Chapter 4

Time Series Analysis of Volatility

In Chapter 3 we examined the time series of daily closing prices of banking shares in Thailand, over the period from January 1994 to December 1999, and we found that the seven series of daily returns were correlated. The objective in this chapter is develop an appropriate statistical model for describing the stochastic volatility of the share prices.

We removed the correlation between the seven series of banking share daily returns by using principal components analysis, giving a new set of seven uncorrelated series. The component with the largest variance (PC1) is approximately the average of the seven share prices, and this component accounted for 67% of the total variance. In contrast, the component with the smallest variance (PC7) is approximately the difference between the BBL and the TFB share prices, and this component accounted for less than 3% of the total variance.

The component with the smallest variance is important because it enables an investor to construct a portfolio based on banking shares with minimal risk, and thus provide protection against economic reversals. The PC7 component comprises one share owned in the BBL and one borrowed in the TFB. Thus if both shares drop by the same amount, the investor does not make any loss. Similarly, the investor does not make any profit when both shares rise in value. However, the investor makes a profit when BBL shares rise and TFB shares decline, and the investor makes a loss when the reverse happens.

In Chapter 3 we saw strong evidence that the variances in the banking share returns increased substantially during the period from 1994 to 1999, and were relatively high from 1996 to 1999. In the present chapter we first model this changing volatility using general autoregressive conditional hereroscedasticity (GARCH) model fitting, and then we apply time series analysis to the volatility series. We do this for both the PC1 and PC7 principal components.

4.1 Comparison of PC1 and PC7

Figure 4.1 shows a graph of an index constructed by summing the share prices in the seven banking institutions. This graph clearly shows the downslide in the banking share index from early 1996 until mid 1998.

Figure 4.2 shows the corresponding graph of an index constructed by subtracting the price of a TFB share from the price of a BBL share. This graph shows a different pattern to the PC1 index, and fluctuates more, although both indexes achieved minimal values near the end of 1998.

