Applications of the Natural Cubic Spline Function to Estimate Financial Volatilities and Time-varying Correlations of the ASEAN-5 Financial Time Series

Jetsada Laipaporn

A Thesis Submitted in Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Research Methodology

Prince of Songkla University

2021

Applications of the Natural Cubic Spline Function to Estimate Financial Volatilities and Time-varying Correlations of the ASEAN-5 Financial Time Series

Jetsada Laipaporn

A Thesis Submitted in Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Research Methodology

Prince of Songkla University

2021

Thesis Title	Applications of the Natural Cubic Spline Function to Estimate Financial Volatilities and Time-varying Correlations of the ASEAN-5 Financial Time Series
Author	Mr. Jetsada Laipaporn
Major Program	Research Methodology

Major Advisor:

Don McNeil
……………………………………………..

(Emeritus Prof. Dr. Don McNeil)

Co-advisor

……………………………………………..

(Asst. Prof. Dr. Phattrawan Tongkumchum)

Examining Committee:

……………………………………Chairperson

(Assoc. Prof. Dr. Apiradee Lim)

Don McNeil
……………………………………Committee

(Emeritus Prof. Dr. Don McNeil)

……………………………………Committee

(Prof. Dr. Muhammad Hisyam Lee)

……………………………………Committee

(Asst. Prof. Dr. Phattrawan Tongkumchum)

……………………………………Committee

(Asst. Prof. Dr. Rattikan Saelim)

The Graduate School, Prince of Songkla University, has approved this thesis as fulfillment of the requirements for the Doctor of Philosophy Degree in Research Methodology.

……………………………………………..

(Prof. Dr. Damrongsak Faroongsarng)

Dean of Graduate School

This is to certify that the work here submitted is the result of the candidate's own investigations. Due acknowledgements have been made of any assistance received.

Don McNeil
……………………..…...............…Signature
(Emeritus Prof. Dr. Don McNeil)
Major Advisor

…………………..……………………... Signature
(Mr. Jetsada Laipaporn)
Candidate

I hereby certify that this work has not been accepted in substance for any degree, and is not being currently submitted in candidature for any degree.

……………….………………………... Signature

(Mr. Jetsada Laipaporn)

Candidate

ชื่อวิทยานิพนธ์ การประยุกต์ใช้ฟังก์ชั่นเส้นโค้งเสมือนพหุนามกำลังสามที่มีเงื่อนไขขอบแบบธรรมชาติเพื่อประมาณการความผันผวนทางการเงินและสหสัมพันธ์ที่เปลี่ยนแปลงตามเวลาของอนุกรมเวลาทางการเงินจากห้าประเทศในภูมิภาคอาเซียน

ผู้เขียน นายเจษฎา ไหลภาภรณ์

สาขาวิชา วิธีวิทยาการวิจัย

ปีการศึกษา 2563

บทคัดย่อ

วิทยานิพนธ์นี้นำเสนอวิธีการในการสำรวจเสถียรภาพและการเชื่อมโยงทางการเงินที่เกิดขึ้นในภูมิภาคอาเซียน ฟังก์ชั่นเส้นโค้งเสมือนพหุนามกำลังสามที่มีเงื่อนไขขอบแบบธรรมชาติและใช้ปมแบบควอนไทล์ ถูกประยุกต์ใช้กับอนุกรมเวลาทางการเงิน ซึ่งประกอบด้วยดัชนีตลาดหลักทรัพย์ อัตราแลกเปลี่ยนเทียบกับดอลลาร์สหรัฐ และอัตราแลกเปลี่ยนตามดัชนีค่าเงิน ในช่วงสองทศวรรษ ตั้งแต่ 1 มกราคม 2544 ถึง 31 ธันวาคม 2563 ของห้าประเทศในอาเซียน ได้แก่ ไทย สิงคโปร์ มาเลเซีย อินโดนีเซีย และฟิลิปปินส์

การศึกษานี้ใช้วิธีการประมาณการค่าความผันผวนทางการเงินของอนุกรมเวลาที่ศึกษาด้วยการประยุกต์ใช้ฟังก์ชั่นเส้นโค้งเสมือนพหุนามกำลังสามที่มีเงื่อนไขขอบแบบธรรมชาติและใช้ปมแบบควอนไทล์จำนวน 22 ปม โดยมีระยะห่างระหว่างปมประมาณหนึ่งปีทำการ ผลการศึกษาแสดงให้เห็นว่าในช่วงที่เกิดวิกฤตการณ์การเงินโลก ความผันผวนทางการเงินที่ประมาณการจากฟังก์ชั่นเส้นโค้งเสมือนพหุนามกำลังสามที่มีเงื่อนไขขอบแบบธรรมชาติมีค่าเพิ่มสูงขึ้นอย่างเห็นได้ชัด สะท้อนให้เห็นความไม่มีเสถียรภาพทางการเงินในช่วงเวลาที่เกิดวิกฤต นอกจากนี้การจำลองสถานการณ์ด้วยวิธีมอนติคาร์โลแสดงให้เห็นว่า ความผันผวนทางการเงินที่ประมาณการจากฟังก์ชั่นเส้นโค้งเสมือนพหุนามกำลังสามที่มีเงื่อนไขขอบแบบธรรมชาติ แสดงให้เห็นรูปแบบของความผันผวนได้ชัดเจนกว่าการปรับค่าความผันผวนจากตัวแบบการ์ช(1,1)ให้เรียบ ตามวิธีการจากการศึกษาในอดีต

ในการสำรวจการเชื่อมโยงทางการเงินในภูมิภาค การศึกษานี้เลือกประมาณการค่าสัมประสิทธ์สหสัมพันธ์ที่เปลี่ยนแปลงตามเวลาของดัชนีตลาดหลักทรัพย์จากห้าประเทศในภูมิภาคอาเซียนด้วยวิธีการใหม่ที่มีพื้นฐานจากแนวคิดความแปรปรวนร่วมทางอ้อมและความผันผวนทางการเงินที่ประมาณการจากฟังก์ชั่นเส้นโค้งเสมือนพหุนามกำลังสามที่มีเงื่อนไขขอบแบบธรรมชาติ ค่าสัมประสิทธ์สหสัมพันธ์ที่

เปลี่ยนแปลงตามเวลาที่ประมาณการได้ แสดงให้เห็นว่า ในช่วงที่เกิดวิกฤตการณ์ทางการเงินโลก ดัชนีตลาดหลักทรัพย์ทั้ง 5 ประเทศมีการเคลื่อนไหวเปลี่ยนแปลงไปในทิศทางเดียวกัน และหลังการประกาศใช้พิมพ์เขียวในการก่อตั้งประชาคมเศรษฐกิจอาเซียน ในปี 2550 ดัชนีตลาดหลักทรัพย์ทั้ง 5 ประเทศมีความเชื่อมโยงกันมากขึ้น ชี้ให้เห็นถึงความเชื่อมโยงทางการเงินที่เกิดขึ้นในภูมิภาค มากไปกว่านั้น การจำลองสถานการณ์แสดงให้เห็นว่า ค่าสัมประสิทธ์สหสัมพันธ์ที่เปลี่ยนแปลงตามเวลาที่ประมาณค่าตามวิธีการนี้แสดงให้เห็นรูปแบบการเปลี่ยนแปลงของค่าสัมประสิทธิ์สหสัมพันธ์ที่เปลี่ยนแปลงตามเวลาได้ดีกว่า วิธีการประมาณการอีกสามวิธี ได้แก่ วิธีการประมาณการค่าสัมประสิทธ์สหสัมพันธ์เคลื่อนที่ถอยหลัง วิธีการประมาณการค่าสัมประสิทธ์สหสัมพันธ์เคลื่อนที่กึ่งกลาง และตัวแบบสหสัมพันธ์ที่มีพลวัตรตามเงื่อนเวลา

วิทยานิพนธ์นี้ ยังสำรวจการใช้เกณฑ์การเลือกตัวแบบ ในการกำหนดจำนวนปมแบบควอนไทล์ของฟังก์ชั่นเส้นโค้งเสมือนพหุนามกำลังสามที่มีเงื่อนไขขอบแบบธรรมชาติ สำหรับการประมาณการค่าความผันผวนการการเงิน การสำรวจสถานการณ์แสดงให้เห็นว่า การตรวจสอบไขว้นัยทั่วไป เป็นเกณฑ์การเลือกปมที่ดีกว่าเกณฑ์อื่นอีก 3 เกณฑ์ ได้แก่ เกณฑ์ข้อมูลของอาคาอิเกะ เกณฑ์ข้อมูลแบบเบย์เซียน และการตรวจสอบไขว้นัยทั่วไปที่มีการดัดแปลง เมื่อประมาณการค่าความผันผวนของอนุกรมเวลาอัตราแลกเปลี่ยนเทียบกับดอลลาร์สหรัฐ และอัตราแลกเปลี่ยนตามดัชนีค่าเงินของประเทศในภูมิภาคอาเซียนทั้ง 5 ประเทศ ด้วยฟังก์ชั่นเส้นโค้งเสมือนพหุนามกำลังสามที่มีเงื่อนไขขอบแบบธรรมชาติ และใช้จำนวนปมแบบควอนไทล์ที่เลือกด้วยการตรวจสอบไขว้นัยทั่วไป ผลการศึกษาแสดงให้เห็นว่า อัตราแลกเปลี่ยนเทียบกับดอลลาร์สหรัฐมีความผันผวนมากกว่าอัตราแลกเปลี่ยนตามดัชนีค่าเงิน ซึ่งสะท้อนให้เห็นอิทธิพลของดอลลาร์สหรัฐที่มีต่อเสถียรภาพของอัตราแลกเปลี่ยนเทียบกับดอลลาร์สหรัฐ นอกจากนี้ความผันผวนของอัตราแลกเปลี่ยนเทียบกับดอลลาร์สหรัฐยังถูกกระทบจากนโยบายอัตราแลกเปลี่ยนที่ใช้ในช่วงเวลานั้นด้วย

การประยุกต์ใช้ฟังก์ชั่นเส้นโค้งเสมือนพหุนามกำลังสามที่มีเงื่อนไขขอบแบบธรรมชาติในการประมาณการค่าความผันผวนทางการเงิน และค่าสัมประสิทธิ์สหสัมพันธ์ที่เปลี่ยนแปลงตามเวลาสามารถประยุกต์ใช้ในการสำรวจเสถียรภาพทางการเงินและความเชื่อมโยงทางการเงินในภูมิภาคอาเซียนได้ดี และสามารถนำไปประยุกต์ใช้เพื่อประมาณการค่าความผันผวนทางการเงิน และค่าสัมประสิทธ์สหสัมพันธ์ที่เปลี่ยนแปลงตามช่วงเวลาของอนุกรมเวลาทางการเงินอื่นๆ ได้

Thesis Title Applications of the Natural Cubic Spline Function to Estimate Financial Volatility and Time-varying Correlation of the ASEAN-5 Financial Time Series

Author Mr. Jetsada Laipaporn

Major Program Research Methodology

Academic Year 2020

ABSTRACT

This thesis introduced methods to investigate financial stability and financial integration in ASEAN. The natural cubic spline function with quantile knots was applied to financial time series, including the stock market indexes, the bilateral exchange rates to the United States dollar, and the effective exchange rates during two decades from January 1, 2001 to December 31, 2020 of the ASEAN-5, including Thailand, Singapore, Malaysia, Indonesia, and the Philippines.

Initially, this study estimated financial volatilities of the investigated series by applying the natural cubic spline function with 22 quantile knots, that had an approximately one trading-year interval between them. The results showed that during the global financial crisis the estimated natural cubic spline volatilities dramatically increased, reflecting an instability in time of crisis. In addition, the Monte Carlo simulation demonstrated that the natural cubic spline volatility revealed more precise volatility's pattern than the smoothing GARCH (1,1) volatility method introduced in previous study.

To investigate the financial integration in this region, this study alternatively estimated time-varying correlation coefficients of the ASEAN-5 stock indexes with a new method based on the indirect covariance concept and the natural cubic spline volatility. The estimated time-varying correlation coefficients consequently exhibited that in time of

the global financial crisis these five stock market indexes were more likely to change in the same direction and after the declaration of the ASEAN Economic Community blueprints in 2007, these stock market indexes had stronger linkages, indicating the emerging financial integration in this region. Moreover, the simulation showed that the time-varying correlation coefficients estimated following this method, better in revealing varying patterns of time-varying correlation coefficients than the other three estimators, consisting of the backward rolling correlation coefficient estimator, the centered rolling correlation coefficient estimator, and the dynamic conditional correlation model.

This thesis also investigated the use of model selection criteria in selecting a number of quantile knots for the natural cubic spline function in the financial volatility estimation. The simulation presented that the Generalized Cross-Validation was a superior criterion than the other three candidates including the Akaike's Information Criteria, the Bayesian Information Criteria, and the modified Generalized Cross-Validation. Then, the volatilities of the bilateral exchange rates and the effective exchange rates of the ASEAN-5 were estimated by the natural cubic spline function with a number of quantile knots selected by the Generalized Cross-Validation. The results showed that the bilateral exchange rate generally had higher volatility than the effective exchange rate, reflecting the influence of the United Stated dollar on the stability of the bilateral exchange rates and volatility of bilateral exchange rate was impacted by its concurrent adopted exchange rate policy.

Applying the natural cubic spline function for estimating financial volatility and time-varying correlation coefficients was found practical to investigate financial stability and financial integration in the ASEAN and could be broadly adopted for estimating financial volatility and time-varying correlation coefficients of the other financial time series as well.

Acknowledgements

I would like to express my gratitude to my thoughtful and beloved advisors, my major advisor Emeritus Professor Dr. Don McNeil, and my co-advisor Assistant Professor Dr. Phattrawan Tongkumchum, for their continuous encouragement, helpful assistance, side by side training, and attentive guidance and on this thesis. I would like to express my appreciation to the other thesis examining committee, the chairperson Associate Professor Dr. Apiradee Lim, the external examiner from University Technology Malaysia Professor Dr. Muhammad Hisyam Lee, and the internal examiner Assistant Professor Dr. Rattikan Saelim, for their fruitful comments and recommendations that enormously improved my thesis.

In addition, I would like to express my thankfulness to Dr. Thomas Fung from Macquarie University in Australia for his helpful advice on coding in R program. I would like to show gratitude to my friends and colleagues for your smiles, sweets, helps, and encouragements along the way of this long journey. Finally, I would like to express thanks to my family for their endless supports throughout this study.

Jetsada Laipaporn

Contents

Contents (cont.)

Contents (cont.)

Contents (cont.)

List of Tables

List of Figures

List of Figures (cont.)

List of Abbreviations

AEC	The Association of Southeast Asian Nations Economic Community
AIC	Akaike's Information Criteria
ARCH	The Autoregressive Conditional Heteroscedasticity model
ASEAN	The Association of Southeast Asian Nations
ASEAN-5	Five founder countries of the ASEAN consisting of Indonesia, Malaysia, the Philippines, Singapore, and Thailand
BER	Bilateral exchange rate to United States dollar
BIC	Bayesian Information Criteria
DCC	The dynamic conditional correlation model
GARCH	The General Autoregressive Conditional Heteroscedasticity model
GARCH (1,1)	The simplest form of the General Autoregressive Conditional Heteroscedasticity model
GARCH-NCS	The smoothing GARCH (1,1) volatility method using the natural cubic spline function
GCV	General Cross-Validation
IDR	The Indonesian rupiah
JKSE	The Indonesia Exchange index or Jakarta Stock Exchange index
KLSE	The Bursa Malaysia index or Kuala Lumpur Stock Exchange index
MAR	The multivariate adaptive regression model
MAE	The mean absolute error
MGCV	Modified General Cross-Validation

List of Abbreviations (cont.)

