&= \Delta t \sigma^2 Var\left[B_1\right] = \Delta t \sigma^2.
\end{aligned} \tag{2.10}$$

On the other hand, the variance of R is calculated by

$$Var[R] \approx \frac{1}{n-1}\sum_{i=1}^{n}\left(R_i - \bar{R}\right)^2; \qquad \bar{R} = \frac{1}{n}\sum_{i=1}^{n} R_i. \tag{2.11}$$

Therefore, the final equation for σ is obtained by equating Eq.(2.10) and Eq.(2.11):

$$\begin{aligned}
Var[R] = \Delta t \sigma^2 &\approx \frac{1}{n-1}\sum_{i=1}^{n}\left(R_i - \bar{R}\right)^2 \\
\Delta t \sigma^2 &\approx \frac{1}{n-1}\sum_{i=1}^{n}\left(R_i - \bar{R}\right)^2 \\
\sigma^2 &\approx \frac{\frac{1}{n-1}\sum_{i=1}^{n}\left(R_i - \bar{R}\right)^2}{\Delta t} \\
\sigma &\approx \sqrt{\frac{\frac{1}{n-1}\sum_{i=1}^{n}\left(R_i - \bar{R}\right)^2}{\Delta t}},
\end{aligned} \tag{2.12}$$

where $\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(R_i - \bar{R}\right)^2}$ is standard deviation of the log return.

For the calculation of parameters of the model with jump, the method of moment is used. The method was probably first introduced by Johannes (Johannes, 1999). Valachy (Valachy, 2004) applied this method to the currencies exchange rate from the Central European (CE) region. Note that the jump sizes are assumed to be normally distributed

$\mathcal{N}(0, \delta^2)$. Let us assume that all parameters are constant. The formula for diffusion part of the Eq.(2.5) is

$$\sigma^2 = \sigma_T^2 - \lambda\delta^2, \tag{2.13}$$

where σ^2 is the diffusion of the continuous part, σ_T^2 is the total diffusion (or 2^{nd} moment), and δ^2 is the variance of jump sizes.

Under the assumption of constant jump intensity, the proposed estimation procedure is as follows:

■ Estimate parametrically the drift μ

$$E[\ln S_{t+\Delta t} - \ln S_t] \approx \frac{1}{n}\sum_{i=1}^{n}\left(\ln\left(\frac{S_{t+\Delta t}}{S_t}\right)\right). \tag{2.14}$$

■ Estimate λ and δ^2 based on the calculation of moments, the ratio of the 4^{th} and 6^{th} moment will give us the estimate of δ^2. Consequently, the estimate of λ will be

$$\hat{\lambda} = \frac{4^{th} Moment}{3(\delta^2)^2}. \tag{2.15}$$

The particular moments are calculated as follows:

$$4^{th} Moment = \frac{1}{n}\sum_{i=1}^{n}\left(\ln\left(\frac{S_{t+\Delta t}}{S_t}\right)\right)^4 = 3\lambda(\delta^2)^2,$$

$$6^{th} Moment = \frac{1}{n}\sum_{i=1}^{n}\left(\ln\left(\frac{S_{t+\Delta t}}{S_t}\right)\right)^6 = 15\lambda(\delta^2)^3.$$

■ The estimation of σ can be completely identified by subtracting the $2^{nd} Moment$ estimated nonparametrically from constant volatility, this means

$$\sigma^2 = 2^{nd} Moment - \hat{\lambda}\delta^2, \tag{2.16}$$

where $2^{nd} Moment = \frac{1}{n}\sum_{i=1}^{n}\left(\ln\left(\frac{S_{t+\Delta t}}{S_t}\right)\right)^2$.

2.1.4 Error calculation

In order to compare the results by the two models, the error between the empirical data and the simulated data are considered. In this study, the Average Relative Percentage Error (ARPE) is used to compare the results which is similar to the research by Maekawa and his research team (Maekawa et al., 2008). The ARPE is defined as

$$ARPE = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{x_i - s_i}{x_i}\right| \times 100, \tag{2.17}$$

where x_i is empirical data, s_i is simulated data and m is data size. The better result is measured in term of the lower value of the ARPE.

2.2 Stability Analysis

This section explains polynomials fit based on least squares method and all necessary definitions and theories for stability analysis.

2.2.1 Polynomial Fits

It is well known that for the data set of size n, one can try to fit it with the polynomial function of degree m which is defined as

$$f(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_2x^2 + a_1x + a_0. \tag{2.18}$$

In order to fit the data set (x_i, y_i), $i = 1, 2, 3, ..., n$, by the polynomial function $y = f(x)$ of degree m, the parameters $a_0, a_1, a_2, ..., a_m$ need to be estimated. Based on the least squares error, the parameters are estimated by the linear system of $m + 1$ equations as follows:

$$\begin{bmatrix} n & \sum x_i & \cdots & \sum x_i^m \\ \sum x_i & \sum x_i^2 & \cdots & \sum x_i^{m+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum x_i^m & \sum x_i^{m+1} & \cdots & \sum x_i^{2m} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \vdots \\ \sum x_i^m y_i \end{bmatrix}. \tag{2.19}$$

It is indeed the polynomial function (2.18) with parameters estimated by (2.19) is the model of our interest. Hence, from now on, the polynomial function could also be referred to as *polynomial*, *polynomial model* or just *model*. Moreover, the goodness of fit of the model can be measured by value of R-squared (R^2) and defined by

$$R^2 = \frac{\sum_{i=1}^{n}(\hat{y}_i - \bar{y})^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2},$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and y_i, $\hat{y}_i$ are the empirical data and modeled data, respectively.

2.2.2 The definitions and theorems in stability analysis

In order to study stability of rubber price, the stability of the fixed points of polynomial (2.18) will be considered. Here are the definitions and theories which involved in this study.

Definition 2.2 (Cobweb plot)**.** A cobweb plot is a visual tool used in the dynamical systems to investigate the qualitative behaviour of one-dimensional iterated functions.

Definition 2.3 (Fixed point)**.** A point p is called a *fixed point or equilibrium point* of $x_{n+1} = f(x_n)$, or more simply a fixed point of $f(x)$, if $f(p) = p$.

In general, if an equilibrium p of f is nonzero, one can always has a zero as an equilibrium of $g(x) = f(p - x) - p$. Therefore, instead of considering the equilibrium p of f, one can consider the equilibrium zero of g.

Definition 2.4 (Stability)**.** An equilibrium $x = 0$ is said to be:

(a) *stable* if for any positive scalar ϵ there exists a positive scalar δ such that $|x_0| < \delta$ implies $|x_n| < \epsilon$ for all $n \geq 0$.

(b) *asymptotically stable* if in addition $x_n \to 0$ as $n \to \infty$.

(c) *unstable* if there exists an $\epsilon > 0$ such that for every $\delta > 0$ there exists an $|x_0| < \delta$, $|x_n| \geq \epsilon$ for some $n > 0$.

In case f is a linear function of the form $f(x) = ax + b$, the following theorems give criteria for the stability of the fixed point.

Theorem 2.5 (Stability of dynamical linear equations)**.** *The fixed point of the equation $y = ax + b$ is stable if $|a| < 1$ or unstable if $|a| > 1$.*

Proof: [Marotto, 2006].

Theorem 2.6 (Oscillation of dynamical linear equations)**.** *For $a > 0$, all solutions of $y = ax + b$ are monotonic (step) converge or diverge, but for $a < 0$ all solutions oscillate around the fixed point.*

Proof: [Marotto, 2006].

In case f is non-linear function that is f is a polynomial of degree greater than or equal to two, the following theorem gives a criterion for the stability of the fixed points.

Theorem 2.7 (Stability and Oscillation of Non-linear equations)**.** *For any fixed point p of $x_{n+1} = f(x_n)$, if $|f^{'}(p)| < 1$ then p is locally stable, but if $|f^{'}(p)| > 1$ then p is unstable. Also, if $f^{'}(p) < 0$, then solution oscillates locally around p, but if $f^{'}(p) > 0$ they do not.*

Proof: [Marotto, 2006].

Besides, the criteria given by theorems 2.5-2.7 can also be confirmed numerically by cobweb graphing. In fact, for a given iterated function $x_{n+1} = f(x_n)$ where $f : \mathbb{R} \to \mathbb{R}$, with an initial value x_0, the cobweb graphing will be created by the following steps:

1. Find the point on the function curve with an x-coordinate of x_0. This has the coordinates $(x_0, f(x_0))$.

2. Plot horizontally across from the point in step 1 to the equilibrium line. This has the coordinates $(f(x_0), f(x_0))$.

3. Plot vertically from the point on the equilibrium line to the function curve. This has the coordinates $(f(x_0), f(f(x_0)))$.

4. Repeat from step 2 as required.

For the examples of the cobweb feature, we can see in Figures 2.1, 2.2, 2.3 and 2.4 which the inward directions to the fixed points mean that the fixed points are stable and the outwards directions mean that the fixed points are unstable.

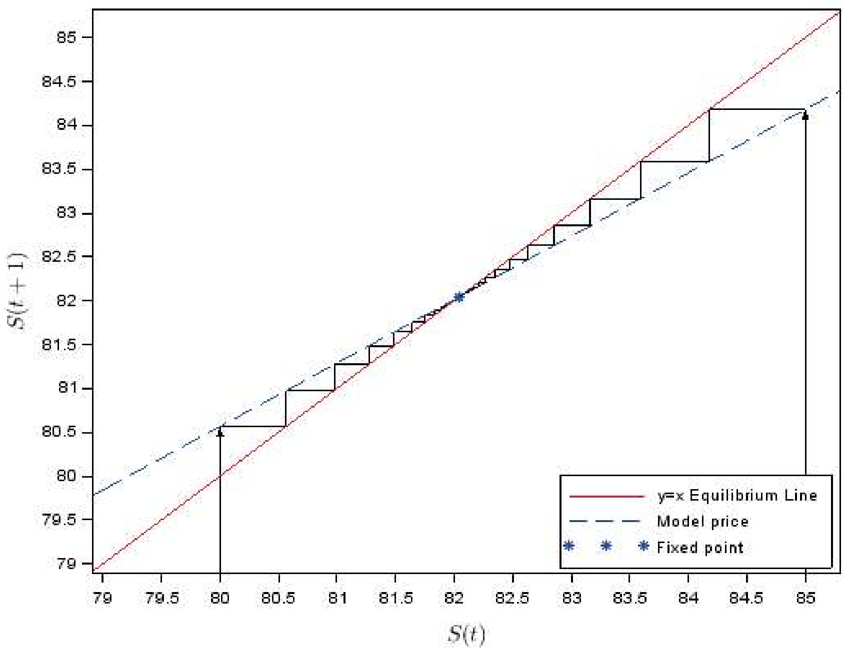


Figure 2.1 The cobweb step convergence.

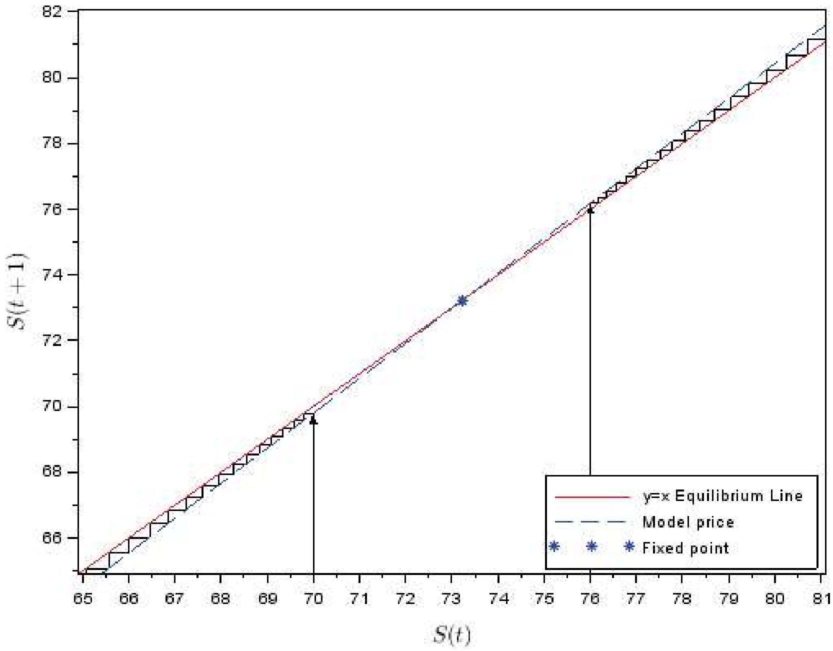


Figure 2.2 The cobweb step divergence.