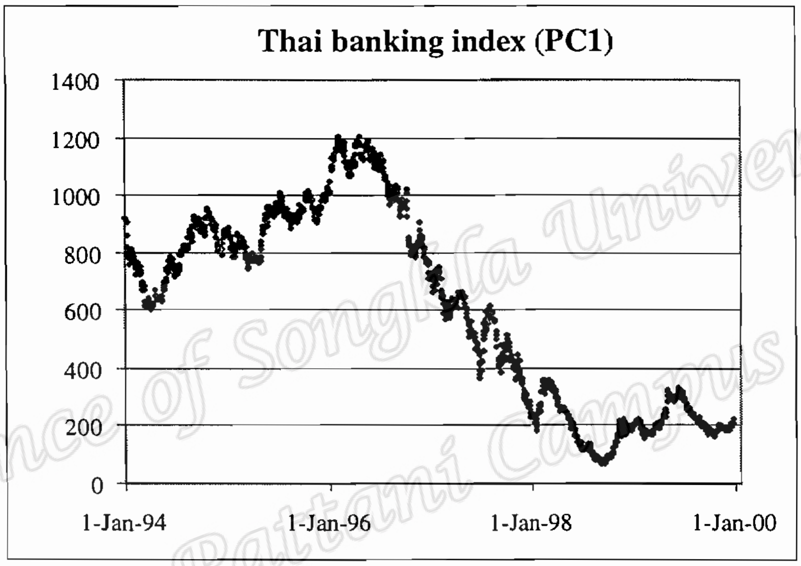

Figure 4.1: Time series of banking share index based on principal component 1

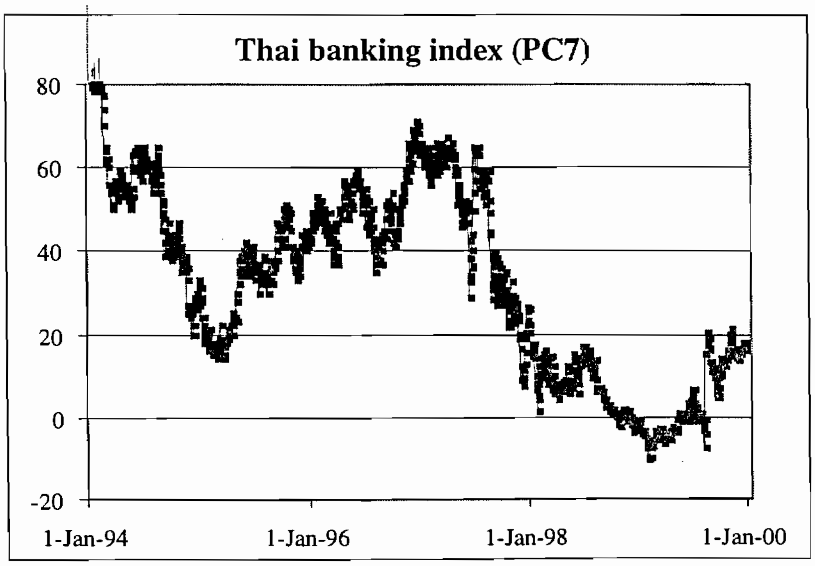

Figure 4.2: Time series of banking share index based on principal component 7

Figure 4.3 shows the relation between the two indexes. The points plotted in Figure 4.3 are clustered according to seven periods, with parallel lines fitted to the points within the groups. The largest cluster corresponds to the three years 1996 to 1999, while the other six clusters each comprise a six-month period in the earlier years. Figure 4.4 shows the centroids of these clusters.