MYR	The Malaysian ringgit
NCSV	The method applying the natural cubic spline function to estimate financial volatility
NCSV-TVC	The time-varying correlation coefficient estimated by the natural cubic spline function
PHP	The Philippine peso
PSE	The Philippine Stock Exchange index
RMSE	The root mean squared error
SET	The Stock Exchange of Thailand index
SGD	The Singapore dollar
Spline-GARCH	The Spline General Autoregressive Conditional Heteroscedasticity model
STI	The Strait Time index
THB	The Thai Baht
USD	The United States dollar

Chapter 1

Introduction

This study proposed alternative methods, which employed the natural cubic spline function, for investigating financial stability and financial integration of the Association of Southeast Asian Nations or ASEAN focusing on the financial time series of five ASEAN founder countries or the ASEAN- 5, which were Singapore, Thailand, Malaysia, Indonesian and the Philippines.

1.1 Background and rationale

At the 13th ASEAN summits in Bali, ASEAN had an initiative to establish the ASEAN Economic Community or AEC to reduce economic disparity within its members and to gain more attractive to the investor. Following the implement of the AEC blueprints, ASEAN consequently liberalized capital transaction to accelerate the financial integration in the region (Shimizu, 2014).

Although financial integration eased capital allocation and lower cost of capital, unfortunately financial integration caused financial interdependency that induced risk spillover or financial contagion (Prukumpai and Sethapramote, 2018). Thus, financial distresses in the global financial liberalization era continuously impacted on the stability of ASEAN economy (Click and Plummer, 2005). Moreover, financial crisis induced considerable financial instability which caused a dramatic loss in confidence of the investors (Chiang *et al.*, 2007). Therefore, the information for monitoring financial stability and financial integration in this region has been demanded increasingly.

Generally, financial volatility indicates a stability of the financial market. It determines possible range of the uncertain financial returns and shows the possible losses or gains, reflecting the opportunity of investing in that financial asset (Poon, 2005). One of widely used volatility models in academic world is the General Autoregressive Conditional Heteroscedasticity or GARCH model proposed by Bollerslev (1986).

A basic form of the GARCH model named as the GARCH (1,1) model, was employed to estimated financial volatilities of the ASEAN financial time series in many studies (Saejiang *et al.*, 2001; Do *et al.*, 2009; Kabigting and Hapitan, 2011; Awalludin and Saelim, 2016; Awalludin *et al.*, 2018). Its estimated volatility was capable to indicate uncertain variation of the financial time series. The GARCH (1,1) model also became a part of the dynamic conditional correlation model or the DCC model that was employed for examining co-movement between the observed financial time series in several studies (Engle and Sheppard, 2001; Engle, 2002; Chiang *et al.*, 2007; Dimitriou and Kenourgios, 2013; Yin *et al.*, 2017).

Since the estimated volatility for GARCH (1,1) was high fluctuated and could not provide an obvious changing pattern of volatility, therefore some studies alternatively employed a spline function to visualize clearer changing patterns of financial volatility. For examples, Awalludin and Saelim (2016) applied a natural cubic spline function to smooth the GARCH (1,1) volatility in order to obtain less frequent predictor of daily volatility that exhibit changing pattern in financial volatility of some stock indexes in the Indonesian stock market. Likewise, Engle and Rangel (2008) combined a quadratic spline function with the number of knots selected by model selection criteria as a low-frequency component of their Spline-GARCH model. Consequently, the Spline-GARCH model was employed for determining the cyclical pattern of financial volatility in many studies (Liu *et al.*, 2015; Karali and Power, 2013). Although the application of spline function was practical to exhibit uncertain movement of financial volatility and capable to indicate the long run stability of financial market, the use of spline function following these existing methods had some disadvantages.

In case of Awalludin and Saelim (2016), their method based on the estimates of the GARCH (1,1) model. Unfortunately, literatures showed that the GARCH (1,1) model had some drawbacks. Do *et al.* (2009) addressed that the GARCH (1,1) model has been insufficient to model the volatility of the financial time series that followed an asymmetric heteroscedastic process. While, a single outlier can make the GARCH (1,1) estimates overestimated without any supported evidence (Farida, 2016). Moreover, the GARCH (1,1) model was ineffective to accommodate leveraged effect and to exhibit a long-run temporal dependence (Hansen and Lunde, 2005; Bentes, 2015). In case of Engle and Rangel (2008), they employed a quadratic spline function, which was less flexible than a natural cubic spline function. Consequently, their volatility model was theoretically less effective in capturing changing patterns of financial volatility.

So, in order to examine financial stability and financial integration of the ASEAN economic community during 2001-2020, this study proposed a method that applied the natural cubic spline function to estimate financial volatilities and time-varying correlation coefficients of the ASEAN-5 financial time series including stock market indexes, the bilateral exchange rates to the United States dollar (USD) and the effective exchange rates. This study additionally conducted the Monte Carlo simulations to assess performance of the proposed method in exhibiting changing pattern of financial volatility and time-varying correlation coefficients by comparing to the performance of existing methods.

To select the number of knots which was critical to flexibility of a natural cubic spline function, this study initially applied a subjective selection and consequently conducted the Monte Carlo simulation to investigate the use of model selection criteria and determined a proper criterion for designating the number of quantile knots of the natural cubic spline function.

1.2 Research objectives

This study offers alternative methods for estimating financial volatilities and time-varying correlation coefficients of the ASEAN-5 financial time series. The specific objectives are as follows:

1) To apply the natural cubic spline function to estimate financial volatilities of the ASEAN-5 financial time series consisting of stock market indexes, bilateral exchange rates to USD, and effective exchange rates, and assess performance of this financial volatility estimation method.

2) To apply the natural cubic spline function to estimate time-varying correlation coefficients for examining co-movements of the ASEAN-5 stock market indexes and assess performance of this time-varying correlation coefficients estimation method.

3) To investigate the use of model selection criteria in determining a proper number of knots for the natural cubic spline function in financial volatility estimation and apply to estimate financial volatilities of the bilateral exchange rates to USD and the effective exchange rates of the ASEAN-5.

1.3 Scope of the research

This study focuses on applying the natural cubic spline function for estimating financial volatilities and time-vary correlations coefficients of the ASEAN-5 financial time series. The financial time series of interests comprises of the stock market indexes, the bilateral exchange rates to USD and the effective exchange rates during two decades from January 1, 2001 to December 31, 2020.

To assessing performance of the financial volatility estimation method using the natural cubic spline volatility, abbreviated as the NCSV method, three Monte Carlo simulations were conducted. This first simulation was to assesse performance of the application of the natural cubic spline function with a number of quantile knots and compared to the performance of the smoothing GARCH (1,1) volatility method, suggested by Awalludin and Saelim (2016). Since this existing method also employed a natural cubic spline to reveal changing pattern of financial volatility, consequently this study selected the smoothing GARCH (1,1) volatility method comparing to the NCSV method. This study abbreviated the smoothing GARCH (1,1) volatility method as the GARCH-NCS method.

The second simulation was to assess the performance of applying the natural cubic spline function to estimate time-varying correlation coefficient and compared to the performance of the other three time-varying correlation coefficient estimators that frequently used for investigating co-movement between financial time series in previous studies (Engle, 2002; Chiang *et al*., 2007; Billio and Caporin, 2010; Dimitriou and Kenourgios, 2013; Wang and Xie, 2013; Tiwari *et al*. 2016; Yin *et al*., 2017; Rey and Nivoix, 2018). The other three estimators included the backward rolling correlation coefficient estimator, the centered rolling correlation coefficient estimator, and the dynamic conditional correlation model.

Lastly, the third Monte Carlo simulation was to investigate the use of four candidate model selection criteria that were frequently employed for determining a number of knots of a spline function in previous studies (Lewis and Stevens, 1991; Chen *et al*., 1997; Engle and Rangel, 2008; Engle *et al*., 2013; Montoya *et al*., 2014; Lee *et al*., 2018; Conrad and Kleen, 2020). These four candidate model selection criteria included the Akaike's Information Criteria (*AIC*), Bayesian Information Criteria (*BIC*), General Cross-Validation (*GCV*), and Modified General Cross-Validation (*MGCV*).

1.4 Conceptual framework

This study began with data preparation. The daily indexes of the stock market indexes, the bilateral exchange rates to USD, and the effective exchange rates of the ASEAN-5 are gathered for calculating daily returns and using these returns for further analysis. The analysis of study is separated into four parts as showed in Figure 1.1.

The preliminary analysis was to reveal the changing patterns of the financial volatilities of the ASEAN-5 stock market indexes, the bilateral exchange rates to the USD, and the effective exchange rates. The procedures in the preliminary analysis was to smooth GARCH (1,1) volatility using the natural cubic spline function following the GARCH-NCS method.

To fulfil the first objective, the analysis part I was to apply the natural cubic spline function with the number of quantile knots to estimate volatilities of the stock market indexes, the bilateral exchange rate, and the effective exchange rates of the ASEAN-5. Then the Monte Carlo simulation I was conducted to assess performance of the NCSV method in revealing the changing pattern of financial volatility and comparing to performance of the GARCH-NCS method.

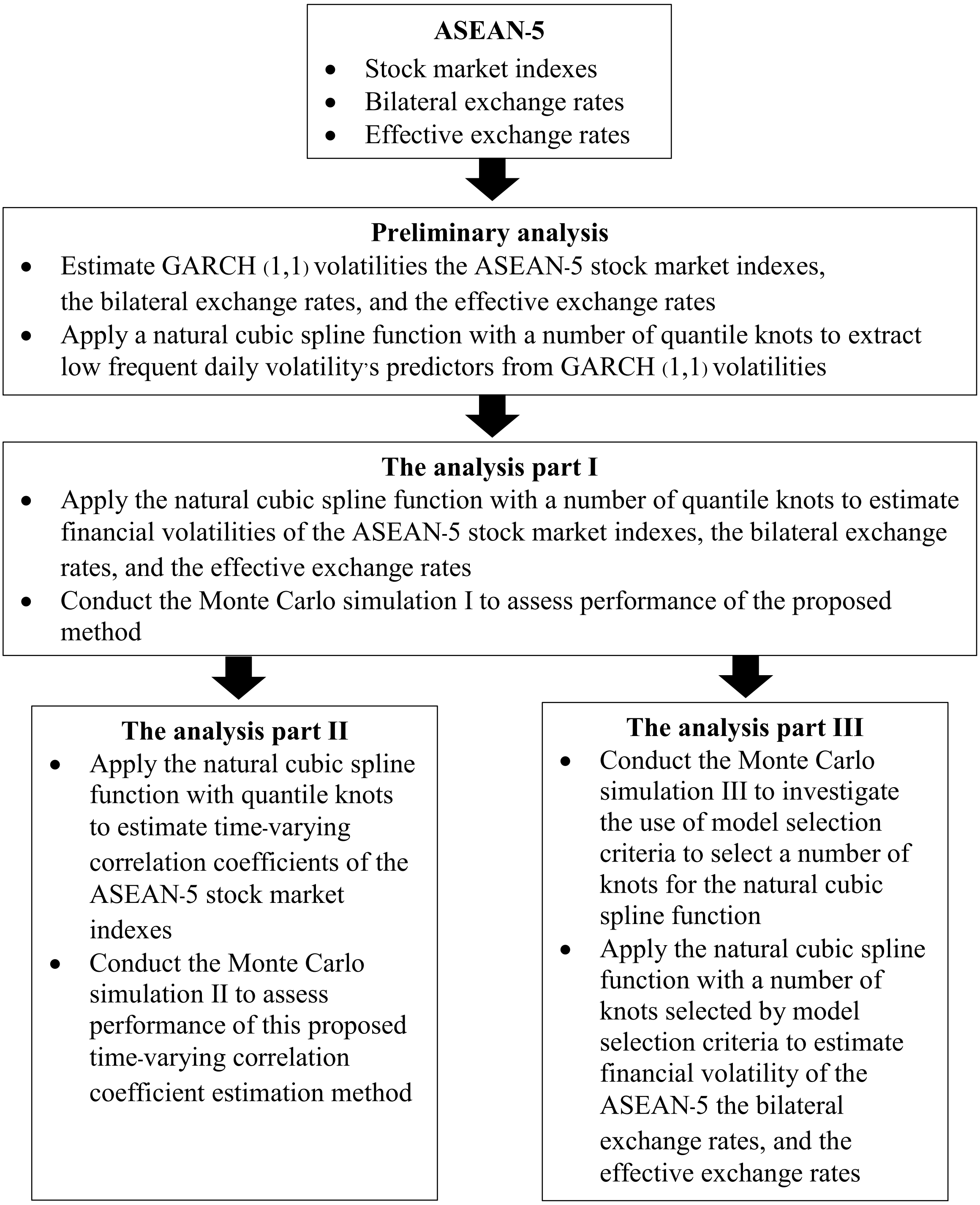


Figure 1.1 Diagram of analysis process

For the second objective in examining co-movements of the ASEAN-5 stock market indexes, the analysis part II was to apply the natural cubic spline function with a number of quantile knots to estimate the time-varying correlation coefficients of the ASEAN-5 stock market indexes. Consequently, the Monte Carlo simulation II was conducted to assess performance of this time-varying correlation coefficient estimation method, comparing to the other three estimators, including the backward rolling correlation coefficient estimator, the centered rolling correlation coefficient estimator, and the dynamic conditional correlation model.

To fulfill the last objective, the analysis part III was to investigates the use of the model selection criteria in selection a number of quantile knots for the natural cubic spline function for estimating financial volatility following the NCSV method. The candidate model selection criteria included the Akaike's Information Criteria (*AIC*), Bayesian Information Criteria (*BIC*), General Cross-Validation (*GCV*), and Modified General Cross-Validation (*MGCV*). Then, the most appropriated model selection criterion was applied to the natural cubic spline function for estimating financial volatilities of the bilateral exchange rates and the effective exchange rates of the ASEAN-5.

1.5 Literature review

1.5.1 Financial volatility modeling

Volatility reflects the variation of the financial asset's returns. Fama (1965) showed that if the stock market follows the efficient-market hypothesis, the changes of the stock prices will constantly vary and they will not depend on their previous changes values. By this hypothesis, the returns of financial asset were possibly assumed having normal distributed with constant variance or had constant volatility (Fama, 1970).

However, that efficient-market hypothesis could be held only for the short horizontal of returns series (Fama, 1970). For the longer time series, even the returns of that asset were still independent but their variation or their variance was inconstant (Fama and French, 1988). That inconstant variance violated the efficient-market hypothesis and effected to the predicted volatility by a simple statistical measurement.

Not only the studies of Fama (1970) and Fama and French (1988) showed the relation between the characteristic of financial asset returns and their volatility, the reviews in Cont (2001), Poon and Granger (2003), and Engle (2004) also indicated that financial asset's returns have inconstant variance and the variances of those returns are conditional to point of time. Additionally, Cont (2001) showed that volatility of the financial asset returns was clustering as stated in Mandelbrot (1963) that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes".

To measure financial volatility, if the efficient-market hypothesis was held, financial volatility over the period of interest is simply measured by a statistical measurement like standard deviation or variance of the returns of financial asset in that period. The volatility like this is assumed constant over that period. Poon (2005) named this constant volatility as unconditional volatility, since this volatility is unconditional to the point of times. Moreover, instead of using volatility in term of variance, volatility is usually presented in term of standard deviation, which is easier to understand when comparing to the observed returns.

Different to the unconditional volatility, which is simply measured by variance or standard deviation of the return series, the time-varied or conditional volatility is modeled in various ways. Engle (1982) firstly employed squared returns as proxies for daily volatility modeling. He assumed daily volatility was conditional to its previous values following the behavior of squared returns, which were highly autocorrelated. His model, Autoregressive Conditional Heteroscedasticity (ARCH) model thus effectively explained the volatility clustering phenomenon (Poon, 2005).

The General Autoregressive Conditional Heteroscedasticity (GARCH) model was extended from the ARCH model by adding the moving average term (Bollerslev, 1986). Hence, it has become more flexible and more predictability than its originated model (Engle, 2001). Consequently, there have been many studies based on the ARCH and GARCH approaches (Engle, 2001; Poon and Granger, 2003).