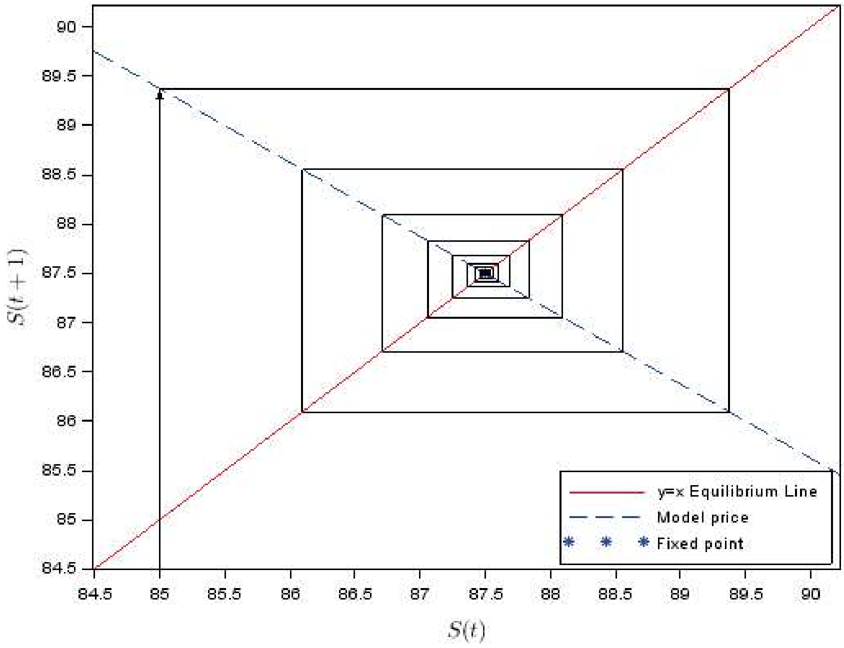


Figure 2.3 The cobweb oscillated convergence.

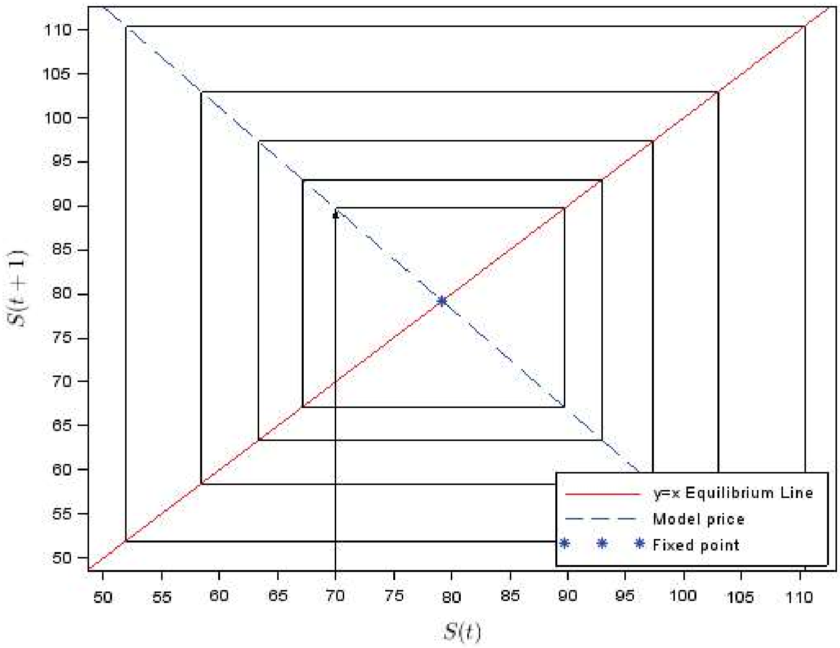


Figure 2.4 The cobweb oscillated divergence.

Chapter 3

Results

This chapter provides the simulation and the stability analysis results. In the simulation results, there are two subsections which are jump behavior in USS price and comparison of the simulation results. For the stability analysis results, there are three subsections which are the analytical results, the numerical results and note on stability of fixed point.

3.1 The simulation results

This section includes jump testing in the USS price and the simulation results.

3.1.1 Jump behavior in USS price

Before fitting the USS price with the model with jump, the jump behavior of the price is confirmed by the Bipower Variation test (Barndorff-Nielsen and Shepard, 2004). The test can be performed as following. Let Y_t be the log price of an asset and Y_δ be the discretization version of Y_t, then the log return can be seen as

$$y_j = Y_{j\delta} - Y_{(j-1)\delta}, j = 1, 2, ..., \lfloor t/\delta \rfloor \tag{3.1}$$

where $\delta > 0$ is a time interval and $\lfloor t/\delta \rfloor$ is the largest integer that does not exceed t/δ. Barndorff-Nielsen and Shepard (Barndorff-Nielsen and Shepard, 2004) introduced the

Table 3.1 The result of jump testing in USS price.

$[Y_\delta]_t$	$\{Y_\delta\}_t^{[1,1]}$	$\mu_1^{-2}\{Y_\delta\}_t^{[1,1]} - [Y_\delta]_t \leq 0$
0.8584	0.4370	-0.1720

calculating of Quadratic Variation (QV), $[Y_\delta]_t$ by

$$[Y_\delta]_t = \sum_{j=1}^{\lfloor t/\delta \rfloor} y_i^2, \tag{3.2}$$

and the Bipower Variation (BV), $\{Y_\delta\}_t^{[1,1]}$ by

$$\{Y_\delta\}_t^{[1,1]} = \sum_{j=2}^{\lfloor t/\delta \rfloor} |y_{j-1}||y_j|, \tag{3.3}$$

where $[Y_\delta]_t$ and $\{Y_\delta\}_t^{[1,1]}$ are said to be the realized quadratic and bipower variations, respectively. If the data has jump, the difference between the realized quadratic and bipower variations will be non-positive, i.e.,

$$\mu_1^{-2}\{Y_\delta\}_t^{[1,1]} - [Y_\delta]_t \leq 0, \tag{3.4}$$

where $\mu_1 = E[|u|] = \sqrt{2/\pi} \simeq 0.79788$ and $u \sim \mathcal{N}(0,1)$. The results in Table 3.1 have shown that the value of

$$\mu_1^{-2}\{Y_\delta\}_t^{[1,1]} - [Y_\delta]_t = -0.1720 \leq 0.$$

It means that there are jumps in this USS price.

3.1.2 Comparison of simulation results

To compare the results obtained by the continuous model

$$S_{t_i+\Delta t} = S_{t_i} \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta B_{t_i+\Delta t}\right), \tag{3.5}$$

and the model with jump

$$S_{t_i+\Delta t} = S_{t_i} \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta B_{t_i+\Delta t} + \sum_{i=N_{t_i+1}}^{N_{t_i+\Delta t}} Y_i\right), \tag{3.6}$$

with all parameters of both models are calculated separately. The parameters $\mu = 0.0310$ and $\sigma = 0.3499$ for the continuous model (3.5) are calculated from formulas (2.9) and (2.12), respectively. While the parameters $\mu = -0.0001$, $\sigma = 0.0137$, $\lambda = 0.0907$ and $\delta^2 = 0.0627$ of the model with jump (3.6) are calculated from formulas (2.14), (2.16) and (2.15), respectively. In simulation, 10000 trajectories for each models ((3.5) and (3.6)) are generated. Then, the results by both models are compared by ARPE:

$$ARPE = \frac{1}{m}\sum_{i=1}^{m}\left|\frac{x_i - s_i}{x_i}\right| \times 100, \tag{3.7}$$

where x_i is empirical data, s_i is simulated data and m is data size. The smaller error indicates the better model. The average of ARPE are 54.78% and 47.54% for the model (3.5) and (3.6), respectively. In term of average of ARPE, we can see that the model with jump provides a little better estimate than the continuous model. As an example, the price trajectories with ARPE $= 17.35\%$ from model (3.5) and ARPE $= 17.25\%$ from model (3.6) are shown in Figures 3.1 and 3.2.

However, it is noticed that the average of ARPE of both models are quite large comparing to the result by Maekawa and his research team (Maekawa et al., 2008) which is $= 36.46\%$ for the continuous model, and $= 28.95\%$ for model with jump with sample size 50955. In fact, the variation of the result is affected by random number and parameters of the model. Indeed, the effect of randomness can be reduced by simply increase the sample size. So that the most potential effect should be on the choices of parameters. In particular, Hull (Hull, 2000) suggested that the volatility in

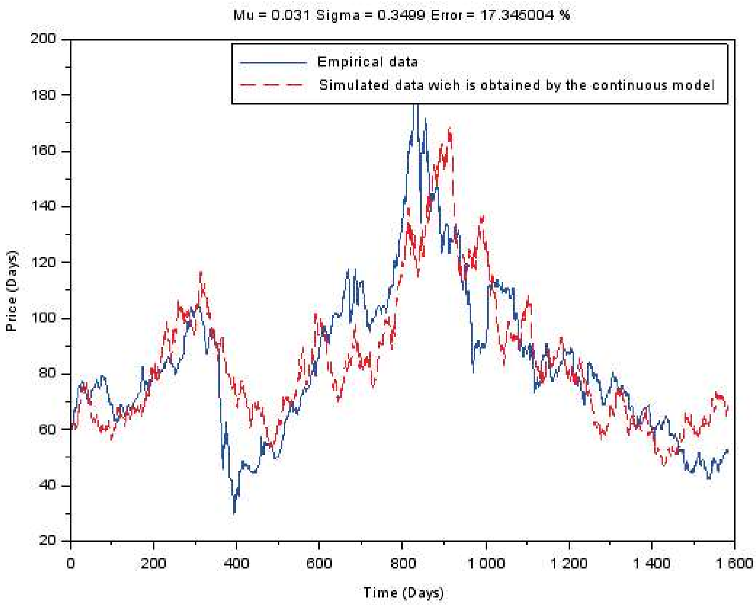


Figure 3.1 The simulated trajectory which is obtained by continuous model with calculated parameters.

the real market could vary from 15% to 60%. A question arises whether there is a more appropriately volatility other than the one obtained by calculation. For simplicity, the Brute-force method can be applied for this problem.

Brute-force method (Paar and Pelzl, 2009) is a straightforward approach based on statement of problem and definitions of concept involved. In this case, the method is applied in searching for appropriate parameters μ and σ in some given intervals that would give a better estimate for the continuous model and the model with jump as follows:

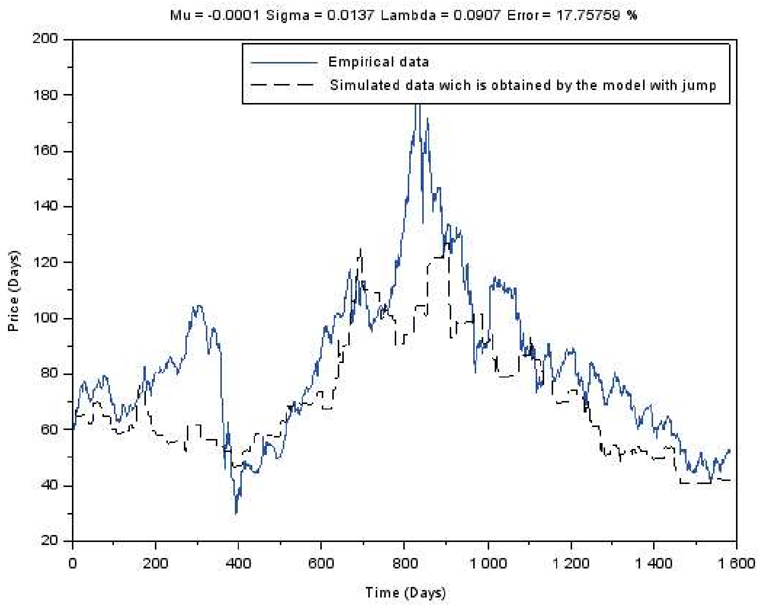


Figure 3.2 The simulated trajectory which is obtained by the model jump with calculated parameters.