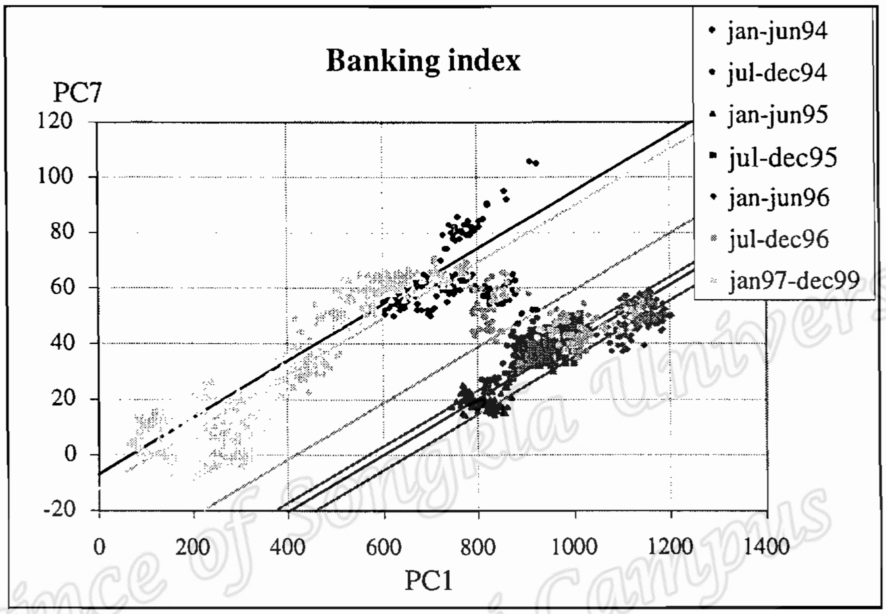

Figure 4.3: Scatter plot of banking share of principal component

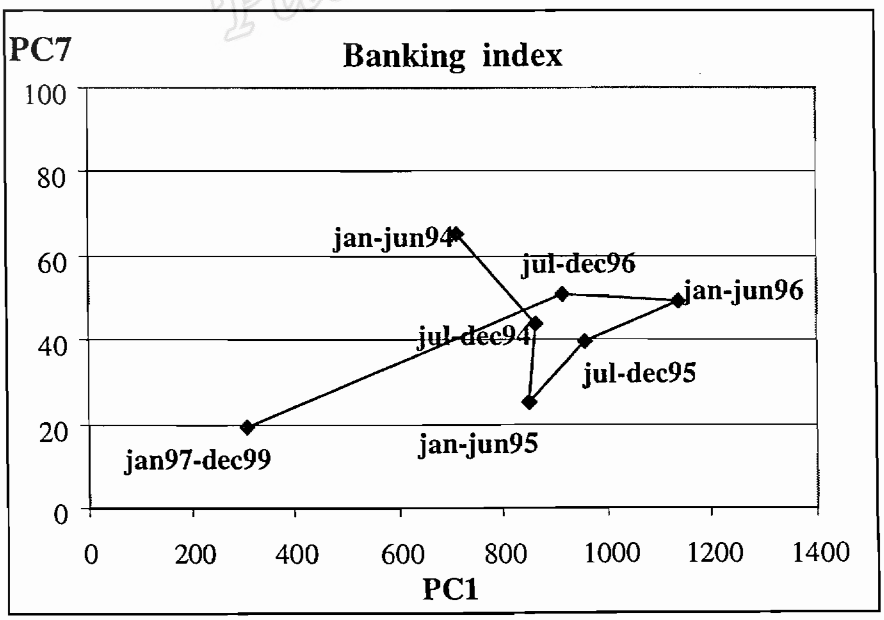

Figure 4.4: Centroids of clusters shown in Figure 4.3

Figure 4.3 shows that the linear relation fits quite well to the points within each cluster. The overall r-squared is 0.79. Figure 4.4 shows how the character of the linear linear relation progressed over the seven periods before reaching a stable situation over the last years.

Figures 4.5 and 4.6 show the daily returns based on the two indexes.

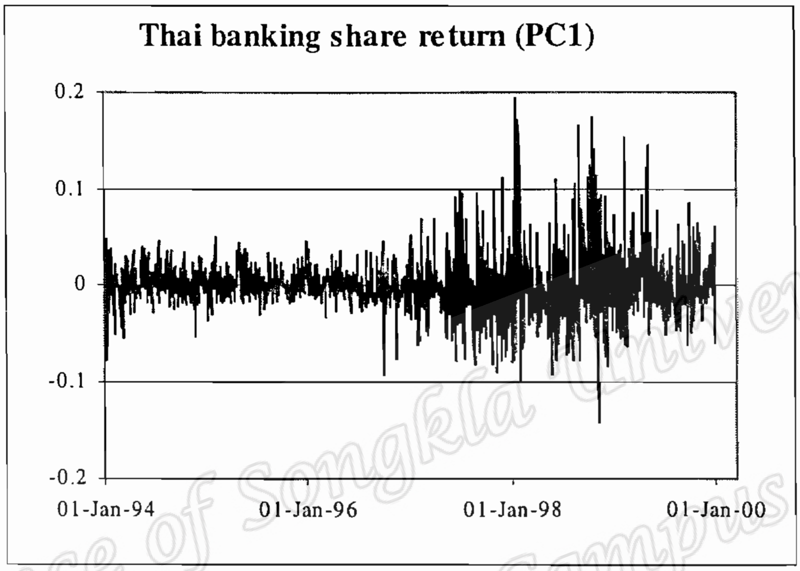

Figure 4.5: Time series of banking share return based on PC1

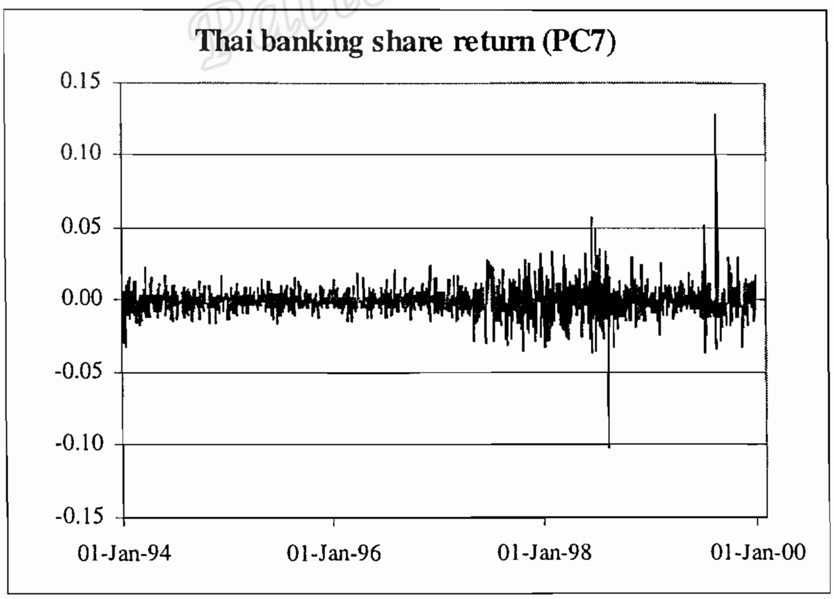

Figure 4.6: Time series of Banking share return based on PC7

These returns are defined as the proportionate movements in each index from the previous trading day. The denominator in each case is the total value of the asset at risk. Thus if the share index price on trading day t is defined as $\sum a_j Y_j$, the return on this index on trading day t is given by $\sum a_j (Y_j - Y_{j-1}) / (\sum |a_j| Y_{j-1})$.