The GARCH (1,1) model is the simplest case of GARCH model. It includes three components, which are a constant long-term variance, an autoregressive parameter and a moving average parameter (Bollerslev, 1986; Engle, 2001). The GARCH (1,1) model assumes that the observed returns are independent and their distribution is normal with mean equal to zero and conditional on its variance. This model is usually parameterized as follows.

$$\sigma_t^2 = \left(1-\alpha-\beta\right)V_L + \alpha r_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{1.1}$$

where r_t and σ_t^2 are observed return and estimated conditional variance at day t, respectively. A long-term variance denoted by V_L, which is constant over the period of interest. The parameter α is a measure of the influence of the most recent return value and β is a smoothing constant (Bollerslev, 1986; Engle, 2001).

This model is fitted to the squared values of the observed returns by maximizing the likelihood of the T observations. Using the formula for the probability density function of this normal distribution, the likelihood, $L(\alpha,\beta)$, is thus

$$L(\alpha,\beta)=\prod_{t}^{T}\left[\frac{1}{\sqrt{2\pi\sigma_t^2}}\exp\left(\frac{r_t^2}{\sigma_t^2}\right)\right] \tag{1.2}$$

Saejiang *et al.* (2001) applied GARCH (1,1) model to investigate the variation of the returns series from the weighted portfolio of major banking shares in Thailand and the estimated volatility was well fitted to the time series from January 1994 to December 1999. Do *et al.* (2009) also employed GARCH (1,1) model to study the behavior of the volatility of five stock markets in ASEAN, including Thailand, Malaysia, Indonesia, the Philippines and Vietnam, in the period from July 28, 2000 to October 31, 2008. The GARCH (1,1) model was well fitted to the volatility of those stock markets but less preferred than the Glosten-Jagannathan-Runkle GARCH (1,1) or GJR-GARCH (1,1), the other expansion of GARCH model, which assumed volatility followed an asymmetric heteroscedastic process.

Furthermore, Kabigting and Hapitan (2011) modeled the volatility of the indexes of stock market and the exchange rates of five ASEAN countries, namely Philippines, Indonesia, Thailand, Malaysia, and Singapore. The GARCH model of each financial asset successfully indicated the volatility clustering. Awalludin and Saelim (2016) applied the GARCH (1,1) model to estimate the volatility of seven stock in Jakarta Stock Exchange, Indonesia. Their finding showed that the GARCH (1,1) was less capability to exhibit the variation pattern of the daily volatility. Otherwise, there were some drawbacks when applying the GARCH (1,1) model. Farida (2016) reported that a single outlier caused the followed overestimated volatility without the actual evidence. Similarly, Hansen and Lunde (2005) also showed that the GARCH (1,1) model was obviously inferior to models that could accommodate leveraged effect.

1.5.2 Proxies for daily volatility

The widely used proxies for daily volatility modeling has been the squared and absolute values of the returns series (Poon and Granger, 2003), since calculating variance by using zero instead of a mean value of the observed returns has provided more accurate volatility (Figlewski, 1997). The statistical properties of sample mean, especially for small samples size, make it inappropriately represent the estimate of the true mean. By that ways, the volatility has modeled as a function of the squared or absolute values of the returns series.

Beside, Ding *et al.* (1993) indicated that the absolute returns were the appropriated proxy of the volatility because they had a long memory property that kept the effect of fluctuation continuing persistent over a long period. Moreover, Poon and Granger (2003) summarized that there were some empirical studies, showing that the volatility models based on the absolute returns provided better volatility forecasts than models based on squared returns. However, as following the review of Poon and Granger (2003), the majority of volatility models were the models based on squared returns.

1.5.3 Time-varying correlation coefficient

Time-varying correlation coefficient is an indicator for assessing the relationship between two observed time series. Ding *et al.* (2014) and Meric *et al.* (2012) employed a time-varying correlation coefficient for revealing the inter-dependent behavior and the co-movement pattern between the financial series. Time-varying correlation coefficient is useful for identifying the linkage between the stock markets. The high degree of time-varying correlation of two stock markets indicates the strong linkage between them (Forbes and Rigobon, 2002) and this linkage reflects how much the markets are integrated (Goetzmann *et al.*, 2005).

Since time-varying correlation coefficient is likely to increase during a highly volatile period (Syllignakis and Kouretas, 2011), declines occur during the bull markets and rises in the bear markets (Forbes and Rigobon, 2002). The time-varying correlation that rapidly increased indicate financial contagion, while the continuously high in level of the time-varying correlation implied an interdependency of the investigated financial time series (Chiang *et al.*, 2007)

As time-varying correlation coefficient reflects the co-movement of financial time series, it has become a critical factor in the portfolio selection model (Markowitz, 1952; Elton and Gruber, 1997; Fabozzi *et al.*, 2002). Since investing in the stocks having a strong co-movement patterns raises the risk of an investment portfolio, investors thus often try to diversify their portfolio by distributing their investment into stocks or stock markets that have a low degree of time-varying correlation coefficient. However, there is no ideal approach to estimate the time-varying correlation coefficient (McMillan, 2019).

The most straightforward approach is the rolling correlation coefficient estimator. This estimator is practically used for investigating the evolution of time-varying correlation coefficient. By changing the size of the rolling window, it capably highlights the pattern of correlation coefficient changes both in long-term and short-term of the market dynamics (Billio and Caporin, 2010; Wang and Xie, 2013).

The rolling correlation coefficient is the Pearson's correlation of asset returns in the time-specified window, which roll step by step throughout the interest period. This estimator is simple and prominent in applied finances. Studies such as Engle (2002), Wang and Xie (2013) and Tiwari *et al.* (2016) have employed this estimator as a benchmark to compare the performance against the more sophisticated estimators. The rolling correlation ($\rho_{xy,t}$) between two return series, r_x and r_y over the time span from $t-s$ to $t-1$, where t is time, and s is the size of the time-specified window, is expressed as follows,

$$\rho_{xy,t} = \frac{\sum_{i=t-s}^{t-1} \left(r_{x,i} - \overline{r}_x\right)\left(r_{y,i} - \overline{r}_y\right)}{\sqrt{\sum_{i=t-s}^{t-1} \left(r_{x,i} - \overline{r}_x\right)^2 \sum_{i=t-s}^{t-1} \left(r_{y,i} - \overline{r}_y\right)^2}} \tag{1.3}$$

The mean of each return series, which is approximately zero, denoted by $\overline{r}_x$ and $\overline{r}_y$, respectively. This estimator is known as the backward rolling correlation coefficient estimator or historical correlation coefficient estimator since this estimator used data between time span, $t-s$ to $t-1$, to estimate the correlation coefficient at time t. The size of time spans frequently set at 250 trading-days or one trading-year, which was capable to capture the short-term pattern of the time-varying correlation (Meric *et al*, 2012; Wang and Xie, 2013; Tiwari *et al*., 2016; McMillan, 2019).

However, the study of Rey and Nivoix (2018) used the centered rolling correlation coefficient estimator instead, because it provides a more accurate result than the backward correlation coefficient estimator. This estimator is formulated as follows.

$$\rho_{xy,t} = \frac{\sum_{i=t-0.5s}^{t+0.5s} \left(r_{x,i} - \overline{r}_x\right)\left(r_{y,i} - \overline{r}_y\right)}{\sqrt{\sum_{i=t-0.5s}^{t+0.5s} \left(r_{x,i} - \overline{r}_x\right)^2 \sum_{i=t-0.5s}^{t+0.5s} \left(r_{y,i} - \overline{r}_y\right)^2}} \tag{1.4}$$

To estimate the correlation coefficient at time t where s is the size of the time-specified window, the centered rolling correlation coefficient estimator differently estimates the subsample between t minus half of s and t plus half of s. Erb *et al.* (1994) showed that this estimator is the best suited for use in portfolio management. Unfortunately, the rolling correlation coefficient estimator uses sub-sample data. Thus, it is impossible to assess the time-varying correlation coefficient at every point in time for the whole observed period.

Another approach for estimating time-varying correlation coefficient is the dynamic conditional correlation or DCC model. This model was introduced by Engle (2002) and is widely used in the academic world. The DCC model provides the robustness analysis of time-varying correlation coefficient by allowing conditional asymmetries in both volatility and correlation and overcomes the heteroscedasticity problem (Dimitriou and Kenourgios, 2013). Contrast to rolling correlation coefficient estimator, the DCC model uses the full sample for estimating (Yin *et al.*, 2017). Thus, it capably provides the estimates at every point in time for the whole investigated period (Chiang *et al.*, 2007).

The DCC model assumes that series of financial returns are conditionally multivariate normal with zero expected value and covariance matrix H_t. The covariance matrix H_t is parameterized as

$$H_t = D_t C_t D_t \tag{1.5}$$

The matrix D_t is the $n \times n$ diagonal matrix of time-varying standard deviation derived from the univariate GARCH models ($\sqrt{h_{i,t}}$) on the i^{th} diagonal,

$i = 1,2,3,...,n$ and n is number of series. The matrix C_t is the matrix of time-varying correlation.

Engle (2002) proposed the two-step procedure for the DCC model estimation. The first step is estimating the univariate GARCH model for each return series. While the second step is applying the standardized residuals or volatility-adjusted returns ($\varepsilon_{i,t} = r_{i,t} \big/ \sqrt{h_{i,t}}$) to estimate the parameters of the conditional correlation as follows.

$$C_t = \left(diag\left(Q_t\right)\right)^{1/2} Q_t \left(diag\left(Q_t\right)\right)^{1/2} \tag{1.6}$$

The element of Q_t is given by $q_{xy,t}$, where $x, y = 1,2,3,...,n$ and $x \neq y$,

$$q_{xy,t} = \left(1-\alpha-\beta\right)\overline{q}_{xy} + \alpha\varepsilon_{x,t-1}\varepsilon_{y,t-1} + \beta q_{xy,t-1} \tag{1.7}$$

The element $\overline{q}_{xy}$ is the unconditional covariance between standardized residuals of series, x and y. The parameters α and β are the new and the decay coefficients, respectively. In addition, the q_{xy} is obtained from $\left(diag\left(Q_t\right)\right)^{1/2}$ which was equal to $diag\left(1\big/\sqrt{q_{11,t}},...,1\big/\sqrt{q_{nn,t}}\right)$. Then time-varying correlation ($\rho_{xy,t}$) in the matrix C_t has the form as below:

$$\rho_{xy,t} = \frac{q_{xy,t}}{\sqrt{q_{xx,t}q_{yy,t}}} \tag{1.8}$$

Let the parameters in D_t be denoted by θ and the additional parameters in C_t be denoted by ϕ. The log-likelihood can be rewritten as the sum of a volatility part and a correlation part.

$$L\left(\theta,\phi\right) = L_v\left(\theta\right) + L_c\left(\theta,\phi\right) \tag{1.9}$$

The two-step procedure for the DCC model estimation is by first maximizing the likelihood $L_v(\theta)$ to find $\hat{\theta} = \arg\max\{L_v(\theta)\}$ and then using $\hat{\theta}$ as given for maximizing the correlation part, $\max_{\phi}\left\{L_c\left(\hat{\theta},\phi\right)\right\}$. The $L_v(\theta)$ is equivalent to the sum of univariate GARCH likelihoods which are separately maximized.

Although the estimation of the DCC model is less complicated and easier to estimate than the other multivariate models (Engle and Sheppard, 2001), this estimator cannot display the precise pattern of changes in correlation coefficient which can be easily obtained by the rolling method (McMillan, 2019).

1.5.4 Natural cubic spline function

The natural cubic spline function is simply piecewise cubic polynomial that are linear in the distant past and future and practically fitted to the dependent variable for extracting the variation pattern of that series (Wahba, 1975). In financial context, the natural cubic spline function has been widely used as an interpolation technique to estimate yield curve of the financial assets (Engle and Russell, 1998).

Recently, Awalludin and Saelim (2016) employed the natural cubic spline function to study the behavior of financial volatility of Indonesia stocks from July 12, 2007 to September 29, 2015. That study used the spline function to smooth the GARCH (1,1) estimates and showed that these smoothed GARCH (1,1) estimates had capability to visualize varying pattern of the financial volatility.

Silverman (1985) described that using splines in non-parametric regression was an attractive and flexible approach to curve estimation. Furthermore, Suits *et al.* (1978)

showed that spline function was used as a device for approximating the shape of curvilinear function without the necessity of pre-specifying the mathematical form of the function.

Additionally, Engle and Rangel (2008) also applied spline function to volatility modeling. Their Spline-GARCH model assumed that the financial volatility could be separated into two components, (1) the slow-moving component as the unconditional volatility and (2) the high-moving component as the conditional volatility. Consequently, in this model the exponential quadratic spline function represented the low-frequency volatility while the conditional volatility follows the GARCH process. They introduced Spline-GARCH model for modeling volatility of macroeconomic variables of 48 countries and found that the low-frequency volatility estimated by Spline-GARCH model was effective to determine a stability of macroeconomic across countries.

However, before estimating the parameters of the natural cubic spline function, the numbers of knots and their locations have to be identified. To choose number of knots, there are several criteria which could be categorized into two approaches (Wongsai, *et al.*, 2017). The first approach was a subjective selection which based on professional judgement, while the second approach was an automatic selection which based on the knot selection criterion.

Based on professional judgement, Awalludin and Saelim (2016) subjectively selected 8 knots and located them with equally spacing for modeling the volatility of Indonesia stocks during 1,990 trading days. Whereas, previous studies that used the spline-GARCH model and the GARCH-MIDAS model alternatively determined the number of knots by utilizing information criteria such as Akaike's Information Criteria (*AIC*) and

Bayesian Information Criteria (*BIC*) (Engle and Rangel, 2008; Engle *et al.,* 2013; Lee *et al.,* 2018; Conrad and Kleen, 2020).

Engle and Rangel (2008) used an information criterion to choose an optimum numbers of knots with respect to the cyclical pattern within the long-run trend of volatility and then similarly located those knots with equally-space interval. Number of knots used in Engle and Rangel (2008) varied from 3 to 15 knots and trading day between each spacing varies from 167 to 11,682 trading day. The number of knots and relevant information for modeling the volatility of the stock market indexes of five ASEAN countries, namely Indonesia, Malaysia, the Philippines, Singapore and Thailand from Engle and Rangel (2008) showed in Table 1.1.

Instead of employing information criteria, Breiman (1993) followed the Step-Wise procedure in univariate curve fitting in Smith (1982) by setting the pool of knots in advance and then applied the statistical linear model theory in knots deletion framework to select the most predictive model. Breiman (1993) also proposed the cross validation as criteria to select the model, different to F test used in Smith (1982).

Likewise, Montoya *et al.* (2014) compared various knots selection criteria and found that generalized cross-validation or *GCV* introduced by Craven and Wabha (1979) is more suitable method in selecting the number of knots for the penalized regression spline model. Likewise, Chen *et al.* (1997) and Lewis and Stevens (1991) utilized the modified generalized cross-validation or *MGCV* proposed by Friedman (1991) for selecting the number of knots of the multivariate adaptive regression or MARS model. They found that this approach provided a parsimonious time series model that exhibited a changing pattern of the time series data.

Table 1.1 The number of knots and relevant information for modeling the volatility of the stock market indexes of five ASEAN countries

Index	Starting year	Trading Days	Number of knots	Trading days per knot
Jakarta Stock Exchange Index	1983	5,204	15	347
Bursa Malaysia Index	1980	6,057	14	433
The Philippines Stock Exchange Index	1986	4,580	13	352
Stock Exchange of Singapore Index	1965	9,917	7	1,417
Stock Exchange of Thailand Index	1975	7,271	12	606

Source: Engle and Rangel (2008)

1.5.5 ASEAN economic community

In 1967, five countries in the Southeast Asia region consisting of Indonesia, Malaysia, the Philippines, Thailand, and Singapore established the Association of Southeast Asian Nations (ASEAN) and the other five countries, which were Brunei, Vietnam, Laos, Myanmar and Cambodia, later became the ASEAN members in the following years.

After the financial crisis in 1997 and the global financial crisis in 2007, ASEAN set up an initiative to establish a free trade area in order to eliminate trade barriers and support regional integration (Ahmed and Singh, 2016). Consequently, the blueprint of the ASEAN community has been declared as a masterplan to establish a single market of goods and services as well as capitals and skilled labors (Rillo, 2018; Ponziani, 2019).