1. Set the intervals of parameters μ and σ for the simulations.

2. Simulate the price trajectories with all pairs of μ and σ in step 1. and calculate the average ARPE for each pairs of μ and σ.

3. Collect the pairs of μ and σ that provide the minimum average ARPE.

In particular, the interval $[-0.05, 0.05]$, increasing by 0.01, is given for parameter μ. Together with suggestion by Hull (Hull, 2000), the interval $[0.05, 0.60]$, increasing by 0.01, is given for parameters σ. The results give $\mu = 0.01$ and $\sigma = 0.15$ for the continuous model (3.5) with a better estimate of average ARPE $= 33.39\%$ and the best estimate of ARPE $= 19.19\%$. For the model with jump (3.6) searching by the Brute-force method gives $\mu = 0.02$ and $\sigma = 0.05$ resulting the average ARPE $= 35.67\%$ with the best estimate of ARPE $= 22.41\%$.

Concerning the model with jump, another parameter which is also important is the intensity λ indicating the average number of jumps in the period. The value of $\lambda = 0.0907$ obtained by formula (2.15) seems to be quite small comparing to the empirical plot in Figure 1.3 in Chapter 1. Therefore, with $(\mu, \sigma) = (0.02, 0.05)$ obtained by the Brute-force method, it is also interesting that whether other value of λ would give a better result. In searching for an appropriate λ, the parameters (μ, σ) is kept fixed and then varies values of λ starting from 1 stepping up by 0.5 and found that $\lambda = 3$ gives the least average of ARPE $= 31.80\%$. Table 3.2 shows comparing of average ARPE from both models with the calculated parameters and the parameters obtained by Brute-force method.

Table 3.2 The comparison of average ARPE from both models with differences choices of parameters.

	Calculated parameters	Parameters by Brure-force
The continuous model	54.78%	33.39%
The model with jump	47.54%	31.80%

3.2 The stability analysis

In this section, the USS price (S_t, S_{t+1}) in some short period of times are fitted by polynomials of degree one, two and three. Then, the model (polynomial) $y = f(x)$ that is most fitted in term of the highest values of R^2 is chosen. Recall that the real number p such that $p = f(p)$ is a fixed point of f. Consequently, the intersections of the model $y = f(x)$ and the equilibrium line $y = x$ gives fixed points of the model. However, fixed points of the model can also be calculated theoretically as will be described in the following section.

3.2.1 The analytical results

This section first gives methods of obtaining fixed points of the models for all cases: linear, quadratic and cubic polynomials. After that the criteria for stability behavior of those fixed points are also given. According to the polynomial (2.18) of degree m, the linear, quadratic and cubic models can be written as $f(x) = ax+b$, $f(x) = ax^2+bx+c$ and $f(x) = ax^3 + bx^2 + cx + d$, respectively where a, b, c and d are real numbers. The method of locating the fixed points x^* such that $x^* = f(x^*)$ of those three models are described as follows.

Firstly, for the linear model $f(x) = ax + b$, if x^* is a fixed point of f then $x^* = ax^* + b$ or

$$x^* = \frac{b}{(1-a)}, a \neq 1. \tag{3.8}$$

As stated in theorem 2.5, the fixed point x^* is stable if $|a| < 1$ and unstable if $|a| > 1$.

Secondly, for the quadratic model $f(x) = ax^2 + bx + c$, if x^* is a fixed point of f then $x^* = a(x^*)^2 + bx^* + c$. Applying the quadratic formula to obtain x^* and then

$$x_1^* = \frac{-(b-1) + \sqrt{(b-1)^2 - 4ac}}{2a}, \tag{3.9}$$

$$x_2^* = \frac{-(b-1) - \sqrt{(b-1)^2 - 4ac}}{2a}. \tag{3.10}$$

As stated in theorem 2.7, the fixed point x^* is locally stable if $|f'(x^*)| < 1$ and unstable if $|f'(x^*)| > 1$. In particular, the fixed point x^* is locally stable if $|2ax^* + b| < 1$ and unstable if $|2ax^* + b| > 1$.

Finally, for the cubic model $f(x) = ax^3 + bx^2 + cx + d$, if x^* is a fixed point of f then $x^* = a(x^*)^3 + b(x^*)^2 + cx^* + d$ or

$$(x^*)^3 + \frac{b}{a}(x^*)^2 + \frac{(c-1)}{a}x^* + \frac{d}{a} = 0. \tag{3.11}$$

Recall the cubic formula (Dunham, 1990) to find all fixed points. If x_1, x_2, x_3 be the solutions of the cubic equation $f(x) = \alpha x^3 + \beta x^2 + \gamma x + \omega$, then it can be factorize as following:

$$\alpha x^3 + \beta x^2 + \gamma x + \omega = \alpha(x - x_1)(x - x_2)(x - x_3) = 0.$$

Together with

$$q = \frac{9\alpha\beta\gamma - 27\alpha^2\omega - 2\beta^3}{54\alpha^3},$$

$$r = \sqrt{\left(\frac{3\alpha\gamma - \beta^2}{9\alpha^2}\right)^3 - q^2},$$

$$s = \sqrt[3]{q + r},$$

$$t = \sqrt[3]{q - r},$$

the three solutions of $f(x) = \alpha x^3 + \beta x^2 + \gamma x + \omega$ can be calculated as follows:

$$\begin{aligned}
x_1 &= s + t - \frac{\beta}{3\alpha}, \\
x_2 &= -\frac{1}{2}(s + t) - \frac{\beta}{3\alpha} + \frac{\sqrt{3}}{2}(s - t)i, \\
x_3 &= -\frac{1}{2}(s + t) - \frac{\beta}{3\alpha} - \frac{\sqrt{3}}{2}(s - t)i,
\end{aligned}$$

where i is an imaginary unit which is $i^2 = -1$.

In particular, the solution of Eq.(3.11) can be calculated by formulas of x_1, x_2 and x_3 with $\alpha = 1, \beta = \frac{b}{a}, \gamma = \frac{c-1}{a}$ and $\omega = \frac{d}{a}$.

As stated in theorem 2.7, the fixed point x^* is locally stable if $|f'(x^*)| < 1$ and unstable if $|f'(x^*)| > 1$. In particular, the fixed point x^* is locally stable if

$$|3a(x^*)^2 + 2bx^* + c| < 1$$

and unstable if

$$|3a(x^*)^2 + 2bx^* + c| > 1.$$

3.2.2 The results by graphing

It should be noticed here that, according to Hat Yai market, the number of price in each month is ranging from 11 to 20 points. This is because there is no price announced for trading on weekend and other national holidays. In this section, the daily price of each month starting from January 3, 2007 to February 27, 2015 is considered. In fact, the daily price in each month is fitted by all models of linear, quadratic and cubic polynomials. The best fit is chosen corresponding to the highest of R^2. However, the results shown in this section are only from some periods of time that appropriate for rubber tapping. According to the information from Agricultural Research Development Agency (Public organization), the months that are suitable for rubber tapping are January, February and March.

This section contains the iterated graphing, showing stability behavior of the equilibrium price (fixed point) in some particular short period of times. The equilibrium prices are graphically located by the intersection of the equilibrium line $y = x$ and the price line $S_{t+1} = f(S_t)$ where f is the model (linear, quadratic or cubic). It is interesting and so important to realize the behavior of the fixed points whether they are stable or not. If the fixed point is stable, the current price which is located near that fixed point will eventually increasing or decreasing or oscillating to the fixed point. While for the unstable fixed point, the current price will increasing or decreasing or oscillating away from the fixed point.

The results in this study are given as follows. Firstly, the price (S_t, S_{t+1}) is fitted by a linear model $y = ax + b$ giving a stable fixed point (Figure 3.3) and an unstable fixed point (Figure 3.4). Secondly, the price (S_t, S_{t+1}) is fitted by a quadratic model $y = ax^2 + bx + c$ giving two fixed points with one is stable and the other is not (Figure 3.5). Finally, the price (S_t, S_{t+1}) is fitted by a cubic model $y = ax^3 + bx^2 + cx + d$ giving the results of only either one or three fixed points. In the case of one fixed point, the stable and unstable cases are shown in Figures 3.6 and 3.7, respectively. For the case of three fixed points, the results are found either one fixed point is stable while the other two are not (Figure 3.8) or two fixed points are stable while the other is not (Figure 3.9). All the results are shown graphically and also verified by theories.

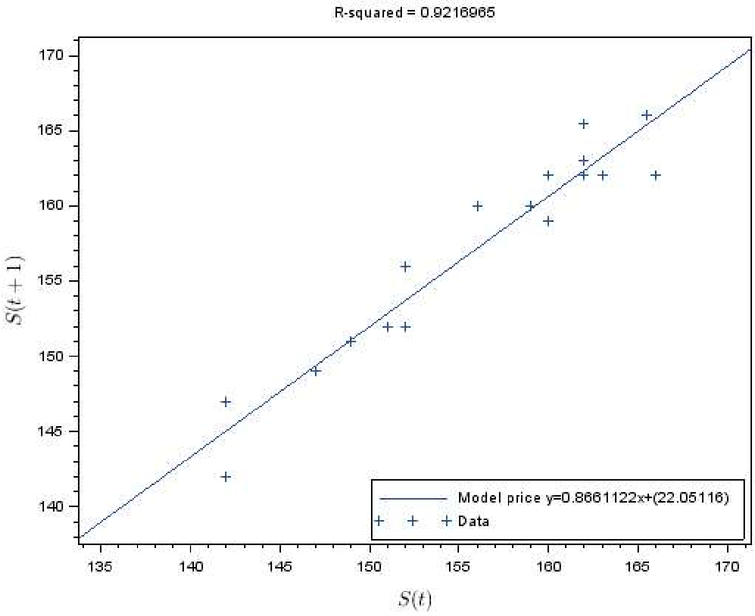
The linear model

It is easy to figure out that the fixed point is stable or unstable by considering the slope of the model. In fact, if the absolute value of the slope of the model is less than 1 then the fixed point is stable otherwise it is unstable.

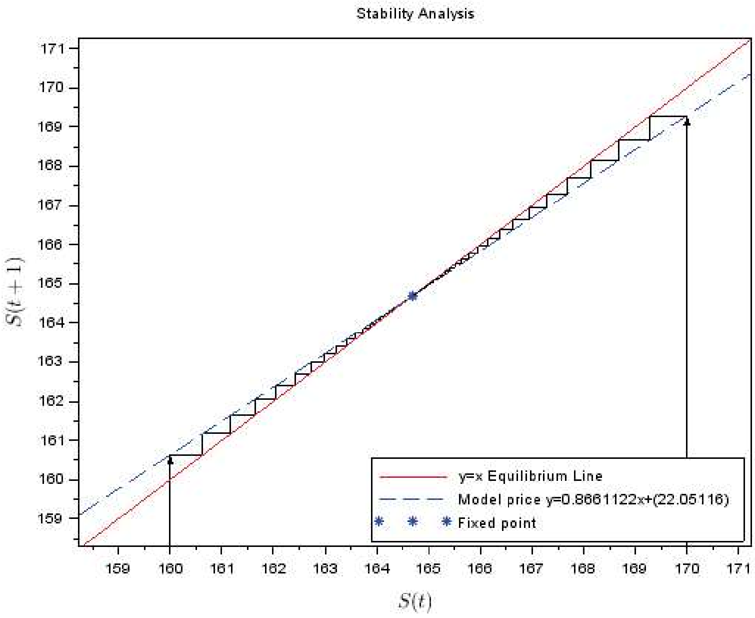
The result given by fitting the whole 17 points of January, 2011 gives a stable fixed point. In fact, the linear model $f(x) = 0.8661x + 22.0512$ with $R^2 = 0.9217$, illustrated in Figure 3.3 (a) is obtained. The fixed point in this case is 164.6988 and it is stable since the absolute value of the slope $|a| = |0.8661| < 1$. The results from linear model fit and stability analysis are shown in Table 3.3. The stability analyse by cobweb graphing in Figure 3.3 (b) showing that in the case of stable equilibrium the solution whose initial values no matter smaller or larger than the equilibrium eventually approaches the equilibrium point.