The returns based on both PC1 and PC7 have constant volatility from 1994 to mid 1997, but the volatility increases quite a lot during the period from mid 1997 to 1999.

Figure 4.7 shows the relation between the returns based on PC7 and PC1.

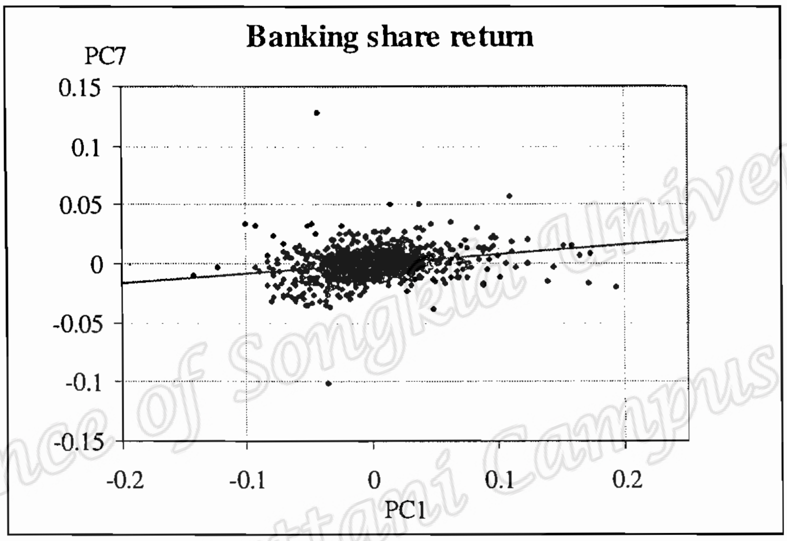

Figure 4.7: Scatter plot showing relation between returns based on PC7 and PC1

The correlation between these returns is 0.054.

Note that there are two large outliers in the graph, one high and one low. The high outlier corresponds to a large jump in the PC7 return, which occurred on the 20th of August 1999. On this day the price of a BBL share remained constant at 61.5, but the price of a TFB share dropped from 62 to 46.25. As a result, the value of the portfolio based on the difference rose from –0.5 to 15.25, an increase of 15.75. The return was thus 15.75/(61.5+62) = 12.75%.

The unusually low outlier in Figure 4.7 occurred on the 11th of August 1998. On this day the price of a BBL share dropped from 31.5 to 27.25 while the price of a TFB share rose from 17.75 to 18.5. As a result, the value of the portfolio dropped by 5, and the corresponding return was –10.15%.

4.2 GARCH (1, 1) model fitting

We now use the GARCH(1,1) model described in Chapter 2 to estimate the volatility series for the returns based on the PC1 and PC7 principal components.

Using the Solver Tool in Microsoft Excel 97 with initial values $\alpha = 0.05$ and $\beta = 0.9$, we obtained the solution $\alpha = 0.059$, $\beta = 0.941$ and $\omega = 0$ (the EWMA model) for the parameters in the volatility series based on the PC1 returns. This series is plotted in Figure 4.8.