In 2015, ASEAN then advanced tighter cooperation in all aspects to establish a single ASEAN community. Furthermore, ASEAN constructed three pillars of the ASEAN community: Economic Community, Political-Security Community and Socio-Cultural Community. They also established the connectivity through the physical connectivity, the regulation connectivity and the people to people connectivity to ensure that ASEAN'S are properly connected.

Due to the blueprints of the ASEAN Economic Community or AEC and the ASEAN connectivity, several implementation plans to establish single market and production base, a high competitive economic region, equitable economic development and full integrations into the global economy were conducted. Since then the financial sectors in this region have become more integrated than ever before, the financial integration consequently made ASEAN gain in low cost of capital and ease to allocate capital (Prukumpai and Sethapramote, 2018).

The stock exchange markets in the ASEAN economic community could be divided into four groups. The first group was the Singapore stock exchange, which was the financial hub for the whole region. The second group was the stock exchange of Thailand and Bursa Malaysia, which were well established domestic market. The third group were the Indonesia exchange and the Philippines exchange, which were still developed markets and the last group were underdeveloped markets in the rest countries (Shimizu, 2014).

Since then, ASEAN has become a safety area against sudden capital outflow (Harvey, 2017) and more attractive to foreign direct investment, especially the ASEAN-5 including Thailand, Singapore, Malaysia, Indonesia, and the Philippines (Tri *et al.*, 2019). Table 1.2 showed that among the ASEAN-5 stock exchange markets, the stock exchange of Thailand had the highest stock traded values in 2019, while the Singapore exchange had the greatest market capitalization.

Table 1.2 The market capitalization and values of stock traded in current billion USD of the ASEAN-5 stock exchange markets in 2019

Stock exchange market	Market capitalization	Values of stocks traded
The Stock Exchange of Thailand	569.2	367.1
Singapore Exchange	697.3	114.2
Bursa Malaysia	403.9	108.6
Indonesia Exchange	523.3	117.9
Philippines Exchange	275.3	29.9

Source: World Bank (2021)

Additionally, the financial infrastructure of the ASEAN-5 economy has been steadily changing over the past fifteen years. This development likely affects the linkages among ASEAN stock market. Moreover, after the global financial crisis, the United State dollar became less stable (Gavranic and Miletic, 2016) and caused instability in the world's monetary system (Staszczak, 2015). This situation possibly affected the exchange rate volatility of the ASEAN-5 currencies.

1.6 Plan of the study

This study started with the application of the natural cubic spline function to estimate financial volatility of the ASEAN-5 financial time series. The Monte Carlo simulation was conducted to demonstrate that the natural cubic spline function was practical to model financial volatility and compared the performance of this volatility estimation method to the smoothing GARCH (1,1) volatility method suggested by previous study (Awalludin and Saelim, 2016). The comparative results from the Monte Carlo simulation as well as the empirical results of the estimated volatility of the ASEAN-5 financial time series were obtained and reported in the first proceeding paper entitled, "Maximum likelihood estimation of non-stationary variance", exhibited in Appendix III.

Then the extend application of the natural cubic spline function on estimation time-varying correlation coefficient of the ASEAN-5 stock market indexes was consequently proposed and presented in the first article entitled, "The time-varying correlation estimator using the natural cubic spline volatility", reported in Appendix I. The Monte Carlo simulation was employed for this article to showed that this proposed method, which applied the natural cubic spline function and the indirect covariance concept, was practical in estimating time-varying correlation coefficients of the investigated financial time series. Furthermore, the Monte Carlo simulation was conducted and provided the comparison between the performance of the proposed method and the performance of the existing time-varying correlation coefficient estimators, comprised of the backward rolling correlation coefficient estimator, the centered rolling correlation coefficient estimator, and the dynamic conditional correlation model.

Since a goodness of fits of the natural cubic spline function was depended on its number of knots. Instead of applying the subjective selection, the candidate model selection criteria, including the Akaike's Information Criteria, Bayesian Information Criteria, General Cross-Validation, and Modified General Cross-Validation, were examined for their performance in determining a proper number of knots for the natural cubic spline function by the Monte Carlo simulation. The comparative simulation results and corresponding empirical results on estimating volatilities the ASEAN-5 exchange rates were obtained and presented in the second proceeding paper entitled “The use of information criteria for selecting number of knots in natural cubic spline volatility estimation” and the second article entitled “Estimating the natural cubic spline volatilities of the ASEAN-5 exchange rates” as exhibited in Appendix IV and II, respectively.

Chapter 2

Methodology

Following the objectives of this study, the financial time series of the ASEAN-5 were examined in order to reveal changing pattern of financial volatility and time-varying correlation which indicated financial stability and integration in ASEAN economic community from 2001 to 2020. The methodology employed in this study was divided into four parts.

The first part presented sources of data and steps on preliminary analysis following the method smoothing GARCH (1,1) volatility by the natural cubic spline function or the GARCH-NCS method. For the second part, the application of the natural cubic spline function was proposed as the alternative method to estimate financial volatilities of the ASEAN-5 financial time series and then was assessed its performance by the Monte Carlo simulation I. In this part, the natural cubic spline function was simply applied with a subjectively selected number of quantile knots.

The third part proposed the method that employed the natural cubic spline function with a number of quantile knots and the indirect covariance concept for estimating time-varying correlation coefficients of the ASEAN-5 stock market indexes and then conducted the Monte Carlo simulation II to assess performance of this time-varying correlation estimator and compare to performance of the other three time-varying correlation coefficient estimators.

The last section described the process to investigate the use of model selection criteria for selecting a number of quantile knots of the natural cubic spline function in financial volatility estimation by the Monte Carlo simulation III and the application of the natural cubic spline function with a number of quantile knots selected by model selection criteria to estimate financial volatilities of the bilateral exchange rates and the effective exchange rates of ASEAN-5. Details of each parts were presented as follows.

2.1 Data and preliminary analysis

Financial time series data of the ASEAN-5 from January 1, 2001 to December 31, 2020 were obtained from two sources. Series of stock market indexes of the ASEAN-5 were collected from Bank of Thailand's website (Bank of Thailand, 2021), while series of the bilateral exchange rates and the effective exchange rates of the ASEAN-5 were gathered from Bank for International Settlements' website (Bank for International Settlements, 2021a; Bank for International Settlements, 2021b).

Regarding further analysis, the logarithm returns according to these time series were calculated as following equation.

$$r_t = \ln\left(x_t\right) - \ln\left(x_{t-1}\right) \tag{2.1}$$

where the logarithm returns denoted by r_t and x_t was the observation of the investigated time series at time t. In the case that stock market index or exchange rate data was not available, the logarithm return was assumed to be equal to the logarithm return in previous day. All return series were examined by the descriptive statistics.

Following the GARCH-NCS method for investigating changing pattern of financial volatility, thus the first step of preliminary analysis was to estimate GARCH (1,1) volatility and then extract volatility's changing pattern as the GARCH-NCS outcomes by applying the natural cubic spline function with 22 quantile knots. The number of quantile knots was set at 22 knots in order to provide interval between knots at approximately 250-trading days. This interval was capable to capture inclusive volatility's changing pattern of financial time series as suggested in previous studies (Meric *et al*., 2012; Wang and Xie, 2013; Tiwari *et al*., 2016; McMillan, 2019). The GARCH-NCS outcomes was plotted and compared to the GARCH (1,1) volatilities and the absolute returns series of the investigated financial time series.

2.2 Applying the natural cubic spline function to estimate financial volatilities of the ASEAN-5 financial time series and the Monte Carlo simulation for assessing performance of this financial volatility estimation method

To reveal volatility's changing pattern of the ASEAN-5 financial time series in order to examine financial stability of the ASEAN economic community during 2001-2020, the natural cubic spline function with a number of quantile knots, abbreviated as the NCSV method, was applied to estimate financial volatility and then performance of this proposed method was assessed by the Monte Carlo simulation.

2.2.1 The financial volatility estimation method using the natural cubic spline function

The financial volatility estimation method using the natural cubic spline function or the NCSV method proposed in this study is to apply the natural cubic spline function to the absolute value of returns series. This estimation model assumed that financial returns is a product of time-varying volatility and random noise as succeeding equation.

$$r_t = s_t \varepsilon_t \tag{2.2}$$

where logarithm returns denoted by r_t. Random noise, denoted by ε_t, is normal distributed with a zero mean and a unit standard deviation, while time-varying volatility, denoted by s_t is parameterized as the natural cubic spline function with respect to time *t* as follows.

$$s_t = \alpha + \beta t + \sum_{k=1}^{p-2} \theta_k \left((t - t_k)_+^3 - \frac{t_p - t_k}{t_p - t_{p-1}} (t - t_{p-1})_+^3 + \frac{t_{p-1} - t_k}{t_p - t_{p-1}} (t - t_p)_+^3 \right) \tag{2.3}$$

This natural cubic spline function includes *p* knots, placed at t_k (*k* = 1, 2, 3, …, *p*) which is a quantile order *k* of time *t* in the interval $[1, T]$. *T* was a number of observations of the return series. Function $(t - t_k)_+$ is a plus function that equal to $t - t_k$ for $t > t_k$ and 0 for otherwise. Parameters of the function (α, β, and θ_k) can be estimated by maximizing the log likelihood function expressed as follows.

$$L(\alpha, \beta, \theta_k) = \sum_{t=1}^{T} \left(-\log(s_t) - \frac{(|r_t|)^2}{2s_t^2} \right) \tag{2.4}$$

The log likelihood function is maximized by the Newton-Raphson method with Marquardt damping factor as succeeding equation.

$$v_{j+1} = v_j - dH_j^{-1} \times w_j \tag{2.5}$$

In the iteration process j, v_j is a $p \times 1$ matrix that contains the estimates of parameters α, β, and θ_k. A Marquardt damping factor for preventing overshooting in the iteration is expressed as d, while w_j and H_j, a $p \times 1$ matrix and a $p \times p$ matrix, contain the first derivative and the second derivative of likelihood functions according to each parameter as follows.

$$w_j = \begin{bmatrix} \frac{\delta L}{\delta \alpha} & \frac{\delta L}{\delta \beta} & \frac{\delta L}{\delta \theta_1} & \cdots & \frac{\delta L}{\delta \theta_k} \end{bmatrix} \tag{2.6}$$

$$H_j = \begin{bmatrix} \frac{\partial^2 L}{\partial \alpha\, \partial \alpha} & \frac{\partial^2 L}{\partial \alpha\, \partial \beta} & \frac{\partial^2 L}{\partial \alpha\, \partial \theta_1} & \cdots & \frac{\partial^2 L}{\partial \alpha\, \partial \theta_k} \\ \frac{\partial^2 L}{\partial \alpha\, \partial \beta} & \frac{\partial^2 L}{\partial \beta\, \partial \beta} & \frac{\partial^2 L}{\partial \beta\, \partial \theta_1} & \cdots & \frac{\partial^2 L}{\partial \beta\, \partial \theta_k} \\ \frac{\partial^2 L}{\partial \alpha\, \partial \theta_1} & \frac{\partial^2 L}{\partial \beta\, \partial \theta_1} & \frac{\partial^2 L}{\partial \theta_1 \partial \theta_1} & \cdots & \frac{\partial^2 L}{\partial \theta_1 \partial \theta_k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 L}{\partial \alpha\, \partial \theta_k} & \frac{\partial^2 L}{\partial \beta\, \partial \theta_k} & \frac{\partial^2 L}{\partial \theta_1 \partial \theta_k} & \cdots & \frac{\partial^2 L}{\partial \theta_k \partial \theta_k} \end{bmatrix} \tag{2.7}$$

Consequently, the standard errors of parameters α, β, and θ_k can be obtained from the squared root of $-diag\left(H_j^{-1}\right)$.

In order to estimate the financial volatility following the NCSV method, the natural cubic spline volatility, of the ASEAN-5 financial time series, a number of quantile knots was subjectively selected based on the assumption that this number of knots had to be sufficient to reveal the changing pattern of financial volatility. Since previous studies suggested that knots with an interval of 250 trading-days were sufficient to reveal the changing pattern of financial volatility (Meric *et al.*, 2012; Wang and Xie, 2013; Tiwari *et al.*, 2016; McMillan, 2019). Therefore, this study set a number of quantile knots to 22 knots with an interval between knots of approximately 250 trading-days for estimating volatility in order to capture inclusive volatility's changing pattern of the investigated financial time series.

2.2.2 The Monte Carlo simulation for assessing performance of the financial estimation method using the natural cubic spline function

This study set the Monte Carlo simulation I to assess the performance of the financial estimation method using the natural cubic spline function or the NCSV method and compare to the performance of the GARCH-NCS method with the condition of various number of quantile knots.

This simulation assumed that volatility with known changing pattern was prior determined, consequently the simulated returns, r_t, were generated as succeeding equation.

$$r_{i,t} = \sigma_{i,t}\varepsilon_t \tag{2.8}$$

t is time ordered from 1 to 5,000, which is nearly 20 years as the investigate series, and $\sigma_{i,t}$ is volatility with known changing patterns. This study applied four sinusoidal function to generate four types of volatilities as following equations.

$$\sigma_{1,t} = 0.01 + 0.2\left(\frac{t}{5000}\right)^2\left(1-\frac{t}{5000}\right)^2 \tag{2.9}$$

$$\sigma_{2,t} = 0.01\left(\sin\left(12\left(\frac{t}{5000}+1\right)\right)\right)\times\left(\cos\left(2\left(\frac{t}{5000}+1\right)\right)\right)\times\left(\sin\left(3\left(\frac{t}{5000}+1\right)\right)\right)+0.015 \tag{2.10}$$

$$\sigma_{3,t} = 0.01\left(\sin\left(24\left(\frac{t}{5000}+1\right)\right)\right)\times\left(\cos\left(8\left(\frac{t}{5000}+1\right)\right)\right)\times\left(\sin\left(3\left(\frac{t}{5000}+1\right)\right)\right)+0.016 \tag{2.11}$$

$$\sigma_{4,t} = 0.002\left(\sin\left(48\left(\frac{t}{5000}+1\right)\right)\right)+0.35\left(\frac{t}{5000}\right)^2\left(0.85-\frac{t}{5000}\right)^2+0.01 \tag{2.12}$$

ε_t is random noise with fat-tailed distribution, generated according to the concept in Huber (1964) as follows.

$$\varepsilon_t = \begin{cases} c + a(z_t - c) & , z_t > c \\ z_t & , -c < z_t < c \\ -c + a(z_t + c) & , z_t < -c \end{cases} \tag{2.13}$$

z_t is normal distributed noise with zero mean and unit standard deviation. c was set as a critical point which was equal to 1.25 and a was a weight for stretching tail of distribution which was equal to 2.5. For each type of volatility, 200 series of simulated returns were generated. Samples of simulated returns in absolute term with respect to each volatility with known changing pattern were showed in Figure 2.1.