Table 3.3 Results from linear model fit and stability analysis of the price in January, 2011.

Model	$f(x) = 0.8661x + 22.0512$
R-squared	$R^2 = 0.9217$
The fixed points	$x^* = 164.6988$
Stability testing	$\|a\| = 0.8661 < 1$
Result	x^* is stable



(a)



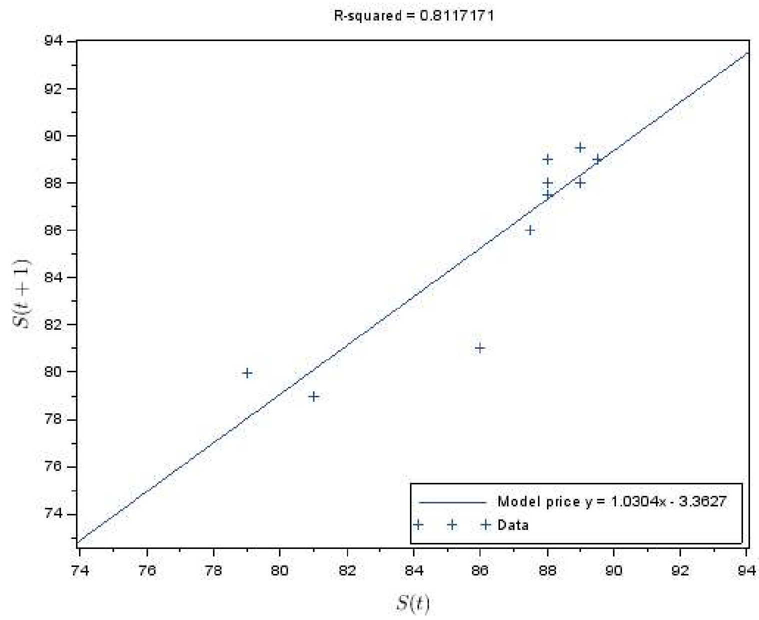
(b)

Figure 3.3 The rubber price of January, 2011 fitted by linear model (a) and the cobweb behavior with initial values $x_0 = 160$ and $x_0 = 170$ (b).

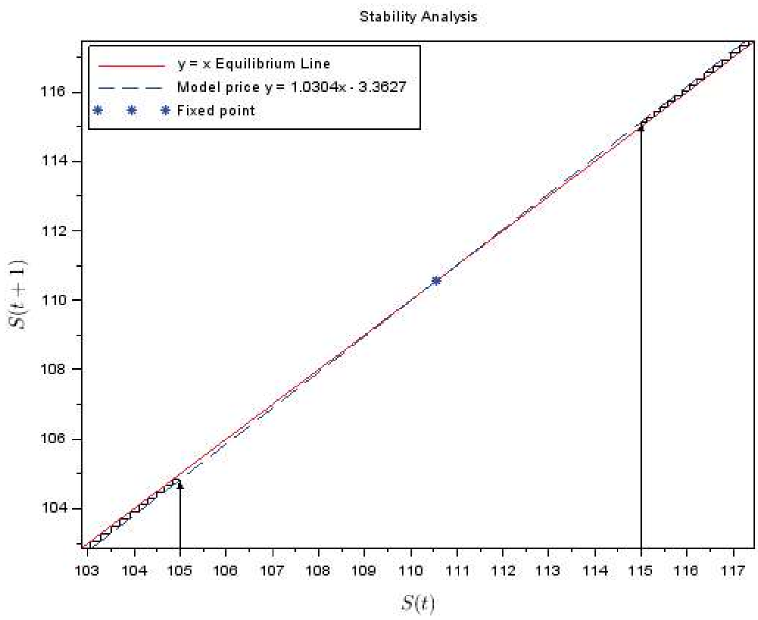
The result given by fitting the whole 11 points of February, 2013 gives an unstable fixed point. In fact, the linear model $f(x) = 1.0304x - 3.3627$ with $R^2 = 0.8117$, illustrated in Figure 3.4 (a) is obtained. The fixed point in this case is 110.5443 and it is unstable since the absolute value of the slope $|a| = |1.0304| > 1$. The results from linear model fit and stability analysis are shown in Table 3.4. The stability analyse by cobweb graphing in Figure 3.4 (b) showing that the solution whose initial values either smaller or larger than the equilibrium eventually moves away from the equilibrium point.

Table 3.4 Results from linear model fit and stability analysis of the price in February, 2013.

Model	$f(x) = 1.0304x - 3.3627$
R-squared	$R^2 = 0.8117$
The fixed points	$x^* = 110.5443$
Stability testing	$\|a\| = 1.0304 > 1$
Result	x^* is unstable



(a)



(b)

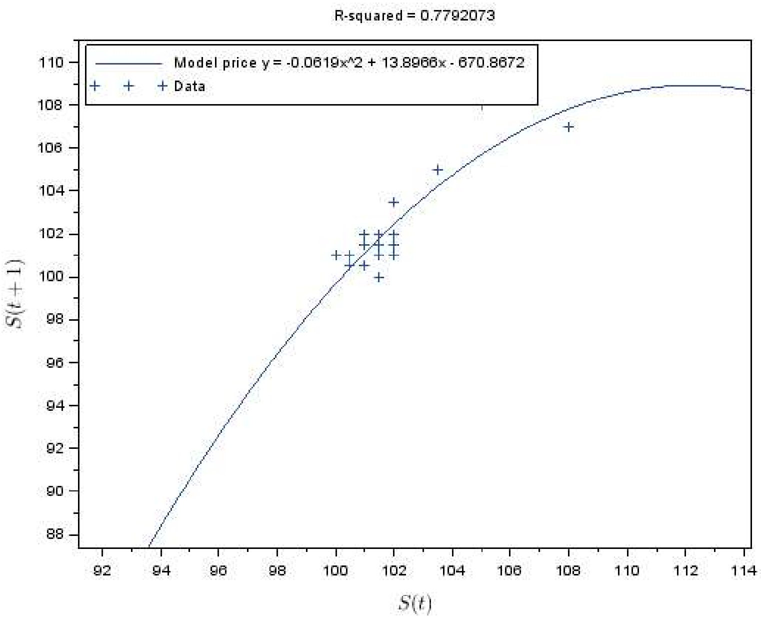
Figure 3.4 The rubber price of February, 2013 fitted by linear model (a) and the cobweb behavior with initial values $x_0 = 105$ and $x_0 = 115$ (b).

The quadratic model

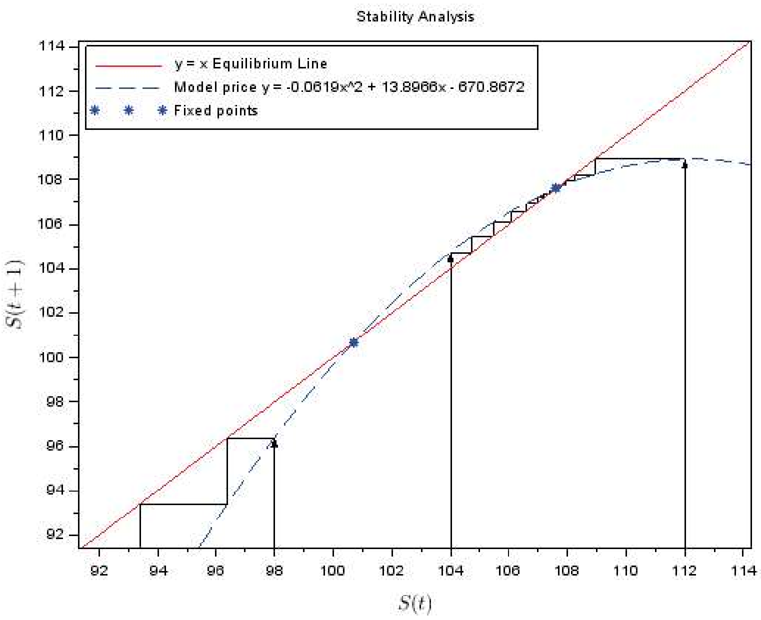
The result given by fitting the whole 20 points of March, 2010 and obtained the quadratic model $f(x) = -0.0619x^2 + 13.8966x - 670.8672$ with $R^2 = 0.7792$, presented in Figure 3.5 (a). The fixed points in this case are $x_1^* = 100.6975$ and $x_2^* = 107.6069$. The fixed point $x_2^* = 107.6069$ is stable as shown in Table 3.5, since the absolute value of the first derivative of $f(x)$ at x_2^* is less than 1. While the fixed point $x_1^* = 100.6975$ is unstable as also shown in Table 3.5, since the absolute value of the first derivative of $f(x)$ at x_1^* is greater than 1. The stability analyse by cobweb graphing in Figure 3.5 (b) showing that the solution whose initial values $x_0 < x_1^*$ moves away from x_1^* while the solution with initial values are between x_1^* and x_2^* approaches the fixed point x_2^*. However, when the initial values $x_0 > x_2^*$ it might be two events. One is approaching to the fixed point x_2^* when the initial values are close to x_2^* and the other is moving way from both fixed points.

Table 3.5 Results from quadratic model fit and stability analysis of the price in March, 2010.

Model	$f(x) = -0.0619x^2 + 13.8966x - 670.8672$
R-squared	$R^2 = 0.7792$
The fixed points	$x_1^* = 100.6975$ and $x_2^* = 107.6069$
Stability testing	$\lvert 2ax_1^* + b\rvert = 1.4278 > 1$ and $\lvert 2ax_2^* + b\rvert = 0.5722 < 1$
Result	x_1^* is unstable and x_2^* is stable



(a)



(b)

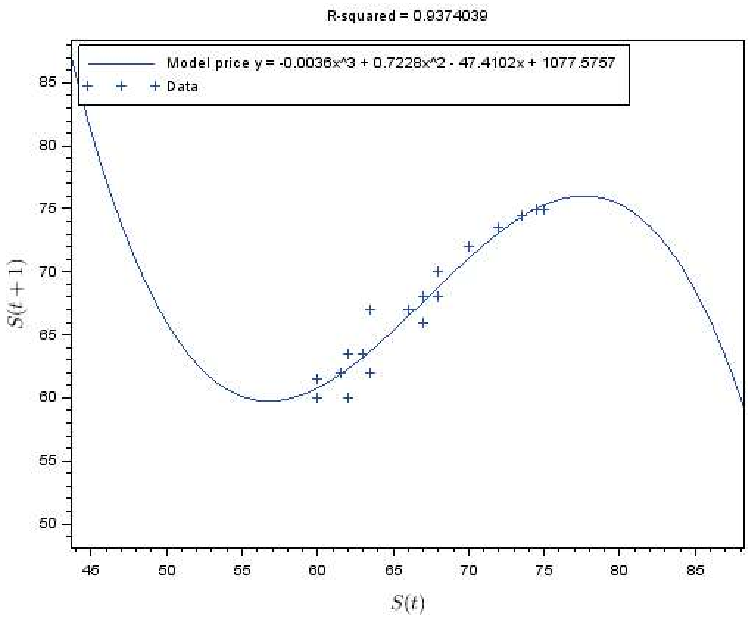
Figure 3.5 The rubber price of March, 2010 fitted by quadratic model (a) and the cob-web behavior with initial values $x_0 = 98$, $x_0 = 104$ and $x_0 = 112$ (b).