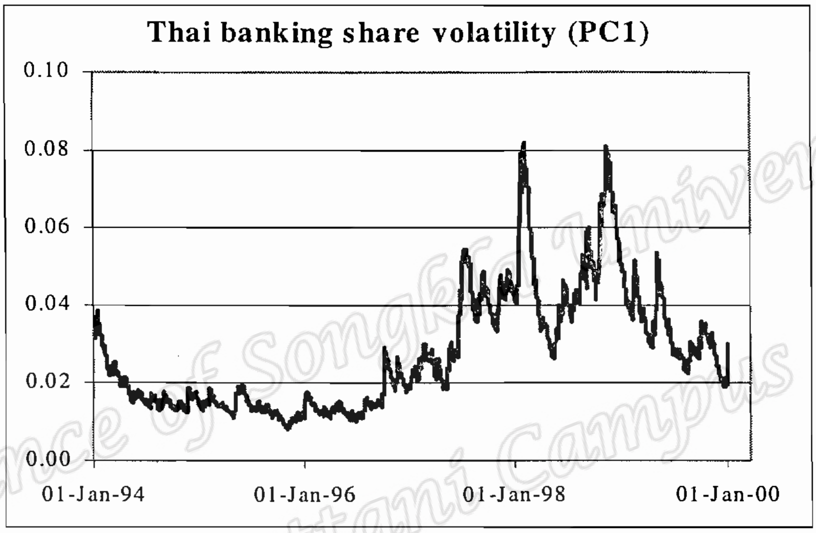

Figure 4.8: The estimated volatility series based on the PC1 returns

This volatility series shows greater variability at higher levels, suggesting the need for a transformation to stabilise this variance. Figure 4.9 shows a plot of the changes in the PC1 volatility from one trading day to the next versus the volatility, where the slope of the least squares fitted line is 0.0048, and the points are spread out above this line. Figure 4.10 shows the same scatter plot after taking natural logarithms of the estimated volatility. In this case the slope of the least squares fitted line is 0.0040, and the points are still spread out above the line.

Ideally, the change in volatility should be independent of its level and the points should be equally spread above and below the line, so taking logarithms may not be the best transformation. Table 4.1 shows the resulting fitted line slopes and corresponding p-values for testing that the slope is 0 (based on the t-test) for various transformations of the volatility series.

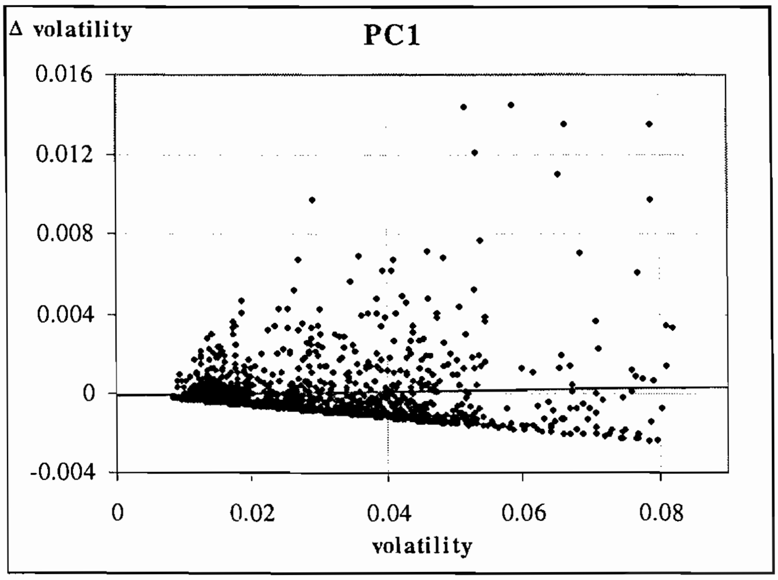

Figure 4.9: Relation between volatility change and volatility level for PC1 return

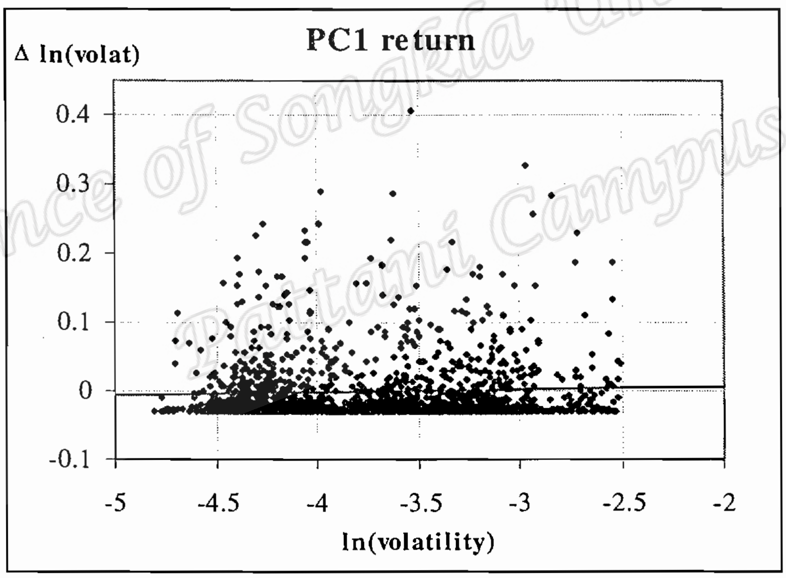

Figure 4.10: Relation between change and level for ln(volatility) of PC1 return

transformation	*slope*	*p-value*
none	0.0048	0.060
square root	0.0041	0.085
cube root	0.0040	0.089
logarithm	0.0040	0.090
reciprocal sq root	0.0045	0.071

Table 4.1: Effect of transformation on slope of line fitted to volatility change versus level for PC1 return

Table 4.1 shows that none of the transformations considered completely removes the relation. However, taking logarithms and cube roots have almost the same effect in minimising the slope. For simplicity, we choose to take logarithms in PC1 and PC7.