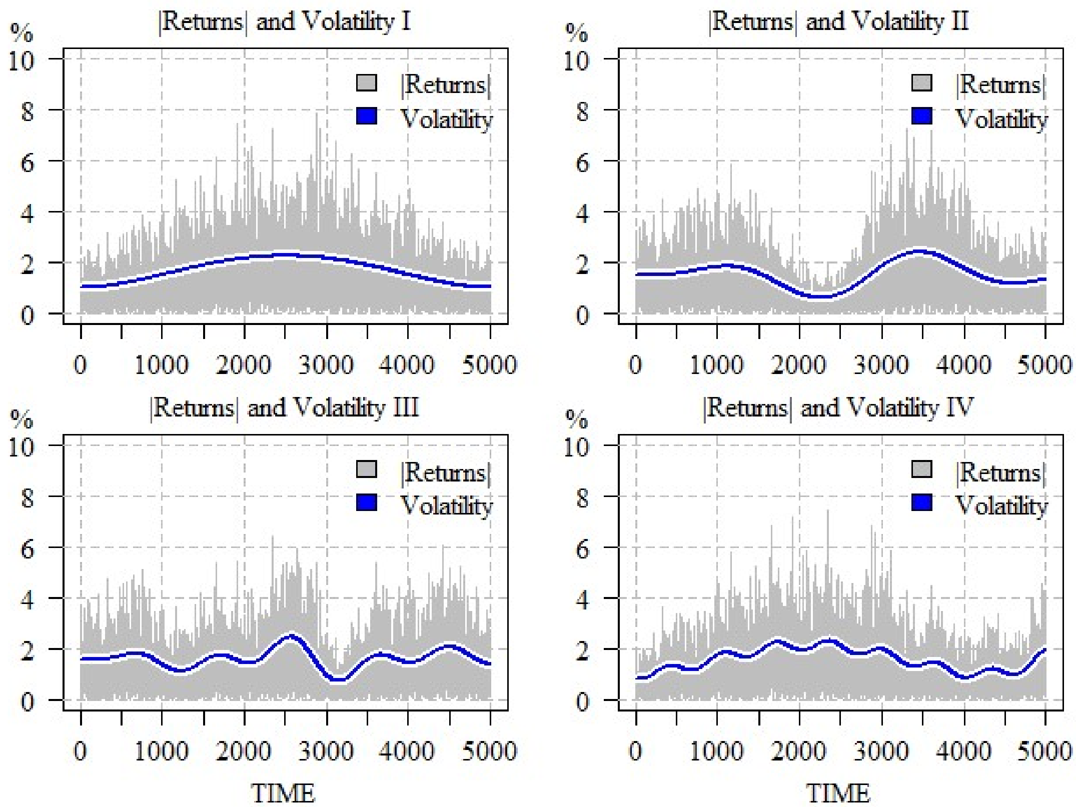


Figure 2.1 Samples of the absolute values of simulated returns series generated from four known volatilities.

Regarding properties of a natural cubic spline function, a number of knots critically effected to the financial volatility estimated by the NCSV method as well as the outcomes of the GARCH-NCS method. Therefore, this study determined a set of several number of quantile knots (k) from 3, 5, 11, 15, 21, 25, 31, 35, 41, 45, and 51 knots. These number of quantile knots were orderly applied with both methods in order to compare performance between these two methods with respect to varying number of knots.

To assess the performance of these two methods, the outcomes of the GARCH-NCS method and the financial volatility estimated by the NCSV method, should be close to the prior-determined volatility's changing pattern. Therefore, this study applied the root mean squared errors $RMSE_{j,k}$ as an indicator which was expressed as following equation.

$$RMSE_{j,k} = \sqrt{\frac{\sum_{t=1}^{T}\left(\sigma_{i,t} - s_{j,k,t}\right)^2}{T}} \tag{2.14}$$

where j was type of changing pattern of prior-determined volatility $\sigma_{j,t}$. The value of k was set as 1 for the outcomes of the GARCH-NCS method and was set as 2 for the financial volatility estimated by the NCSV method. The method that provided the minimum root mean squared errors thus became the superior method in revealing volatility's changing pattern.

2.3 Estimating the time-varying correlation coefficients of the ASEAN-5 stock market indexes by the natural cubic spline function and the Monte Carlo simulation to assess performance of this estimation method

To illustrate co-movements of the ASEAN-5 stock market indexes which indicated financial integration in the ASEAN economic community, the time-varying correlation coefficients estimator using the natural cubic spline function with a number of quantile knots or the NCSV-TVC estimator was proposed and applied for estimating time-varying correlation coefficients of the ASEAN-5 stock market indexes during 2001-2020. Moreover, the Monte Carlo simulation was employed to compare performance of this time-varying correlation estimator to the other three time-varying correlation coefficient estimators which are the backward rolling correlation coefficient estimator, the centered rolling correlation coefficient estimator, and the dynamic conditional correlation or DCC estimator.

2.3.1 The time- varying correlation coefficient estimator using the natural cubic spline function

This study introduces a time-varying correlation coefficient estimator by employing the natural cubic spline function with a number of quantile knots and indirectly derived covariance. Details of this estimator were described as follows.

Suppose there are two returns series, x and y. The correlation coefficient between x and y or ρ_{xy} can be formulated as a following equation.

$$\rho_{xy} = \frac{\operatorname{cov}(x, y)}{\sqrt{\operatorname{var}(x)\operatorname{var}(y)}} \tag{2.15}$$

However, using the identity "variance of a sum and variance of a difference of random variables" as described in Tsay (2012) and Sclove (2013), the covariance between x and y or $\operatorname{cov}(x,y)$ can be indirectly derived as follows.

$$\operatorname{var}(x+y)=\operatorname{var}(x)+\operatorname{var}(y)+2\operatorname{cov}(x,y) \tag{2.16}$$

$$\operatorname{var}(x-y)=\operatorname{var}(x)+\operatorname{var}(y)-2\operatorname{cov}(x,y) \tag{2.17}$$

Subtract equation 2.14 with equation 2.15 then, $\operatorname{cov}(x,y)$ becomes

$$\operatorname{cov}(x,y)=\frac{\operatorname{var}(x+y)-\operatorname{var}(x-y)}{4} \tag{2.18}$$

Assume that this identity continuously holds for time-varying covariance. Then, time-varying covariance between return series x and y can be obtained by the variance of a sum ($x+y$) and a difference ($x-y$) of these two return series.

Let $s_{x,t}$, $s_{y,t}$, $s_{x+y,t}$ and $s_{x-y,t}$ be the financial volatilities estimated by the natural cubic spline function following the NCSV method of the return series x, y, $x+y$, and $x-y$ respectively. Then, the time-varying correlation coefficients between return series x and y ($\rho_{xy,t}$) can be estimated as the following formula.

$$\rho_{xy,t}=\frac{S_{x+y,t}^{2}-S_{x-y,t}^{2}}{4\sqrt{S_{x,t}^{2}S_{y,t}^{2}}} \tag{2.19}$$

The time-varying correlation coefficients estimator using the natural cubic spline function with quantile knots following the NCSV method or the NCSV-TVC estimator was applied to the return series of the ASEAN-5 stock market indexes and then the time-varying correlation coefficients by the NCSV-TVC estimator were plotted and compared to the time-varying correlation coefficients from the other three estimators which are the backward rolling correlation coefficient estimator, the centered rolling correlation coefficient estimator, and the dynamic conditional correlation model.

To estimate time-varying correlation coefficients of the ASEAN-5 stock market indexes, quantile knots of the natural cubic spline function were set at 22 knots. This number of knots had interval between knots approximately 250-trading days. Therefore, the size of rolling window of the rolling estimators was set at 250-trading days as well. This determination of knots interval and size of rolling window was based on previous studies (Meric *et al*., 2012; Wang and Xie, 2013; Tiwari *et al*., 2016; McMillan, 2019).

2.3.2 The Monte Carlo simulation to assess performance of the time- varying correlation estimator using the natural cubic spline function

This study set the Monte Carlo simulation II to assess the performance of the NCSV-TVC estimator by comparing to the other three time varying correlation coefficient estimators. This assessment comprises of three steps. The first one was to generate two series of 4,000 returns ($r_{x,t}$ and $r_{y,t}$) with zero mean, known volatilities ($\sigma_{x,t}$ and $\sigma_{y,t}$) and known correlation coefficient ($\rho_{xy,t}$) as follows.

$$\begin{pmatrix} r_{x,t} \\ r_{y,t} \end{pmatrix} \sim n\left[0, \begin{pmatrix} \sigma_x^2 & \rho_{xy,t}\sigma_x\sigma_x \\ \rho_{xy,t}\sigma_x\sigma_x & \sigma_y^2 \end{pmatrix}\right] \tag{2.20}$$

$$\sigma_{x,t} = 0.01\left(\sin\left(12\left(\frac{t}{4000}+1\right)\right)\right)\times\left(\cos\left(2\left(\frac{t}{4000}+1\right)\right)\right)\times\left(\sin\left(3\left(\frac{t}{4000}+1\right)\right)\right)+0.015 \tag{2.21}$$

$$\sigma_{y,t} = 0.4\left(\frac{t}{4000}\right)^2\times\left(1-\frac{t}{4000}\right)^2+0.01 \tag{2.22}$$

Furthermore, this study generated four types of known correlation coefficients ($\rho_{xy,t}$), consisting of (1) I: Constant, (2) II: Step, (3) III: Periodic, and (4) IV: Non-periodic as following equations.

$$\rho_{xy,t} = (0 \times t) + 0.7 \tag{2.23}$$

$$\rho_{xy,t} = \begin{cases} (0 \times t) + 0.3, & 1 \leq \mathrm{t} \leq 2000 \\ (0 \times t) + 0.7, & 2001 \leq \mathrm{t} \leq 4000 \end{cases} \tag{2.24}$$

$$\rho_{xy,t} = 0.5 + 0.4\left(\cos\left(\frac{2\pi t}{2000}\right)\right) \tag{2.25}$$

$$\rho_{xy,t} = 0.5 + 0.2\left(\cos\left(\frac{2\pi t}{1000}\right)\right) + 0.2\left(\cos\left(\frac{2\pi t}{2500}\right)\right) \tag{2.26}$$

The examples of the absolute values of simulated returns ($r_{x,t}$ and $r_{y,t}$) with their corresponding volatility and the four types of known correlation coefficients were showed in Figures 2.2 and 2.3, respectively.

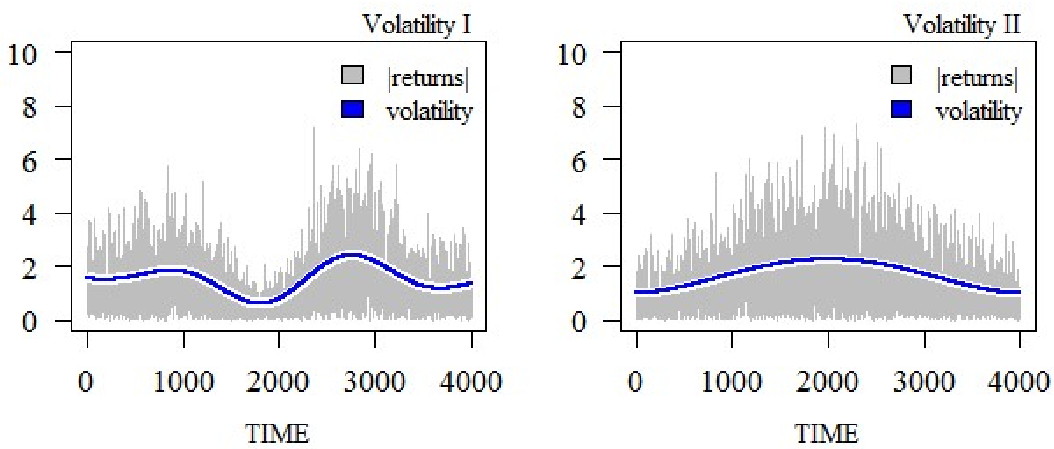


Figure 2.2 Examples of absolute returns with their known volatility

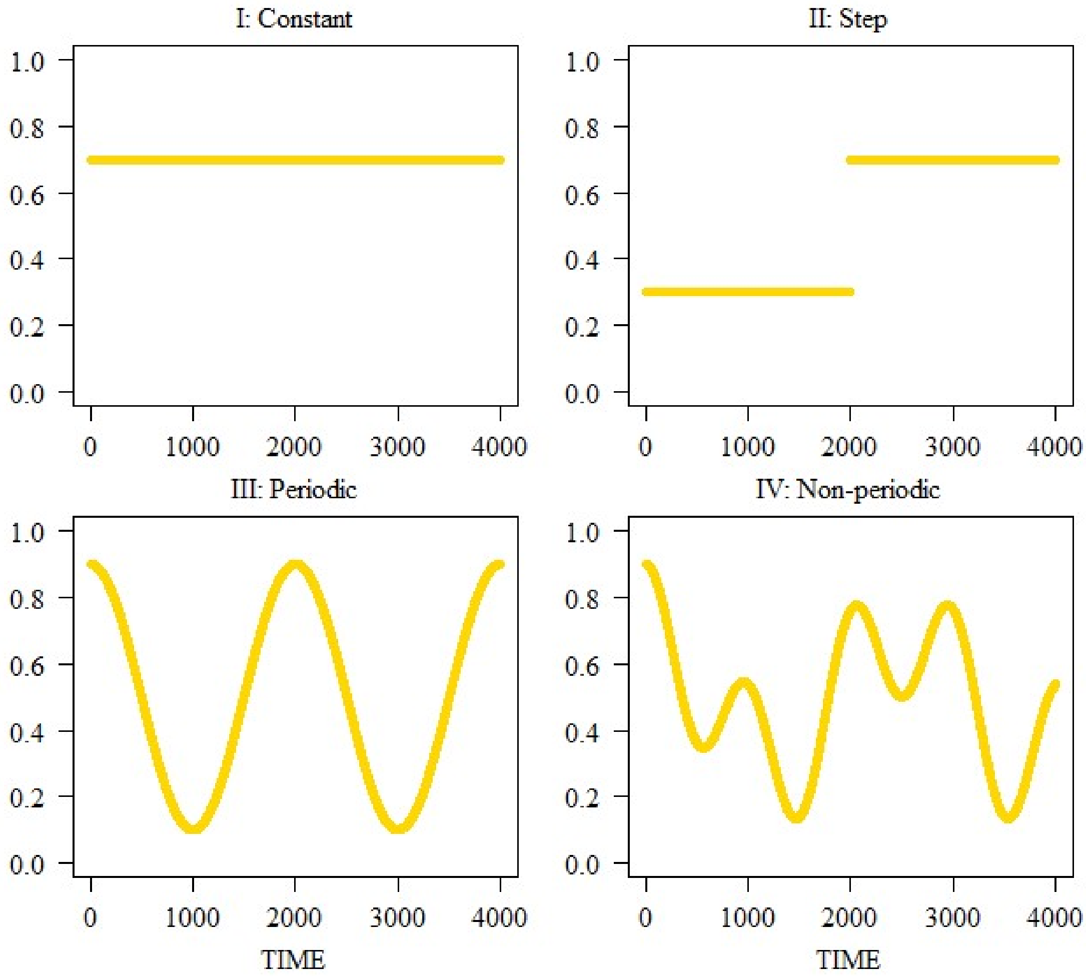


Figure 2.3 Four types of known correlation

The second step of the simulation was to estimate time-varying correlation coefficients by four estimators. This study determined 250-trading days as the time-interval for the estimation of time-varying correlation which are the size of the rolling windows and the interval between quantile knots for the NCSV-TVC estimator.

This selected number of quantile knots applied to the natural cubic spline function was based on previous studies (Meric *et al*., 2012; Wang and Xie, 2013; Tiwari *et al*., 2016; McMillan, 2019). The 250 trading-day interval was almost one trading-year which was capable of capturing the short-term pattern of the time-varying correlation coefficients.

Since the correlation coefficient patterns were pre-specified, the third step was to compare the accuracy of these estimators by using the mean absolute error (MAE) as follows.

$$\mathrm{MAE}_k = \frac{\sum_{t=1}^{T} \left| \rho_{xy,t} - \hat{\rho}_{xy,t} \right|}{T} \tag{2.27}$$

where $\rho_{xy,t}$ and $\hat{\rho}_{xy,t}$ were the known correlation coefficients and the estimated correlation coefficients obtained by the estimators, k was the type of correlation coefficients estimator and T was the number of time-varying correlation coefficients which equal to 4,000.

This study generated 500 pairs of simulated return series, $r_{x,t}$ and $r_{y,t}$, for each type of time-varying correlation. Therefore, the time-varying correlation estimator with a lower MAE_k had a better performance than others.

2.4 Investigating on the use of model selection criteria for selecting the number of quantile knots of the natural cubic spline function to estimate financial volatility and its application on the bilateral exchange rates and the effective exchange rates of ASEAN-5

The straight forward way for determining a number of knots for the natural cubic spline function to estimate financial volatility following the NCSV method was a subjective selection by the professional judgment. This approach might provide bias estimated volatility. Therefore, this study conducted the Monte Carlo simulation to examine the use of the model selection criteria as alternative way to determine an appropriate number of quantile knots for the natural cubic spline volatility estimation.