The cubic model

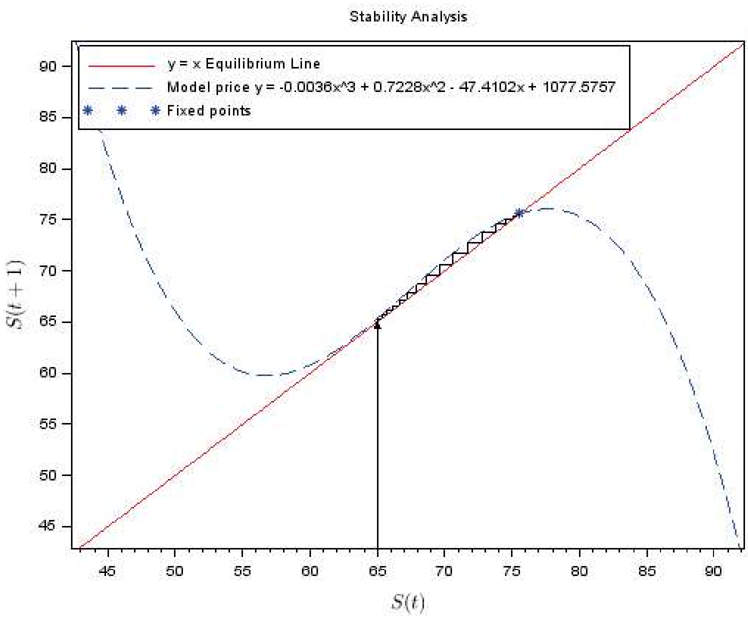
The result given by fitting the whole 20 points of January, 2007 gives one fixed point which is stable. In fact, the cubic model $f(x) = -0.0036x^3 + 0.7228x^2 - 47.4102x + 1077.5757$ with $R^2 = 0.9374$, illustrated in Figure 3.6 (a) is obtained. The fixed point in this case is $x^* = 75.5751$ and it is stable as shown in Table 3.6, since the absolute value of the first derivative of $f(x)$ at x^* is less than 1. The stability analyse by cob-web graphing in Figure 3.6 (b) showing that the solution whose initial value $x_0 = 65$ approaches the fixed point $x^* = 75.5751$.

Table 3.6 Results from cubic model fit and stability analysis of the price in January, 2007.

Model	$f(x) = -0.0036x^3 + 0.7228x^2 - 47.4102x + 1077.5757$
R-squared	$R^2 = 0.9374$
The fixed points	$x^* = 75.5751$
Stability testing	$\lvert 3a(x^*)^2 + 2bx^* + c\rvert = 0.4179 < 1$
Result	x^* is stable



(a)



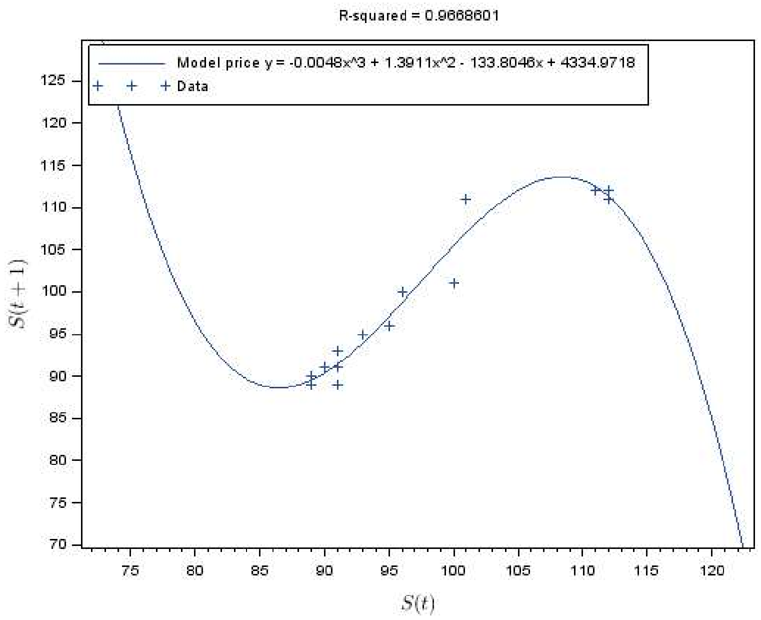
(b)

Figure 3.6 The rubber price of January, 2007 fitted by cubic model (a) and the cobweb behavior with initial value $x_0 = 65$ (b).

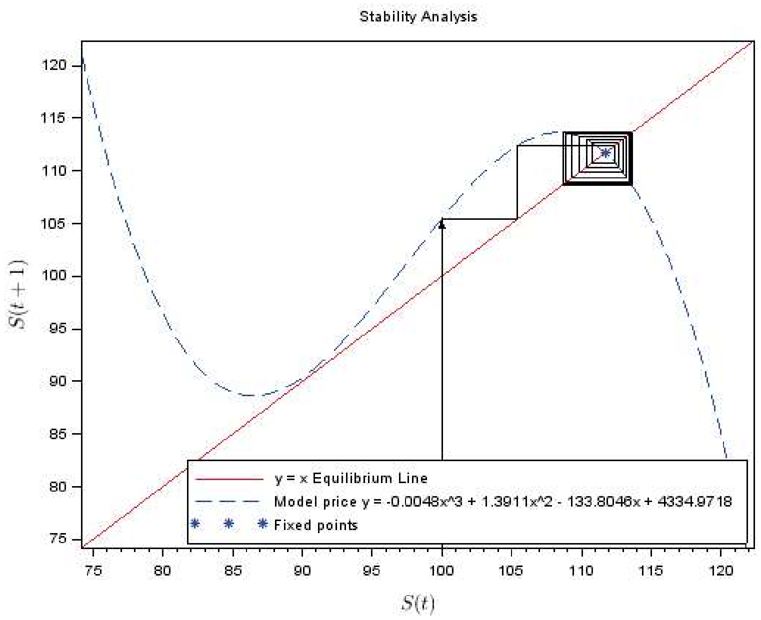
The result given by fitting the whole 17 points of January, 2012 gives one fixed point which is unstable. In fact, the cubic model $f(x) = -0.0048x^3 + 1.3911x^2 - 133.8046x + 4334.9718$ with $R^2 = 0.9667$, presented in Figure 3.7 (a) is obtained. The fixed point is $x^* = 111.7032$ and it is unstable as shown in Table 3.7, since the absolute value of the first derivative of $f(x)$ at x^* is greater than 1. The stability analyse by cobweb graphing in Figure 3.7 (b) showing that the solution whose initial value $x_0 = 100$ oscillates around the fixed point $x^* = 111.7032$.

Table 3.7 Results from cubic model fit and stability analysis of the price in January, 2012.

Model	$f(x) = -0.0048x^3 + 1.3911x^2 - 133.8046x + 4334.9718$
R-squared	$R^2 = 0.9667$
The fixed points	$x^* = 111.7032$
Stability testing	$\lvert 3a(x^*)^2 + 2bx^* + c \rvert = 1.2011 > 1$
Result	x^* is unstable



(a)



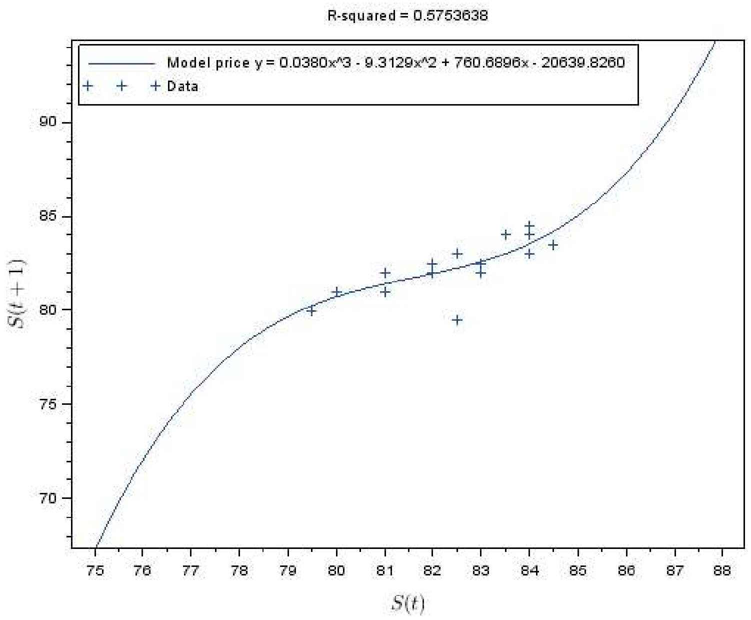
(b)

Figure 3.7 The rubber price of January, 2012 fitted by cubic model (a) and the cobweb behavior with initial value $x_0 = 100$ (b).

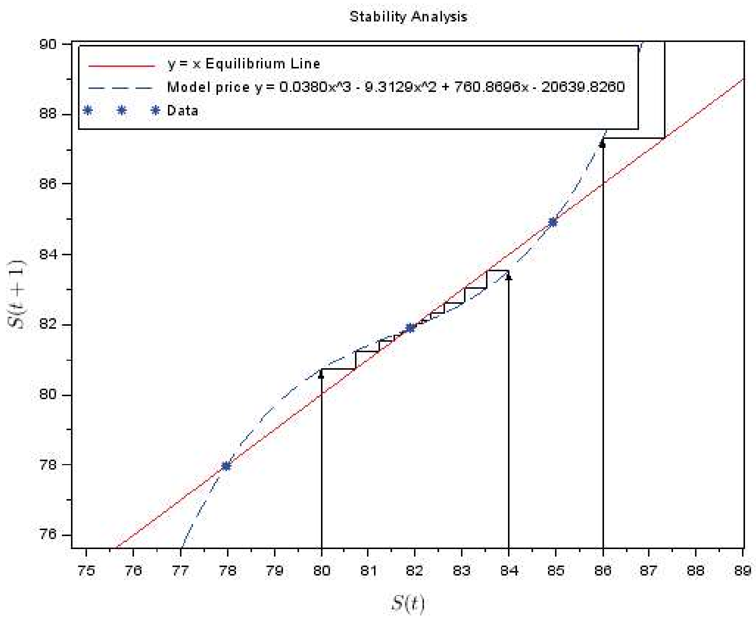
The result given by fitting the whole 17 points of March, 2008 gives three fixed points with one is stable and the other two are not. In fact, the cubic model $f(x) = 0.0380x^3 - 9.3129x^2 + 760.6896x - 20639.826$ with $R^2 = 0.5754$, illustrated in Figure 3.8 (a) is obtained. The fixed points in this case are $x_1^* = 77.9763, x_2^* = 81.8932$ and $x_3^* = 84.9505$. The fixed point $x_2^* = 81.8932$ is stable as shown in Table 3.8, since the absolute value of the first derivative of $f(x)$ at x_2^* is less than 1. While the fixed points $x_1^* = 77.9763$ and $x_3^* = 84.9505$ are unstable as also shown in Table 3.8, since the absolute values of the first derivative of $f(x)$ at x_1^* and x_3^* are greater than 1. The stability analyse by cobweb graphing in Figure 3.8 (b) showing that the solution whose initial values $x_1^* < x_0 < x_3^*$ approaches the fixed point x_2^*. Also in Figure 3.8 (b) showing that the solutions whose initial values no matter smaller or larger than x_1^* and x_3^* move away from the fixed points x_1^* and x_3^*.

Table 3.8 Results from cubic model fit and stability analysis of the price in March, 2008.