Figure 4.11 shows a graph of the natural logarithm of the estimated volatility of PC1. Comparing this series with the one shown in Figure 4.7, the log transformation is effective in stabilising the variance. The graph goes down and up and down again like a roller-coaster.

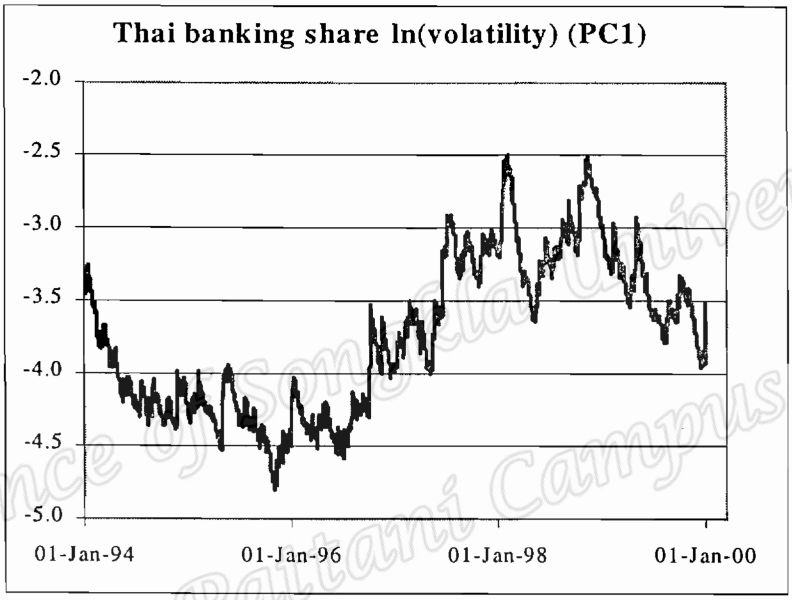

Figure 4.11: The estimated volatility series based on the PC1 returns after a natural logarithm transformation

When this procedure was repeated for the volatility based on the PC7 return, we obtained the solution $\alpha = 0.065$, $\beta = 0.935$ and $\omega = 0$. Figure 4.12 shows this volatility series again after taking natural logarithms. Like the graph of the PC1 volatility series, this series shows a roller-coaster effect, but there are some large in the last two years.

Figure 4.13 shows a scatter plot of the relation between the daily volatility on the PC1 and PC7 portfolio returns. The two series are positively related. The slope of the fitted line is 0.63, and the correlation coefficient is 0.57.

In the next section we model these volatility series using time series analysis.

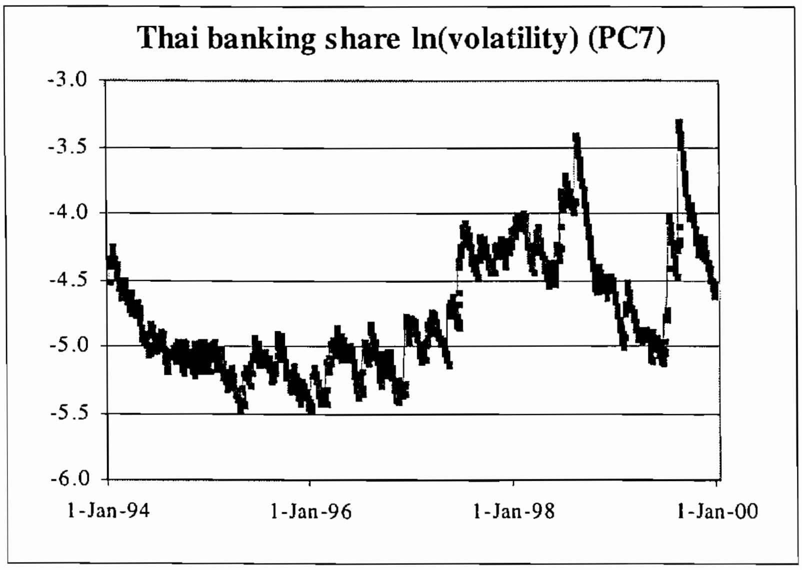

Figure 4.12: The estimated volatility series based on the PC7 returns after a natural logarithm transformation