Four candidate model selection criteria investigated in this study consisting of Akaike's Information Criteria (*AIC*), Bayesian Information Criteria (*BIC*), Generalized Cross Validation (*GCV*), and Modified Generalized Cross Validation (*MGCV*). These four model selection criteria were generally applied for selection a number of knots of the spline function in previous studies (Craven and Wabha, 1979; Friedman, 1991; Lewis and Stevens, 1991; Chen *et al.,* 1997; Engle and Rangel, 2008; Engle *et al.,* 2013 Montoya *et al.,* 2014; Lee *et al.,* 2018; Conrad and Kleen, 2020). The Monte Carlo simulation III for this investigation was set as follows.

This most appropriate model selection criterion should determine a number of knots that provide the best estimated volatility. Suppose true volatility was pre-specified, initially simulated returns series (r_t) were generated as a random noise with zero mean and pre-specified volatility (σ_t) as following equation.

$$r_t = n\left(0, \sigma_t\right) \tag{2.28}$$

This study determined ten types of pre-specified volatilities as the rolling standard deviation calculated from daily returns of ASEAN-5 bilateral exchange rates with two different rolling windows, 60 and 120 trading days per window following a procedure applied in Engle (2001). Note that the rolling standard deviations with wider windows were less fluctuated than the rolling standard deviation with narrower windows.

Then ten sets of 500 series of 1,500 simulated daily returns were generated corresponding to ten types of pre-specified volatility. The number of simulated daily returns for each series was designated conditional to reduce time utilizing in simulation process. Examples of simulated returns series in absolute term and its corresponding pre-specified volatility are showed in Figure 2.4.

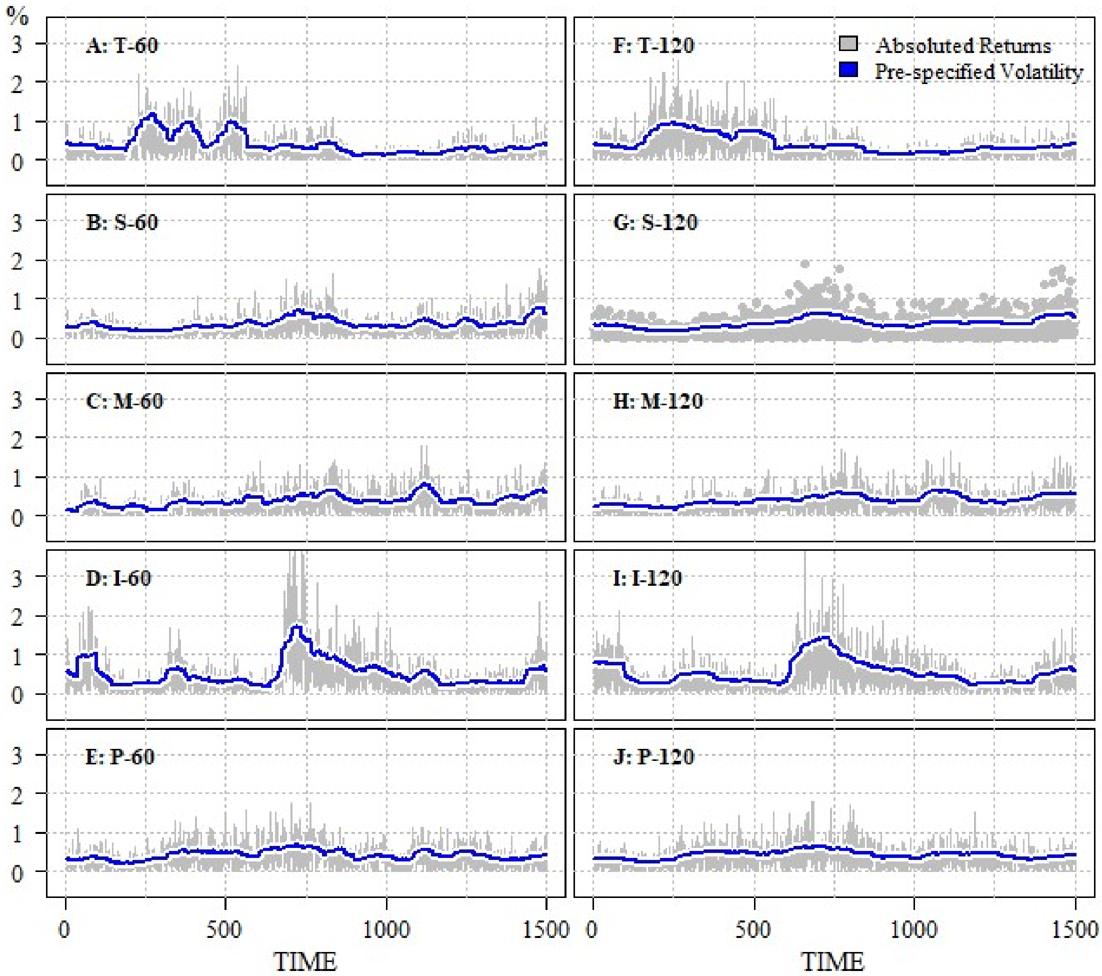


Figure 2.4 Examples of simulated returns series in absolute term and their corresponding pre-specified volatilities

Each model selection criteria were applied to selected a number of quantile knots from a possible set of numbers. This set of numbers initially included three knots, comprised two boundary knots at the first and the last observations and one interior knot in the middle. Moreover, the upper limit of a number of knots in this possible range was set at $T/p > 40$ to ensure that between each knot there were at least 40 observations, equal to number of trading days in one quarter, whereas T was number of observations in simulated returns series and p is a number of knots.

In order to investigate the use of model selection criteria in selecting a number of knots for the natural cubic spline function in the NCSV method, this study determined a number of knots, which provided the volatility model with the least average of $RMSE$, as a benchmark with respect to each pre-specified volatility and then compared to a number of knots selected by the least average of each model selection criterion. Therefore, a model selection criterion that specifies a number of knots closest to the benchmark was a suitable model selection criterion for the natural cubic spline function in the NCSV method.

Each number of quantile knots in the possible range was employed to construct the natural cubic spline volatility model of each simulated returns series. The values of $RMSE$ as well as the values of candidate model selection criteria, AIC, BIC, GCV and $MGCV$, according to each model were then obtained as following formulas.

$$RMSE = \sqrt{\frac{\sum(\sigma_t - s_t)^2}{T}} \tag{2.29}$$

$$AIC = -L + 2p \tag{2.30}$$

$$BIC = -L + p\log(T) \tag{2.31}$$

$$GCV = \frac{T^{-1}\sum_{t=1}^{T}(|r_t| - s_t)^2}{\left(1 - \frac{(p-1)}{T}\right)^2} \tag{2.32}$$

$$MGCV = \frac{T^{-1}\sum_{t=1}^{T}(|r_t| - s_t)^2}{\left(1 - \frac{(p+1)+dp}{T}\right)^2} \tag{2.33}$$

Whereas, s_t and L were estimated values and a log-likelihood value of each volatility model, respectively. The number of observations of return series denoted by T and d in the MGCV formula was a parameter representing the cost of the increased knot in the spline function. The larger number of d tended to signify a fewer number of knots. This study set d equal to 2 following the recommendation in Friedman (1991).

For each pre-specified volatility. Their corresponding values of AIC, BIC, GCV and $MGCV$ were compared to values of $RMSE$. Since $RMSE$ was a benchmark in this comparison, then a model selection criterion, that specifies the number of knots closest to the number indicated by $RMSE$, was the most appropriated criterion for selecting number of quantile knots for the natural cubic spline function in the NCSV method.

Then, the most appropriated model selection criterion was applied to select the number of quantile knots estimate financial volatility of the bilateral exchange rates to USD and the effective exchange rates of the ASEAN-5 following the NCSV method.

To compare the estimated natural cubic spline volatilities of these two types of exchange rates, the volatility ratio was calculated as following formula in order to indicate difference between two exchange rates' volatilities.

$$ratio = \frac{s_{BER}}{s_{EER}} \tag{2.34}$$

where s_{BER} and s_{EER} were the estimated natural cubic spline volatilities of the bilateral exchange rate to USD and the effective exchange rate, respectively.

Chapter 3

Results

This chapter divided the results of the study into four sections. Initially, the first section displayed the ASEAN-5 financial time series investigated in this study and preliminary analysis, then the second section presented the application of the natural cubic spline function with a number of quantile knots to estimate financial volatility of the investigated time series and the performance assessment of the financial volatility estimation method. Furthermore, the third section showed the application of natural cubic spline function with a number of quantile knots on estimating time-varying correlation coefficients of the ASEAN-5 stock market indexes and compared the performance of this time-varying correlation coefficients estimator to the other estimators. The last section illustrated the investigation on the use of model selection criteria for selecting the number of knots of the natural cubic spline function in estimating financial volatility following the NCSV method and its application on estimating financial volatilities of the bilateral exchange rates and the effective exchange rates of the ASEAN-5.

3.1 Preliminary analysis of the ASEAN-5 financial time series

The financial time series from January 1, 2001 to December 31, 2020 of the ASEAN-5 stock market indexes, comprised the stock exchange of Thailand index (SET), the Strait Time index (STI), the Bursa Malaysia index or the Kuala Lumpur index (KLSE), the Indonesia exchange index or the Jakarta stock exchange index (JKSE), and the Philippines stock exchange index (PSE) were retrieved from the website of Bank of Thailand, while bilateral exchange rates in USD (BER) and effective exchange rates (EER) of the ASEAN-5 currencies, consisting of Thai baht (THB), Singapore dollar (SGD), Malaysian ringgit (MYR), Indonesian rupiah (IDR), and Philippines peso (PHP) were obtained from the website of Bank for International Settlement as showed in Figure 3.1.

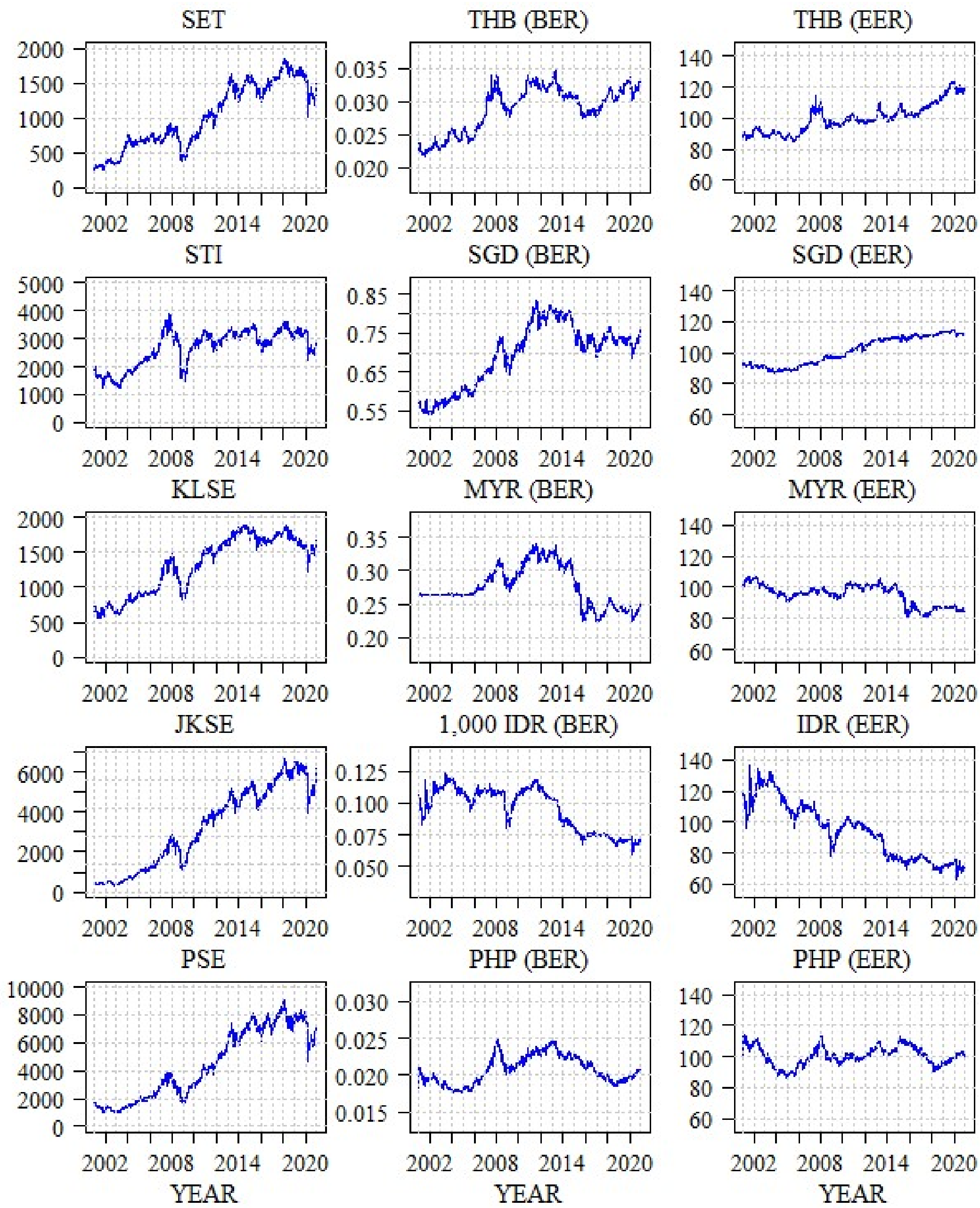


Figure 3.1 Stock market indexes, bilateral exchange rates and effective exchange rates of the ASEAN-5 between January 1, 2001 to December 31, 2020

All stock markets indexes showed the increasing trend during the investigated period. However, all indexes dropped sharply during the 2008 global financial crisis and then gradually increased over the following years, except the Strait Time index (STI). For the period of European debt crisis in 2012, all bilateral exchange rates (BER) precisely appreciated except the Philippines peso. In contrast, the effective exchange rates (EER) were less varied. In the beginning of 2020, the Coronavirus disease (COVID-19) epidemic occurred throughout the world and caused all stock market indexes dropped dramatically.

The bilateral exchange rates showed that the global financial crisis also affected all currencies. Thai baht (THB) and Singapore dollar (SGD) more appreciated, especially between 2001 and 2020. Malaysian ringgit (MYR) and Indonesian rupiah (IDR) were found more depreciated, while Philippine peso (PHP) swung in a narrow range. Additionally, the effective exchange rates of the ASEAN-5 were less fluctuated. However, they reflected the same direction of currency change as their corresponding bilateral exchange rates.

The logarithm returns of corresponding time series were calculated as exhibited in Figure 3.2. For the day that stock market index or exchange rate data was not available, the logarithm return in that day was assume to be the same as the return in previous day. The descriptive statistics of the logarithm returns of the ASEAN-5 financial time series were showed in Table 3.1.

Regards to the return series of the ASEAN-5 stock market indexes, the index of Jakarta Stock Exchange (JKSE) were most fluctuated in the range of -25.78 and 19.07

percent, while the Straits Times Index (STI) were least fluctuated in the range of -8.69 and 7.53 percent.