Model	$f(x) = 0.0380x^3 - 9.3129x^2 + 760.6896x - 20639.8260$
R-squared	$R^2 = 0.5754$
The fixed points	$x_1^* = 77.9763$, $x_2^* = 81.8932$ and $x_3^* = 84.9505$
Stability testing	$\lvert 3a(x_1^*)^2 + 2bx_1^* + c\rvert = 2.0394 > 1$, $\lvert 3a(x_2^*)^2 + 2bx_2^* + c\rvert = 0.5442 < 1$ and $\lvert 3a(x_3^*)^2 + 2bx_3^* + c\rvert = 1.8110 > 1$
Result	x_2^* is stable while x_1^* and x_3^* are unstable



(a)



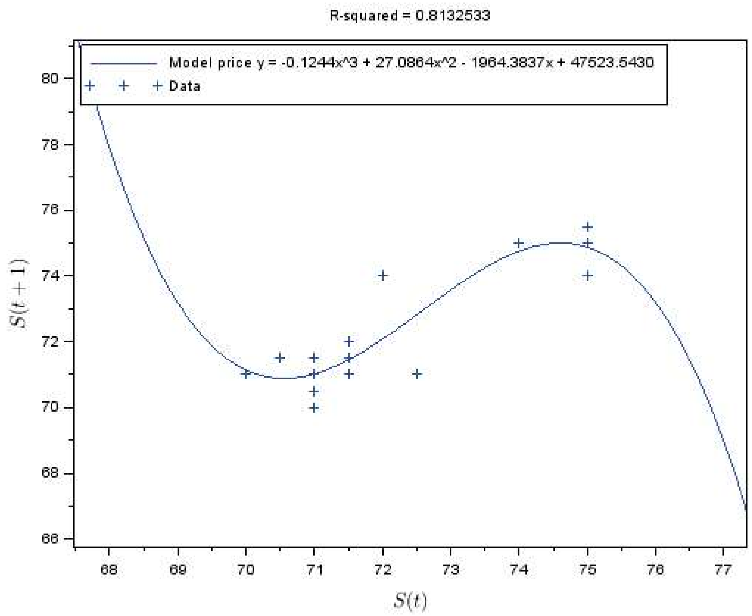
(b)

Figure 3.8 The rubber price of March, 2008 fitted by cubic model (a) and the cobweb behavior with initial values $x_0 = 77$, $x_0 = 80$, $x_0 = 84$ and $x_0 = 86$ (b).

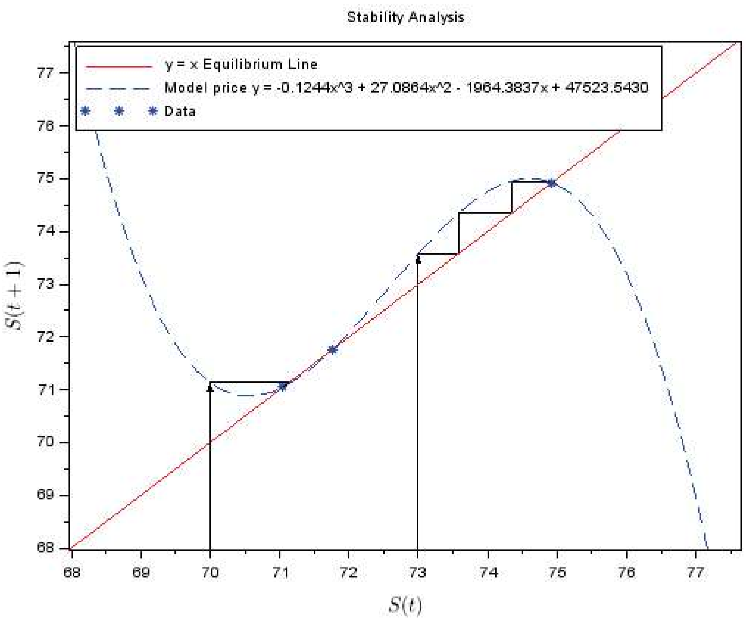
The result given by fitting the whole 18 points of March 2007 gives three fixed points with two are stable and the other is not. In fact, the cubic model $f(x) = -0.1244x^3 + 27.0864x^2 - 1964.3837x + 47523.5430$ with $R^2 = 0.8133$, presented in Figure 3.9 (a) is obtained. The fixed points in this case are $x_1^* = 71.0500$, $x_2^* = 71.7656$ and $x_3^* = 74.9224$. The fixed point $x_2^* = 71.7656$ is unstable as shown in Table 3.9, since the absolute value of the first derivative of $f(x)$ at x_2^* is greater than 1. While the fixed points $x_1^* = 71.0500$ and $x_3^* = 74.9224$ are stable as also shown in Table 3.9, since the absolute values of the first derivative of $f(x)$ at x_1^* and x_3^* are less than 1. The stability analyse by cobweb graphing in Figure 3.9 (b) showing that the solution whose initial values either smaller or larger than x_2^* diverges from the fixed point x_2^*. Also in Figure 3.9 (b) showing that the solutions whose initial values $x_0 = 70$ and $x_0 = 73$ approach x_1^* and x_3^*, respectively.

Table 3.9 Results from cubic model fit and stability analysis of the price in March, 2007.

Model	$f(x) = -0.1244x^3 + 27.0864x^2 - 1964.3837x + 47523.5430$
R-squared	$R^2 = 0.8133$
The fixed points	$x_1^* = 71.0500$, $x_2^* = 71.7656$ and $x_3^* = 74.9224$
Stability testing	$\|3a(x_1^*)^2 + 2bx_1^* + c\| = 0.6553 < 1$, $\|3a(x_2^*)^2 + 2bx_2^* + c\| = 1.2810 > 1$ and $\|3a(x_3^*)^2 + 2bx_3^* + c\| = 0.5207 < 1$
Result	x_1^* and x_3^* are stable while x_2^* is unstable



(a)



(b)

Figure 3.9 The rubber price of March, 2007 fitted by cubic model (a) and the cobweb behavior with initial values $x_0 = 70$ and $x_0 = 73$ (b).

3.2.3 Note on stability of fixed points

The implementation of this results is simple and useful. No matter the equilibrium price stable or not, a farmer can get benefit from it. For the case that the equilibrium price is stable, it can either be monotonic-step (Figure 2.1) or oscillating (Figure 2.3) convergence.

In the case of the equilibrium price is stable and monotone step convergence, the farmer can react corresponding to the current price as follows. If the current price is less than the equilibrium, the farmer should keep their products and wait until the price moves stepwise up to the equilibrium, so that they can earn more. If the current price is greater than the equilibrium, the farmer must immediately sell their products before the price moves stepwise down to the equilibrium. If the equilibrium price is stable and oscillating convergence, it means that the price is swaying above and below and moving toward the equilibrium. In this case, the farmer can sell their products once the price is swaying above the equilibrium.

Similarly, in the case that the equilibrium is unstable, it also can either be monotonic-step (Figure 2.2) or oscillating (Figure 2.3) divergence. In case the equilibrium price is unstable and monotone divergence, the farmer can react corresponding to the current price as following. If the current price is less than the equilibrium, the farmer must immediately sell their products before the price move stepwise down away from the equilibrium. If the current price is greater than the equilibrium, if possible, the farmer can keep their products and wait until the price moving stepwise up to the desirable price. Finally, if the equilibrium price is unstable and oscillating divergence, it means the price swaying above and below and moving away from the equilibrium. In this case a smart and tolerant farmer can make a great profit by selling their product when the price swaying up high above the equilibrium price. Some possible reactions of the farmer corresponding to the behavior of equilibrium price and the current price are shown in Table 3.10.

Table 3.10 Some possible reactions of the farmer corresponding to the equilibrium price x^* and the current price x_0.

	Stable equilibrium		Unstable equilibrium	
	$x_0 < x^*$	$x_0 > x^*$	$x_0 < x^*$	$x_0 > x^*$
Monotonic-step	Keep and wait	Sell immediately	Sell immediately	Keep and wait
Oscillation	Wait until $x_t > x^*, t > 0$	Sell immediately	Wait until $x_t > x^*, t > 0$	Sell immediately or wait for a higher price

Chapter 4

Conclusion and Discussion

This chapter contains summaries of all results and discussion on limitations and possible further works.

4.1 Main results

The two main results of this study are the simulations of the USS price and the stability analysis of the price in a short interval of time. In the simulation part, it was expected that there were jumps in the USS price by observing the changes in the log return. The jump behavior was tested by Bipower variation method (section 3.1.1). The negative value of $\mu_1^{-2}\{Y_\delta\}_t^{[1,1]} - [Y_\delta]_t$ indicating that there were jumps in the price. This means that the model with jump could be applied and would hopefully give a better simulation. In order to verify this, 10000 trajectories of USS price were simulated by both the continuous model (3.5) and the model with jump (3.6). Our conjecture is confirmed by the result in Table 3.2 showing that the model with jump gave a better result in term of a smaller ARPE. In fact, the model with jump gave ARPE 47.54% while the continuous model 54.78%. These results were obtained with parameters calculated by formulae (2.8) - (2.9) and (2.10) - (2.13) for the continuous model and the model with jump, respectively. One can see that the ARPE 47.54% is still large. In order to get a better estimation in term of a smaller ARPE, the Brute-force method is applied in searching for appropriate parameters. The new parameters give a better result for both the continuous model and the model with jump and still the model with jump gives a better result as shown in Table 3.2.

For the stability analysis of the USS price, it was shown that the number of fixed points and their stability behaviors varied by the model fitting and the price as shown in section 3.2.2. By considering the stability of the fixed point and the current price, a farmer can make a decision of whether to sell their products immediately or to keep and wait until it reaches his desirable price. Moreover, no matter whether the fixed point is stable or not a decision with a financial sense can be made as described in section 3.2.3.

4.2 Limitations and further works

In fitting model to the price, the result can be improved by parameters obtained by the Brute-force method. In this study, the parameters used in the models are all constants. However, one can notice that the fluctuation in the log return indicating a non-constant volatility. In fact, the volatility could possibly be a function of time or a stochastic process. Therefore, it is also interesting to consider a model fitting with constant and non-constant parameters.

In the stability analysis, some short intervals of data are considered and fitted with polynomials of degree at most three. However, a question of fitting with polynomials of degree greater than three could arise. Therefore, it is also interesting to try with a higher degree of polynomial. In that case, there would be more equilibrium points to consider and definitely more parameters to estimate.

Finally, concerning the stability of the equilibrium price in the case that it is expecting the price to increase and suggesting the farmer to keep their product and wait for a higher price, it has not been suggested yet that for how long (days) the farmer should wait for. This problem could possibly be solved by watching closely at the price in the real market. However, it should also be noted here that, in most cases of farmers in a developing country like Thailand, they have to make a daily living by selling the product everyday. In this case, there would be some small groups of farmers benefit from the results of this study.

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Appendix

ภาควิชาคณิตศาสตร์และวิทยาการคอมพิวเตอร์

คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย

ขอมอบเกียรติบัตรเพื่อแสดงว่า

จักรพงศ์ โต๊ะหมูด

ได้นำเสนอผลงานและเข้าร่วม

การประชุมวิชาการทางคณิตศาสตร์ประจำปี 2559 ครั้งที่ 21 (AMM 2016)

การประชุมวิชาการคณิตศาสตร์บริสุทธิ์และประยุกต์ประจำปี 2559 (APAM 2016)

ณ คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย

ระหว่างวันที่ 23 – 25 พฤษภาคม 2559

(ศาสตราจารย์ ดร.กฤษณะ เนียมมณี)
ประธานคณะกรรมการจัดการประชุม

(ศาสตราจารย์ ดร.ชิดชนก เหลือสินทรัพย์)
หัวหน้าภาควิชาคณิตศาสตร์และวิทยาการคอมพิวเตอร์

Thailand Natural Rubber Price Simulation: Continuous vs. Jumps Behaviors

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Abstract

Natural rubber is one of the most important agricultural products of Thailand. Since the movement of the natural rubber price is similar to the stock price, it is interesting to study its behavior. Most of the work considered prices as continuous processes which are modeled based on Brownian motion which has continuous sample paths. However, many small and large changes of the price observed by eyes brought to the question of jumps. This study aims to simulate the Unsmoked Sheet Rubber (USS) price in Hat Yai market starting from January 3, 2007 to February 27, 2015 with two models. One is when the price is assumed to be continuous, the other is when it is assumed to have jumps. The results show that the simulation obtained by the continuous model provided an approximately better fit than the model with jumps. It indicated that those large changes observed does not affect the continuous behavior of the studied USS price.