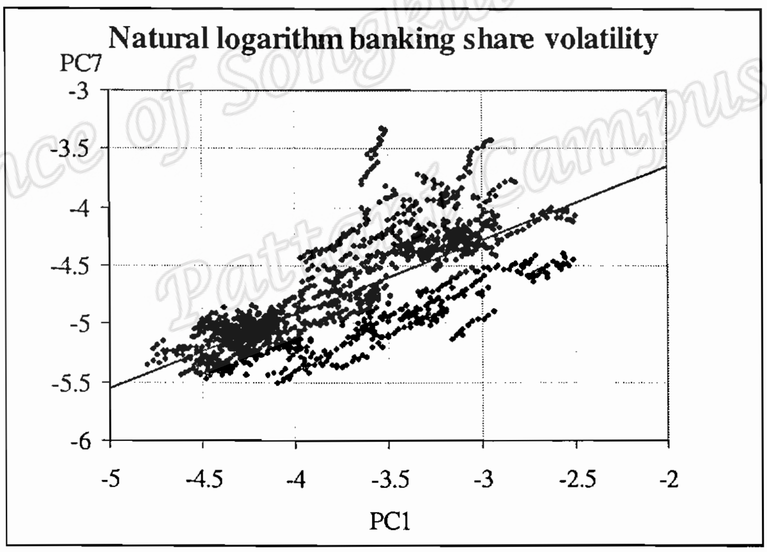

Figure 4.13: Scatter plot of ln(volatility) of PC7 versus ln(volatility) of PC1

4.3 Time series analysis of volatility

Following the method for time series analysis described in Chapter 2, Figure 4.13 shows the result for the natural logarithm of the volatility series based on PC1. This model contains a single harmonic component at the lowest frequency, that is, with period equal to the length of the series, and there is no other trend. The residual series is fitted well by a first-order auto-regression with parameter $a = 0.975$, and a small moving average component at lag equal to 1 trading day. The residuals satisfy the Ljung-Box test for no serial correlation in the residuals.