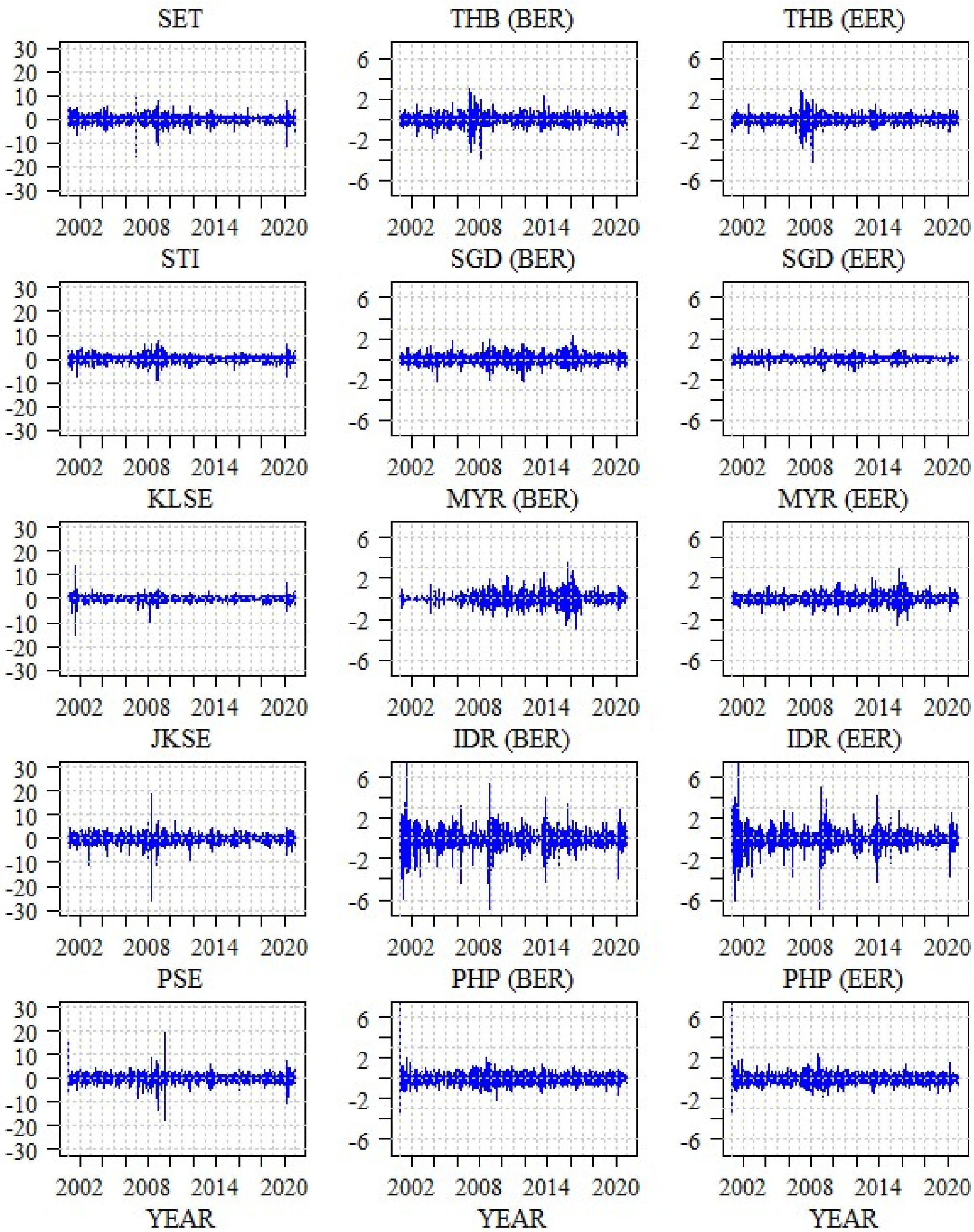


Figure 3.2 Returns series of the stock market indexes, the bilateral exchange rates in USD and the effective exchange rates of the ASEAN-5

Table 3.1 The descriptive statistics of the ASEAN-5 financial return series

Returns (%)	Mean	Minimum	Maximum	Standard deviation	Kurtosis	Skewness
ASEAN-5 Stock market index						
SET	0.048	-16.06	10.57	1.23	16.58	-0.99
STI	0.001	-8.69	7.53	1.09	10.74	-0.53
KLSE	0.019	-15.03	14.19	0.82	49.65	-0.97
JKSE	0.055	-25.78	19.07	1.38	39.17	-1.53
PSE	0.040	-17.65	19.30	1.31	27.51	-0.05
ASEAN-5 Bilateral exchange rates (BER)						
THB	0.011	-3.79	2.90	0.35	13.89	-0.35
SGD	0.008	-2.03	2.21	0.32	6.96	-0.02
MYR	0.001	-2.96	3.61	0.37	11.01	0.30
IDR	-0.004	-6.96	8.80	0.62	24.25	0.08
PHP	0.003	-3.62	14.15	0.40	271.25	7.50
ASEAN-5 Effective exchange rates (EER)						
THB	0.005	-4.15	2.83	0.32	20.43	-0.59
SGD	0.003	-1.27	1.10	0.18	7.74	-0.03
MYR	-0.003	-2.65	3.02	0.32	9.50	0.18
IDR	-0.009	-6.89	9.18	0.58	30.86	0.32
PHP	-0.001	-3.64	13.95	0.38	320.37	8.58

Among the return series of the ASEAN-5 bilateral exchange rates, the Indonesian rupiah (IDR) were most fluctuated in the range of -6.96 and 8.80 percent, while the Singapore dollar were least fluctuated in the range of -2.20 and 2.21 percent.

The same as the return series of the effective exchange rates, the Indonesian rupiah (IDR) were most fluctuated in the range of -6.89 and 9.18 percent, whereas the Singapore dollar (SGD) were least fluctuated in the range of -1.27 and 1.10 percent.

During 2007 to 2009 which was the period that the global financial crisis occurred, the returns of the ASEAN-5 stock market indexes were more fluctuated than the other periods of time. The Thai baht (THB), both bilateral and effective exchange rates, were obviously much fluctuated during that period as well.

In addition, the kurtosis values indicated that all financial returns series likely had fat-tailed distribution indicating inconstant volatility and volatility clustering in these returns series, while the skewness values indicated that all series, except the Philippine peso, likely had symmetric distribution. For the Philippine peso, their kurtosis and skewness values were extremely high indicating the issue of the outliers.

To investigate the changing pattern of financial volatility following the GARCH-NCS method, it had to estimated GARCH (1,1) volatility first and then applied the natural cubic spline function to smooth GARCH (1,1) volatility in order to obtain less fluctuated daily volatility predictors or the GARCH-NCS outcomes which capably exhibited volatility's changing pattern. The GARCH (1,1) volatilities of these time series were estimated and represented by the blue dots as showed in Figure 3.3.

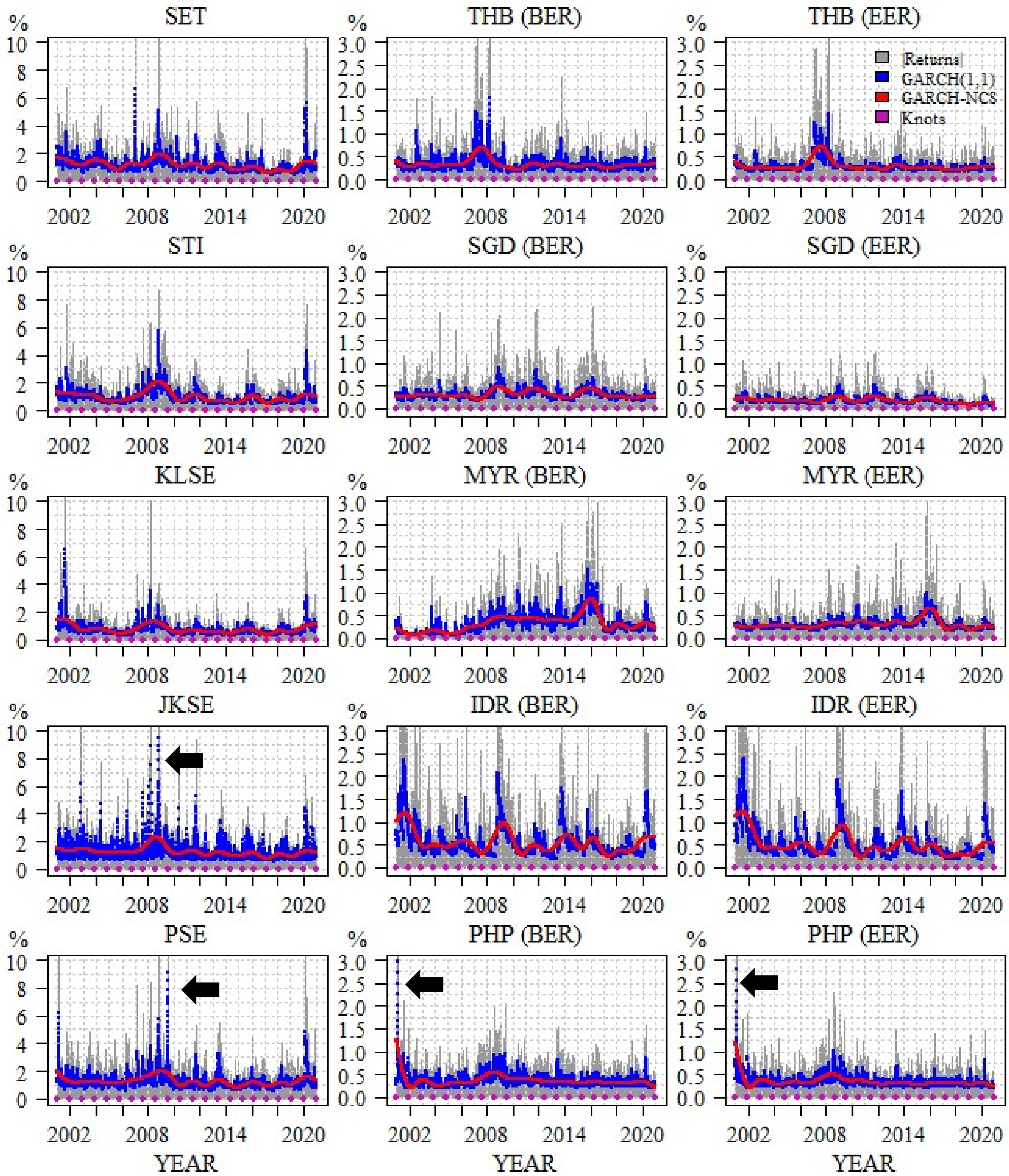


Figure 3.3 The absolute returns, the GARCH (1,1) volatilities, and the GARCH-NCS outcomes of the stock market indexes, the bilateral exchange rates, and the effective exchange rates of the ASEAN-5

Several evidences in this figure indicated a drawback of the GARCH model, whereas a single outlier of financial returns caused the followed overestimated GARCH (1,1) volatility without the supported evidence as the sharp spikes, marked by black arrows in the plots of JKSE, PSE, PHP(BER), and PHP(EER). The number of quantile knots was subjectively determined based on the assumption that it was sufficient to reveal the changing pattern of GARCH (1,1) volatilities. The GARCH-NCS outcomes were showed by the red dots in Figure 3.3. These values were less varied than their corresponding GARCH (1,1) volatilities and capably revealed volatilities' changing patterns of these financial time series.

3.2 Applying the natural cubic spline volatility model with a number of quantile knots to estimate financial volatility of the ASEAN- 5 financial time series and the performance assessment of this financial volatility estimation method

Instead of utilizing the GARCH-NCS method, this study alternatively applied the natural cubic spline volatility or NCSV model to the ASEAN-5 financial time series with a quantile knots and then compared the natural cubic spline volatilities of the investigated time series with their corresponding GARCH-NCS outcome as showed in Figure 3.4.

The natural cubic spline volatilities (blue lines) estimated by the natural cubic spline function with 22 quantile knots following the NCSV method were less frequently varied the same as the GARCH-NCS outcomes (red lines) and also revealed changing patterns of the financial volatilities of the investigated time series. However, with the same number of knots, the natural cubic spline volatilities of several investigated time series were slightly different to the GARCH-NCS outcomes.

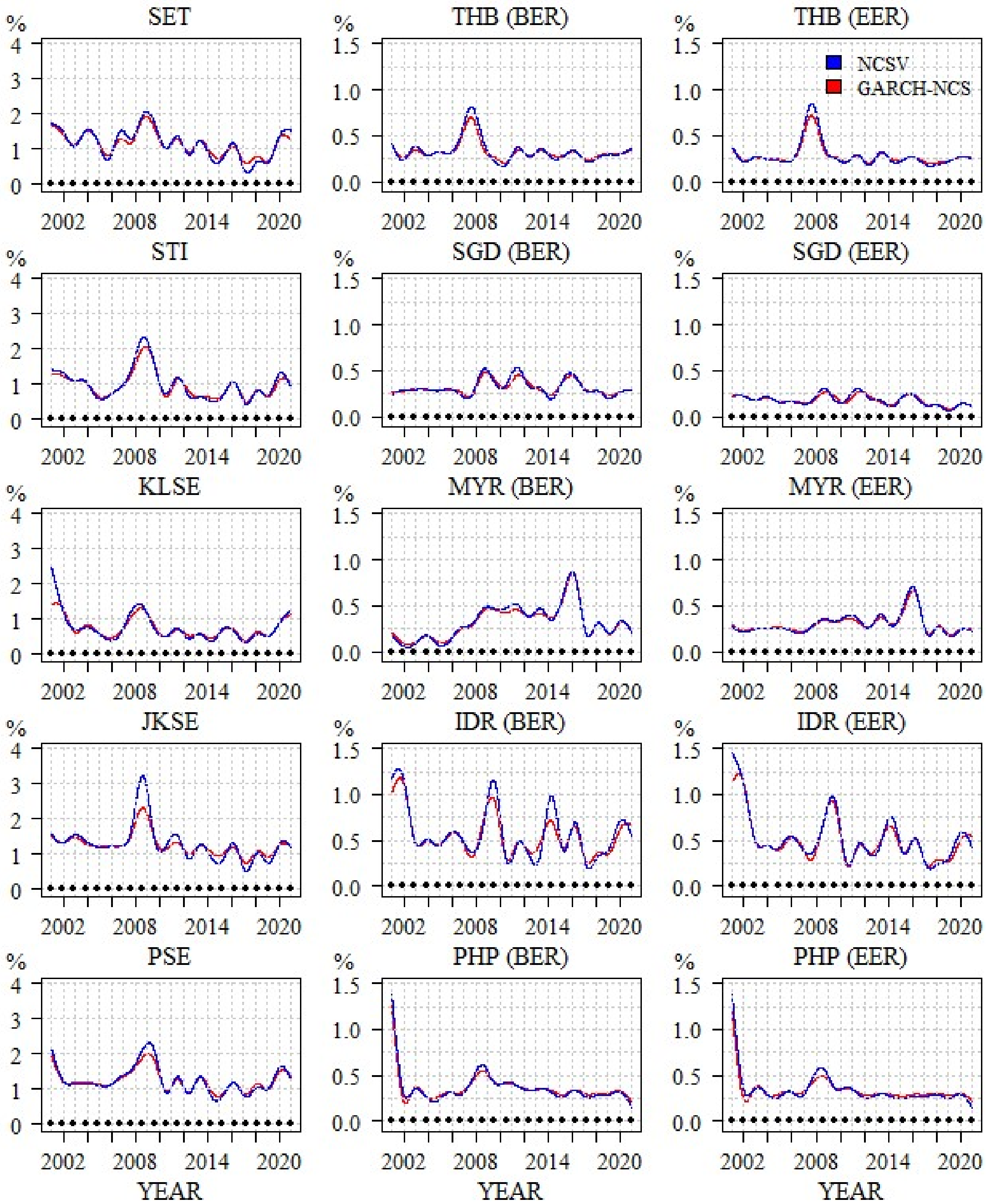


Figure 3.4 The natural cubic spline volatilities estimated by the NCSV method and the GARCH-NCS outcomes of the stock market indexes, the bilateral exchange rates, and the effective exchange rates of the ASEAN-5

As financial volatility cound not be observed directly, so it could not be determined which method was better in representing the true changing patterns. Therefore, the Monte Carlo simulation I was conducted to assess performance of the natural cubic spline volatility estimated by the NCSV method in revealing volatility's changing pattern and compare to the performance of the GARCH-NCS method.

To implement the Monte Carlo simulation I, the natural cubic spline function in the NCSV method and the GARCH-NCS method was applied with 3, 5, 11, 15, 21, 31, 35, and 41 quantile knots to four set of 200 simulated return series categorized by four generated volatilities. Each simulated series contained 5,000 simulated daily returns.