Mathematics Subject Classification: 58J65, 91B25, 91B70

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1 Introduction

Natural rubber is one of the most important agricultural products of Thailand. In fact, Thailand is the world's largest natural rubber export taking turns with Indonesia and Vietnam (Workman, 2015). Rubber plantations are originally most planted in the southern of Thailand. However, they are wildly planted across the country since 1961 according to the government promoting through special policies and programs. The rubber smallholders collect the rubber latex and either sell the latex or rubber sheets to the agents or companies.

The behavior of rubber price is of interest because rubber is one of the important commodities product of Thailand. Since the movement of the natural rubber price is similar to the stock price it is interesting to study its behavior. There are many models in economics and finance that can describe the behavior of the stock price. The continuous model is wildly used for describing the behavior of asset prices which are assumed to be continuous. In this case the model is driven by Brownian motion with constants drift and volatility. However, a shortcoming of this model is that it does not consider the random jumps which can occur any time. The model with jumps is supposed to improve the continuous model in describing the price with jumps. In other word, it is assumed that the behavior of asset prices are not purely continuous. There are many researches which studied the results given by the continuous model (Black and Scholes, 1973; Klein, 1996; Khaled and Samia, 2010) and the model with jumps (Merton, 1976; Kou, 2002; Maekawa et al., 2008; Gondal, 2011; Yan, 2011; El-Khatib and Al-Mdallal, 2012). However, Maekawa and his research team (Maekawa et al., 2008) compared the results between the continuous model and the model with jumps (Kou, 1996) which both models applied with Japanese stock market. The result has shown that the model with jumps outperforms the continuous model. Neupane and Calkins (Neupane and Calkins, 2013) studied the statistical models to capture price volatility of latex type RSS3 in Thailand for the period 2004-2011 where the daily price of latex type RSS3 was modeled by GARCH, GARCH-GJR and EGARCH models. The results showed that the price volatility of RSS3 is strongly persistent and the estimated results are statistically valid. The pricing model for such a jump diffusion model does not have a closed form formula since the market is incomplete (El-Khatib and Al-Mdallal, 2012).

In this study we aim to fit the Thailand rubber price with the continuous pricing model and the model with jump where the parameters are estimated from the historical data and also to compare the results which are obtained by both models.

2 Presentation of the data

We consider the historical data of Unsmoked Sheet Rubber (USS) price in Hat Yai market which is obtained from the webpage of The Thai Rubber Association (The Thai rubber association, 2015) and the data is starting from Jan 3, 2007 to Feb 27, 2015 (see Fig. 1) which is 1584 observations of the USS price. During this period, there is 1584 daily prices which are observed in the official trade day which is around 7 years. The maximum price and minimum price in this period are 186 and 30 baht/kg respectively. In general the return is the difference between the prices of two consecutive days. However, with some nice properties the log return R_i calculated by the difference between the log of today price S_{t_i} and the log of yesterday price S_{t_i-1}:

$$R_i = \log\left(S_{t_i}\right) - \log\left(S_{t_i-1}\right) = \log\left(\frac{S_{t_i}}{S_{t_i-1}}\right), i = 1, 2, 3, \ldots, n \tag{1}$$

is considered instead. The daily log return of the USS price is shown in figure 2. In this figure, the small (dots) and large (stars) changes of the log returns observed by eyes brought to the question of jumps.

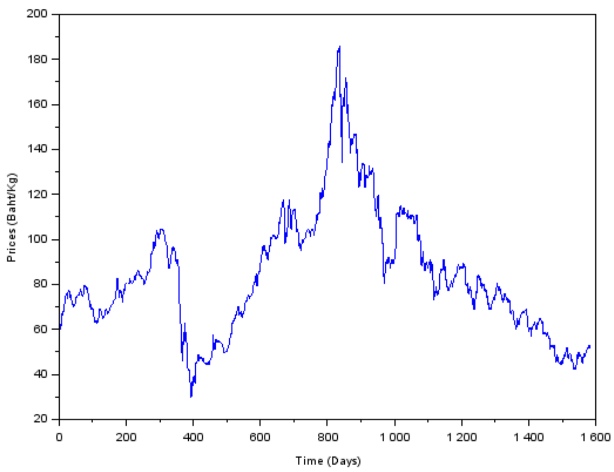

Figure 1: The historical data of USS price (Baht/Kg) start from Jan 3, 2007 to Feb 27, 2015.

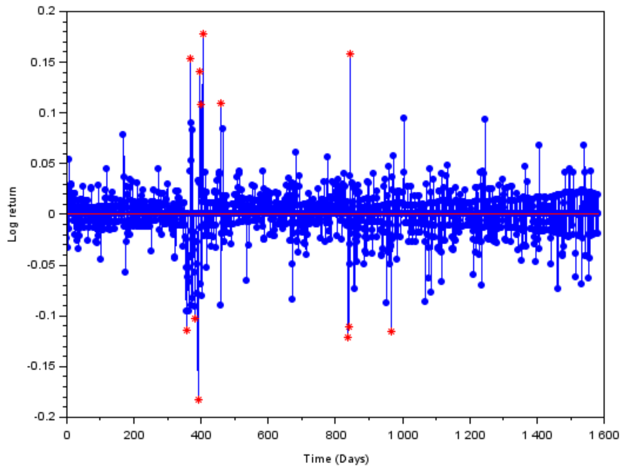

Figure 2: The daily log returns of the USS price.

3 The models

The behavior of the USS price will be considered as two folds. One is that it has a continuous sample path so that it is assumed to behave as similar as the price of the underlying asset of Black-Scholes model (Black and Scholes, 1973):

$$dS_t = \mu S_t dt + \sigma S_t dB_t. \tag{2}$$

The other is that it has jumps so that it is assumed to behave as similar as the price of the underlying asset of Merton Jump Diffusion (MJD) model:

$$dS_t = \mu S_t dt + \sigma S_t dB_t + \left(y_t - 1\right) S_t dN_t. \tag{3}$$

In model (2) the price is driven by Brownian motion B_t where the constants μ and σ are drift and volatility respectively. While the model (3) can be seen as if it is an improvement of model (2) proposed by Merton (Merton, 1976) by adding an independently and identically jump part which has Poisson distribution. Suppose that in a small interval Δt the asset price jumps from S_t to $y_t S_t$. We call y_t an absolute price jump size. Then the relative price jump size is

$$\frac{dS_t}{S_t} = \frac{y_t S_t - S_t}{S_t} = y_t - 1$$

where y_t is assumed to be a nonnegative random variable drawn from lognormal distribution which is $\ln(y_t)$ having independent and identical normal distribution with mean α and variance δ^2. Then, Merton introduced the new equation for describing the behavior of asset price with jump which is driven by the Brownian motion B_t, the Poisson process N_t with constants drift μ and volatility σ in Eq. (3).

To receive the models of asset price from both cases, we need the following proposition (Cont and Tankov, 2004).

Proposition 3.1. (Itô's formula for jump diffusion processes)

Let X be a diffusion process with jumps, defined as the sum of a drift term, a Brownian stochastic integral and a compound Poisson process

$$X_t = X_0 + \int_0^t a_s ds + \int_0^t b_s dB_s + \sum_{i=1}^{N_t} \Delta X_i$$

where a_t and b_t are continuous non anticipating processes with

$$E\left[\int_0^T b_t dt\right] < \infty .$$

Then, for any $C^{1,2}$ function, $f:[0,T]\times\mathbb{R}\to\mathbb{R}$, the process $Y_t = f(t, X_t)$ can be represented as

$$f(t,X_t) - f(0,X_0) = \int_0^t \left[\frac{\partial f}{\partial s}(s,X_s) + \frac{\partial f}{\partial X}(s,X_s)a_s\right]ds$$

$$+\frac{1}{2}\int_0^t \left[\frac{\partial^2 f}{\partial X^2}(s,X_s)b_s^2\right]ds + \int_0^t \left[\frac{\partial f}{\partial X}(s,X_s)b_s\right]dB_s$$

$$+\sum_{\{i\geq 1, T_i\leq t\}} \left[f\left(X_{T_i^-} + \Delta X_i\right) - f\left(X_{T_i^-}\right)\right]. \qquad \text{(a)}$$

Proof. [See Cont and Tankov, 2004]

Now we will solve (3) by applying proposition 3.1 with $f\left(t,\mathrm{S}_t\right)=\ln\left(\mathrm{S}_t\right)$ by (a). When the parameters μ and σ are constant and ω is fixed then

$$a_{\mathrm{t}}=\mu S_t\,,\qquad b_{\mathrm{t}}=\sigma \mathrm{S}_{\mathrm{t}}\,,\qquad \Delta X_i=\left(y_t-1\right)S_t$$

$$\frac{\partial f}{\partial t}\left(t,S_t\right)=0\,,\quad \frac{\partial f}{\partial S_t}\left(t,S_t\right)=\frac{1}{\mathrm{S}_{\mathrm{t}}}\,,\quad \frac{\partial^2 f}{\partial S_t^2}=-\frac{1}{\mathrm{S}_{\mathrm{t}}^2}\,.$$

We substitute all into (a)

$$\ln\left(S_t\right)-\ln\left(S_0\right)=\int_0^t\left[0+\frac{1}{S_s}\mu S_s\right]ds+\frac{1}{2}\int_0^t\sigma^2 S_s^2\left(-\frac{1}{S_s^2}\right)ds$$

$$+\int_0^t\left[\sigma S_s\left(\frac{1}{S_s}\right)\right]dB_s+\sum_{i=1}^{N_t}\left[\ln\left(S_{t^-}+\left(y_t-1\right)S_{t^-}\right)-\ln\left(S_{t^-}\right)\right]$$

$$\ln\left(\frac{S_t}{S_0}\right)=\int_0^t\mu ds-\frac{1}{2}\int_0^t\sigma^2 ds+\int_0^t\sigma dB_s+\sum_{i=1}^{N_t}\left[\ln\left(\frac{S_{t^-}+\left(y_t-1\right)S_{t^-}}{S_{t^-}}\right)\right]$$

$$\ln\left(\frac{S_t}{S_0}\right)=\mu(\mathrm{t}-0)-\frac{1}{2}\sigma^2(\mathrm{t}-0)+\sigma\left(B_t-B_0\right)+\sum_{i=1}^{N_t}\left[\ln\left(y_t\right)\right]$$

$$\ln\left(\frac{S_t}{S_0}\right)=\left(\mu-\frac{1}{2}\sigma^2\right)\mathrm{t}+\sigma B_t+\sum_{i=1}^{N_t}\left[\ln\left(y_t\right)\right]$$

$$\exp\left(\ln\left(\frac{S_t}{S_0}\right)\right)=\exp\left(\left(\mu-\frac{1}{2}\sigma^2\right)\mathrm{t}+\sigma B_t+\sum_{i=1}^{N_t}\left[\ln\left(y_t\right)\right]\right)$$

$$S_t=S_0\exp\left\{\left(\mu-\frac{1}{2}\sigma^2\right)\mathrm{t}+\sigma B_t+\sum_{i=1}^{N_t}\left[\ln\left(y_t\right)\right]\right\}.$$

Hence,

$$S_t=S_0\exp\left\{\left(\mu-\frac{1}{2}\sigma^2\right)\mathrm{t}+\sigma B_t+\sum_{i=1}^{N_t}Y_i\right\} \tag{4}$$

where μ is again the drift, σ is again the volatility, N_t is the counting process which is a Poisson process with intensity λ. The $\sum_{i=1}^{N_t} Y_i$ represents the jump part of the process.

Note that $\lambda = 0$ means that there are no any jumps occur in the process and the solution in Eq. (4) is reduced to be the same as the solution of Eq. (2)

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma B_t\right) \tag{5}$$

Eq. (5) is known as the Geometric Brownian Motion (GBM) and wildly used in financial mathematics.