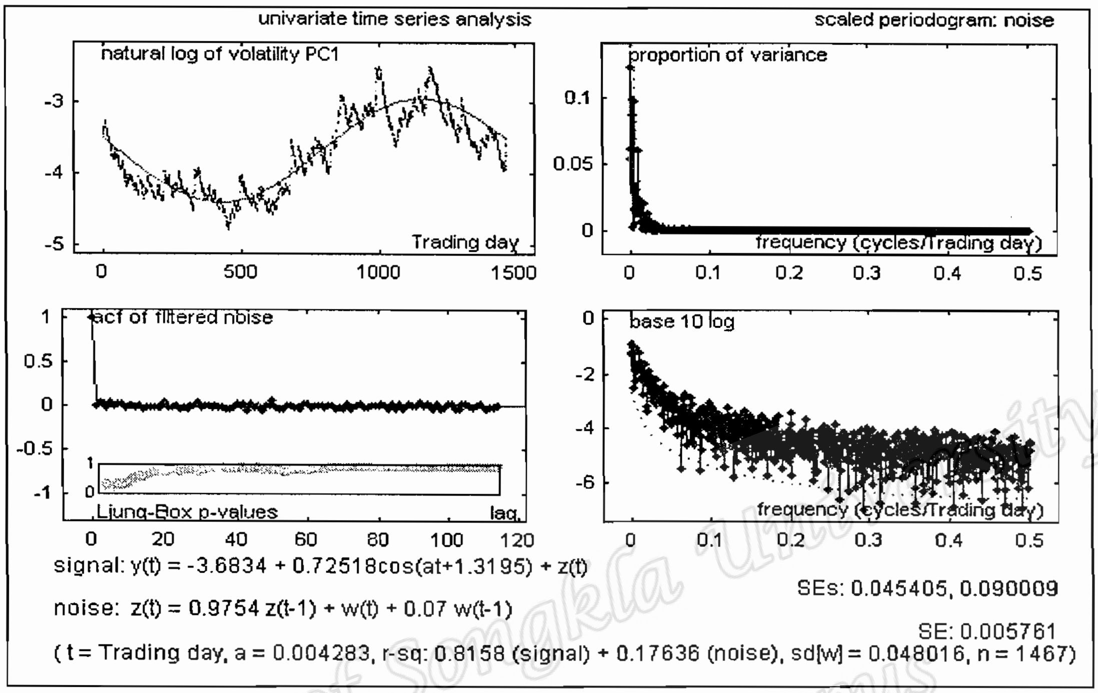

Figure 4.14: Time series analysis of ln(volatility) based on PC1 return

Figure 4.14 show time series analysis of volatility based on the PC1 returns. In this case the fitted autoregressive parameter is 0.9754 and moving average parameter 0.07, the r-squared is 0.816 for the signal and 0.176 for the noise, accounting for a total of more than 99% of the variation in the volatility series.

Figure 4.15 show a similar analysis for the natural logarithm of the volatility series based on the PC7 returns. In this case the fitted autoregressive parameter is 0.989 and the moving average parameter is again 0.07. While the r-squared goodness-of-fit for the signal is much less than for the PC1 volatility (only 0.598), the total r-squared based on the signal and the noise is still high (98.5%).

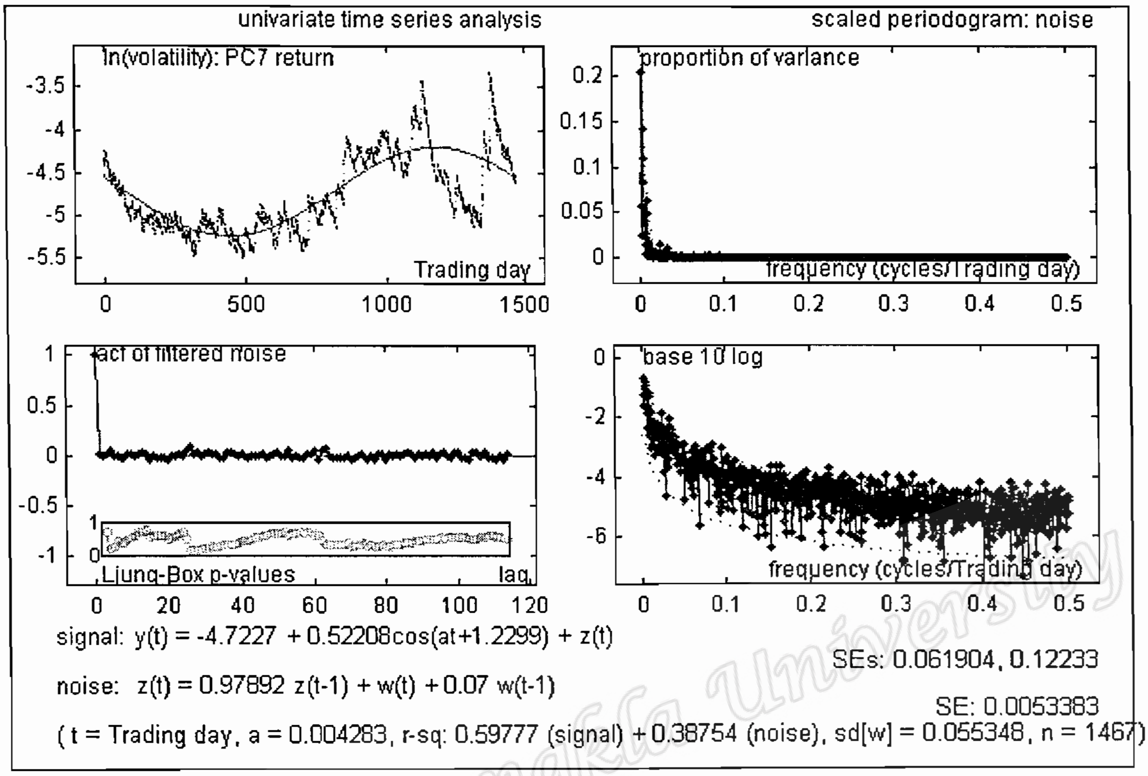

Figure 4.15: Time series analysis of ln(volatility) based on PC7 return