4 Simulations

In order to discretize eq. (4) and (5), assume that we have a fixed set of date $0 = t_0 < t_1 = t_0 + \Delta t < \ldots < t_n = t_{n-1} + \Delta t = T$ with step time Δt. Let us first consider the continuous model (5) with constants μ and σ. For the today price at $t = t_i$, the discretization version of (5) is

$$S_{t_i} = S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t_i + \sigma B_{t_i}\right\}$$

and for the price of the next day $t = t_i + \Delta t$ is

$$S_{t_i+\Delta t} = S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)\left(t_i + \Delta t\right) + \sigma B_{t_i+\Delta t}\right\}.$$

Then, the return can be obtained by

$$\frac{S_{t_i+\Delta t}}{S_{t_i}} = \frac{S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)\left(t_i + \Delta t\right) + \sigma B_{t_i+\Delta t}\right\}}{S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t_i + \sigma B_{t_i}\right\}}.$$

Then we have

$$S_{t_i+\Delta t} = S_{t_i} \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta B_{t_i+\Delta t}\right\} \tag{6}$$

where $\Delta B_{t_i+\Delta t} = B_{t_i+\Delta t} - B_{t_i}$. By the properties of Brownian motion (independent and identically increment) $\Delta B_{t_i+\Delta t} = B_{t_i+\Delta t} - B_{t_i} \overset{d}{=} B_{t_i+\Delta t-t_i} = B_{\Delta t} \overset{d}{=} \sqrt{\Delta t} B_1$ where B_1 is standard normal distribution $N(0,1)$.

For the model with jumps, we also discretize the model in Eq. (4) and use similar computation as above. Then, we also have the today price at $t = t_i$

$$S_{t_i} = S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right) t_i + \sigma B_{t_i} + \sum_{i=1}^{N_{t_i}} Y_i\right\}$$

and the next day price at $t = t_i + \Delta t$ is

$$S_{t_i+\Delta t} = S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)(t_i + \Delta t) + \sigma B_{t_i+\Delta t} + \sum_{i=1}^{N_{t_i+\Delta t}} Y_i\right\}.$$

Then also the return can be obtained by

$$\frac{S_{t_i+\Delta t}}{S_{t_i}} = \frac{S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)(t_i + \Delta t) + \sigma B_{t_i+\Delta t} + \sum_{i=1}^{N_{t_i+\Delta t}} Y_i\right\}}{S_0 \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right) t_i + \sigma B_{t_i} + \sum_{i=1}^{N_{t_i}} Y_i\right\}}.$$

We again apply the mathematical manipulation and the property of exponential function. Then

$$\frac{S_{t_i+\Delta t}}{S_{t_i}} = \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t\right\} \exp\left\{\sigma B_{t_i+\Delta t} - \sigma B_{t_i}\right\} \exp\left\{\sum_{i=1}^{N_{t_i+\Delta t}} Y_i - \sum_{i=1}^{N_{t_i}} Y_i\right\}$$

.

As $\sum_{i=1}^{N_{t_i+\Delta t}} Y_i - \sum_{i=1}^{N_{t_i}} Y_i = \sum_{i=N_{t_i}+1}^{N_{t_i+\Delta t}} Y_i$, then we get

$$S_{t_i+\Delta t} = S_{t_i} \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta B_{t_i+\Delta t} + \sum_{i=N_{t_i}+1}^{N_{t_i+\Delta t}} Y_i\right\} \tag{7}$$

where $N_{t_i} \subset N_{t_i+\Delta t}$ and $N_{t_i+\Delta t}$ has Poisson distribution with parameter $\lambda \Delta t$.

We will simulate the USS price by using Eq. (6) and Eq. (7) and all parameters will be estimate from the historical data of the USS price. To estimate the parameters of the continuous model we consider from the Eq. (6). The first assumption for this model is log return required to be Gaussian distribution this means

$$\frac{S_{t_i+\Delta t}}{S_{t_i}} = \exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta B_{t_i+\Delta t}\right\}.$$

Take the logarithm both sides and we get

$$\ln\left(\frac{S_{t_i+\Delta t}}{S_{t_i}}\right) = \left\{\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta B_{t_i+\Delta t}\right\}.$$

Let $\mathrm{R} = \ln\left(\frac{S_{t_i+\Delta t}}{S_{t_i}}\right)$ represent the log return of asset price as in Eq. (1), then we apply the property of expectation and Brownian motion and we have

$$E\left[R\right] = E\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta B_{t_i+\Delta t}\right] = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t.$$

If there are n observations of the log return ($R_1, \ldots, R_n$) by the law of large number the expectation of the log return can be estimated as

$$E\left[R\right] \approx \frac{1}{n}\sum_{i=1}^{n} R_i.$$

Therefore, parameter μ can be estimated by

$$\frac{1}{n}\sum_{i=1}^{n} R_i \approx \left(\mu - \frac{\sigma^2}{2}\right)\Delta t$$

$$\mu \approx \frac{\frac{1}{n}\sum_{i=1}^{n} R_i}{\Delta t} + \frac{\sigma^2}{2}. \tag{8}$$

For estimating σ, we consider the variance of the log return R with the property of variance and Brownian motion and then we obtain

$$Var\left[R\right] = Var\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta B_{t_i+\Delta t}\right]$$

$$= \Delta t \sigma^2.$$

The usual estimation for the variance is

$$Var\left[R\right] \approx \frac{1}{n-1}\sum_{i=1}^{n-1}\left(R_i - \overline{R}\right)^2 \; ; \; \overline{R} = \frac{1}{n}\sum_{i=1}^{n} R_i$$ is average value of the log return.

Then,
$$Var\left[R\right] = \Delta t \sigma^2 \approx \frac{1}{n-1}\sum_{i=1}^{n-1}\left(R_i - \overline{R}\right)^2$$

$$\sigma^2 \approx \frac{\frac{1}{n-1}\sum_{i=1}^{n-1}\left(R_i - \overline{R}\right)^2}{\Delta t}$$

$$\sigma \approx \sqrt{\frac{\frac{1}{n-1}\sum_{i=1}^{n-1}\left(R_i - \overline{R}\right)^2}{\Delta t}} \tag{9}$$

where $\sqrt{\frac{1}{n-1}\sum_{i=1}^{n-1}\left(R_i - \overline{R}\right)^2}$ is the standard deviation of the log return.

For estimating all parameters of the model with jumps, we apply the method of moment which is called nonparametric estimation introduced by Johannes (Johannes, 2004) and Valachy (Valachy, 2004) applied this method to the currencies exchange rate from the Central European (CE) region. Recall that the jump sizes are assumed to be normally distributed with mean 0 and variance δ^2. Let us assume that all parameters are constants. The formula for diffusion part of the model is

$$\sigma^2 = \sigma_T^2 - \lambda\delta^2 \tag{10}$$

where σ^2 is the diffusion of the continuous part, σ_T^2 is the total diffusion (or 2^{nd} moment), and δ^2 is the variance of jump sizes. Under the assumption of constant jump intensity, the proposed estimation procedure is as follows:

Estimate parametrically the drift μ

$$E\left[\log S_{t+\Delta t} - \log S_t\right] \approx \frac{1}{n}\sum_{i=1}^{n}\left(\log\left(\frac{S_{t_i+\Delta t}}{S_{t_i}}\right)\right). \tag{11}$$

Estimate λ and δ^2 based on the calculation of moments, the ratio of the 4^{th} and 6^{th} moments will give us the estimate of δ^2. Consequently, the estimate of λ will be

$$\hat{\lambda} \approx \frac{4^{th}\,Moment}{3\left(\delta^2\right)^2}.$$

(12) The particular moments are calculated as follows

$$4^{th}\,Moment \approx \frac{1}{n}\sum_{i=1}^{n}\left(\log\left(\frac{S_{t_i+\Delta t}}{S_{t_i}}\right)\right)^4 = 3\lambda\left(\delta^2\right)^2$$

$$6^{th}\,Moment \approx \frac{1}{n}\sum_{i=1}^{n}\left(\log\left(\frac{S_{t_i+\Delta t}}{S_{t_i}}\right)\right)^6 = 15\lambda\left(\delta^2\right)^3.$$

The estimation of σ can be completely identified by subtracting the 2^{nd} moment estimated nonparametrically from constant volatility, this mean

$$\sigma^2 = 2^{nd}\,Moment - \hat{\lambda}\delta^2$$

where $2^{nd}\,Moment \approx \frac{1}{n}\sum_{i=1}^{n}\left(\log\left(\frac{S_{t_i+\Delta t}}{S_{t_i}}\right)\right)^2.$

In order to compare the results, we calculate the error between the empirical data and simulated data as Maekawa and his research team use in their research work (Maekawa et al., 2008). We use the Average Relative Percentage Error (ARPE) which is defined by

$$ARPE = \frac{1}{M}\sum_{i=1}^{M}\left|\frac{x_i - s_i}{x_i}\right|, \tag{13}$$

where M is the sample size, x_i and s_i are empirical data and simulated data respectively.

5 Result

In this section we show the results which are obtained by the simulation of both models. The data is divided into 6 groups in which each of the first 5 groups has 250 observations that according to the data size of one year trading in the market and the last group contains the rest. All parameters of both models are estimated separately. The Table 1 shows the estimated parameters of both models. We simulate 2000 trajectories of the price in each part with Eq. (6) and Eq. (7) then we use formula (13) for calculating the errors which is compared with the real USS price. The Table 2 shows the errors that we get. Under the same Brownian motion, we can see that most of the errors that we obtain by the simulation with the continuous model give the better estimate than the model with jumps.

Table 1. The estimated parameters

	The continuous model		The model with jumps			
	$\hat{\mu}_c$	$\hat{\sigma}_c$	$\hat{\mu}_d$	$\hat{\sigma}_d$	$\hat{\lambda}$	$\hat{\delta}^2$
Data 1	0.2764	0.2275	0.0011	0.0087	0.2049	0.0273
Data 2	-0.2921	0.5449	-0.0019	0.0041	0.2817	0.0678
Data 3	0.6515	0.2469	0.0027	0.0090	0.2279	0.0293
Data 4	-0.0431	0.3843	-0.0005	0.0115	0.1477	0.0593
Data 5	-0.0400	0.3240	-0.0004	0.0076	0.3317	0.0354
Data 6	-0.2614	0.2946	-0.0013	0.0074	0.5001	0.0257

Table 2. The average errors which are obtained by both models (%)

	The continuous model	The model with jumps
Data 1	14.94	16.07
Data 2	42.10	66.63
Data 3	20.61	39.84
Data 4	28.90	26.92
Data 5	22.02	22.83
Data 6	25.15	47.08

In the Table 1 we see that the estimated volatility $\hat{\sigma}_d$ of the model with jumps is very small. In the real market the volatility should not be small value. The typical value of volatility in the real market should be around 15 % to 60 % (Hull and Options, 2000). For this reason we change the estimation of $\hat{\sigma}_d$ to be according to the real market by applying this simple transformation

$$\hat{\sigma}_d = \left(\frac{b-a}{8}\right)\left(x+4\right)+a$$

where x is the Gaussian random number and a , b are the typical value of volatility in the real market which are 15 % and 60 % respectively. Then, we again simulate as the previous one. The Table 3 shows the new errors that we get after we change the estimation of $\hat{\sigma}_d$. We can see that even we change the value of volatility to be according to the real market the continuous model still provide the better errors.

Table 3. The average errors which are obtained by both models after the new estimation of $\hat{\sigma}_d$ (%)

	The continuous model	The model with jumps
Data 1	13.92	24.25
Data 2	42.04	68.53
Data 3	21.82	40.42
Data 4	27.87	32.17
Data 5	21.63	31.62
Data 6	24.18	51.31

6 Conclusion

In this study we provide the simulation of the Thailand natural rubber price by using the continuous model and the model with jumps. The parameters are estimated from the historical price. The results show that the errors which are obtained by the continuous model is smaller than the model with jumps compare with the real USS price. Since the first spotted large change in the USS price we expected the model with jumps would provide the better error but it is not. This maybe because of the jumps in the USS price are not large enough to affect the continuous behavior of this USS price.

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List of Proceeding

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