Table 3.2 presents medians of $RMSE$ according to these two methods and varied number of knots applied for the generated volatility I, II, III, and IV. In general, mostly medians of $RMSE$ from the NCSV model were lower than medians of $RMSE$ from the GARCH-NCS method with respect to varied number of knots. For the generated volatility I, Table 3.2 exhibited that 3 knots provided the lowest median of $RMSE$ from both methods at 0.0010 and 0.0026, respectively. For the generated volatility II, Table 3.2 indicated that 5 knots and 11 knots provided the same lowest median of $RMSE$ from the NCSV model at 0.0015, while 5 knots and 11 knots provided the same lowest median of $RMSE$ from the GARCH-NCS method at 0.0035.

Likewise, for the generated volatility III Table 3.2 indicated that 15 knots provided the lowest median of $RMSE$ from both methods at 0.0023 and 0.0054, respectively. In case of the generated volatility IV, Table 3.2 indicated that 22 knots provided the lowest median of $RMSE$ for both methods at 0.0031 and 0.0059, respectively.

Table 3. 2 Medians of root mean squared errors according to the natural cubic spline volatilities and the GARCH-NCS outcomes with the number of knots applied for the generated volatility I, II, III, and IV

Knots	NCSV				GARCH-NCS			
	I	II	III	IV	I	II	III	IV
3	**0.0010**	0.0702	0.0631	0.0118	**0.0026**	0.0628	0.0619	0.0127
5	0.0011	**0.0015**	0.0423	0.0112	0.0030	**0.0035**	0.0415	0.0124
11	0.0017	**0.0015**	0.0061	0.123	0.0038	**0.0035**	0.0088	0.0131
15	0.0024	0.0020	**0.0023**	0.0050	0.0045	0.0038	**0.0054**	0.0069
21	0.0032	0.0027	0.0029	**0.0031**	0.0052	0.0042	0.0057	**0.0059**
25	0.0038	0.0031	0.0034	0.0035	0.0055	0.0045	0.0061	0.0065
31	0.0046	0.0038	0.0042	0.0041	0.0059	0.0048	0.0063	0.0068
35	0.0052	0.0045	0.0047	0.0048	0.0061	0.0051	0.0066	0.0072
41	0.0063	0.0057	0.0058	0.0056	0.0064	0.0053	0.0070	0.0075

In case of the changing pattern of financial volatility was known, the Monte Carlo simulation showed that the root mean squared error practically indicated the suitable number of knots for both methods and showed that the NCSV model provided the lower $RMSE$ than the GARCH-NCS method, which indicated that the NCSV model was better in revealing changing patterns of generated volatilities than the GARCH-NCS method. With this result, it might imply that the natural cubic spline volatilities from the NCSV model was better characterizing volatilities' changing patterns of the ASEAN-5 financial time series than the outcomes of the GARCH-NCS.

3.3 Applying the natural cubic spline volatility with a number of quantile knots on estimating time-varying correlation of the ASEAN-5 stock market indexes and the assessment of its performance

This study proposed the time-varying correlation coefficient estimator using the natural cubic spline function, abbreviated as the NSCV-TVC estimator, and applied this estimator to the returns series of the ASEAN-5 stock market indexes in order to reveal co-movement pattern which was an evidence of the financial integration occurring in the ASEAN economic community.

The returns series of the ASEAN-5 stock market indexes consisting of SET, STI, KLSE, JKSE and PSE. Therefore, they provided ten pairwise time-varying correlation coefficients. The time-varying correlation coefficients estimated by the NCSV-TVC estimator and the other three estimators, which were the backward rolling correlation coefficients estimator, the centered rolling correlation coefficients estimator, and the dynamic conditional correlation or DCC model, were showed in Figure 3.5.

The estimated time-varying correlation coefficients from the backward rolling correlation coefficients estimator, the centered rolling correlation coefficients estimator, and the NCSV-TVC estimator were less fluctuated and provided more obvious co-movement patterns of the ASEAN-5 stock market indexes than the estimated correlation coefficients obtained from the DCC model.

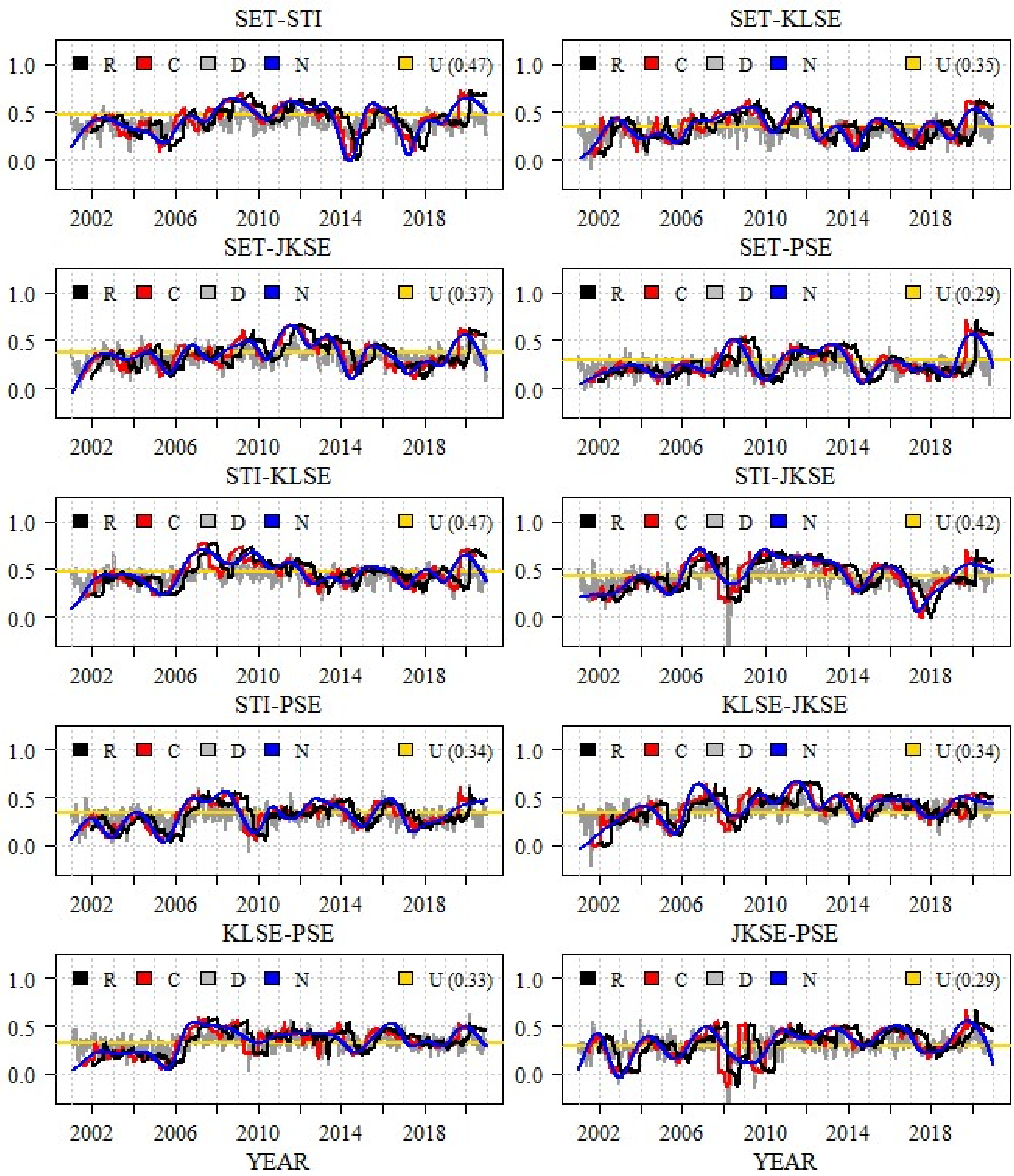


Figure 3.5 Time-varying correlation coefficients of the ASEAN-5 stock market indexes, consisting of backward rolling correlation coefficients (R), centered rolling correlation coefficients (C), the DCC (D), the NCSV-TVC (N), and corresponding unconditional correlation coefficients (U)

The estimated time-varying correlation coefficients obtained from the NCSV-TVC estimator were comparable to the centered rolling correlation coefficients and covered every time points in the period of interests, while the centered rolling correlation coefficients were not applicable especially in the time interval at the beginning and the end of observed period, which was a limitation of this estimator.

The backward rolling correlation coefficients seemly provided the same co-movement patterns of the ASEAN-5 stock market indexes as the centered rolling correlation coefficients, but their position in the observed period were more forward than the position of the center rolling correlation coefficients.

The values in the parenthesis of the topleft legend in each plot were pairwise unconditional correlation coefficients (U) of the ASEAN-5 stock market indexes. These unconditional correlation coefficients indicated that the linkages between the SET-PSE and JKSE-PSE were not reliable as each other's linkages since the unconditional correlation coefficients between the PSE and these two indices were quite low, equal to 0.29 respectively. Contrast to the linkage between the STI and the other indices except for the PSE, their unconditional correlation coefficients were little higher in a range from 0.33 to 0.47.

Compared to the unconditional correlations coefficients, the estimated time-varying correlation coefficients among the ASEAN-5 stock markets obtained from these four estimators were capably separated into five sub-periods, before 2007, 2007-2009, 2009-2012, 2012-2019, and 2019-2020. For the first sub-period, the time-varying correlation coefficients before 2007 of all pair-wises was mostly lower than the level of unconditional correlation coefficients. Then, the time-varying correlations coefficients between 2007-2009, during the period of global financial crisis, raised sharply in the cases of STI-KLSE, STI-JKSE, STI-PSE, KLSE-JKSE and KLSE-PSE, while the time-varying correlations coefficients of SET-STI, SET-KLSE, SET-JKSE, SET-PSE and JKSE-PSE gradually increased. For the third sub-period, the time-varying correlations coefficients among ASEAN-5 stock markets remained higher than the level of unconditional correlation coefficients. However, in the fourth sub-period after the European debt crisis in 2012, there was a decline 2017, which was followed by an increase to the unconditional correlation coefficients level in the fifth sub-period which the COVID-19 began to expose.

Since the estimated time-varying correlation coefficients among the ASEAN-5 stock markets obtained from these four estimators were quite different. To demonstrate that which estimator provided the most appropriated co-movement patterns of the ASEAN-5 stock market indexes, the Monte Carlo simulation II was conducted.

This study generated 500 pairs of simulated returns series for each type of known correlation coefficients pattern and then applied four time-varying correlation coefficient estimators to the simulated return series. Figure 3.6 showed samples of estimated time-varying correlation coefficient according to four estimators and four types of known correlation. The DCC model provided more fluctuated time-varying correlation coefficients than the others, on the other hand the NSCV-TVC estimator provided less fluctuated time-varying correlation coefficients than the others.

Although, the time-varying correlation coefficients estimated by the backward rolling correlation coefficients estimator was able to track the patterns of all types of known correlation coefficients similar to other estimators, considering the position of time its estimated correlation coefficients was obviously located a step ahead of known correlation coefficients pattern.

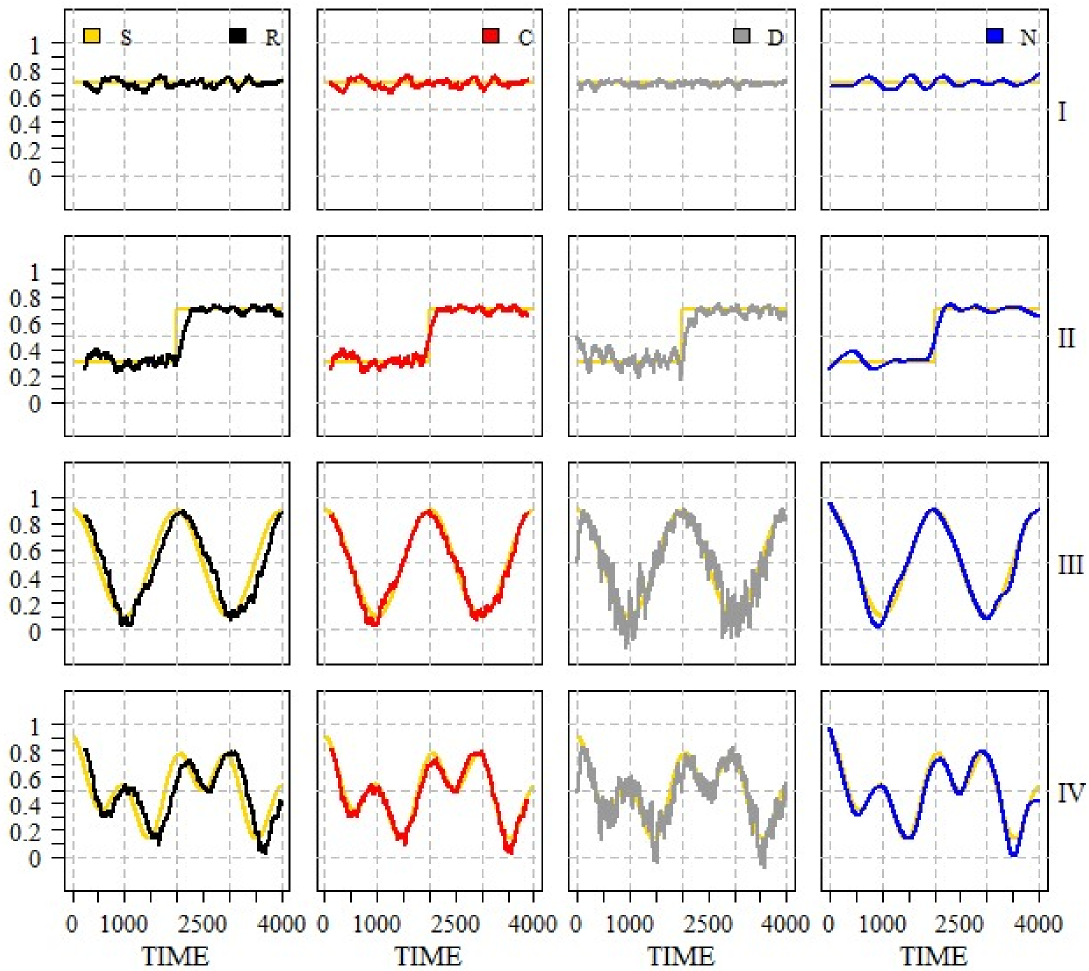


Figure 3.6 Samples of estimated time-varying correlation coefficients according to four types of known correlation coefficients patterns (S), Constant (I), Step (II), Periodic (III), and Non-periodic (IV), and four estimators, the backward rolling correlation coefficient estimator (R), the centered rolling correlation coefficient estimator (C), the DCC model (D), and the NCSV-TVC estimator (N)

Moreover, the backward rolling correlation coefficients estimator and the centered rolling correlation coefficients estimator could not provide the estimated time-varying correlation coefficients for the whole time points in the series of simulated returns. The backward rolling correlation coefficient estimator could not provide the estimated values at the beginning of the observed period, while the centered rolling correlation coefficient estimator could not provide the estimated values at the beginning and the end of the observed period. According to simulation's result of the Monte Carlo simulation II, Table 3.3 summaried mean absolute errors (MAEs) categorized by four correlation coefficient estimators and four types of known correlation coefficient pattern.

Table 3.3 Medians of mean absolute errors (MAEs) categorized by four types of known correlation coefficients patterns, Constant, Step, Periodic, and Non-periodic, and four estimators, the backward rolling correlation coefficient estimator, the centered rolling correlation coefficient estimator, the DCC model, and the NCSV-TVC estimator

Time-varying correlation coefficient estimator	**Known correlation coefficients patterns**			
	Constant	**Step**	**Periodic**	**Non-periodic**
Backward rolling	0.0260	0.0460	0.1075	0.1065
Centerred rolling	0.0261	**0.0408**	0.0385	0.0398
DCC	**0.0152**	0.0501	0.0730	0.0726
NCSV-TVC	0.0264	0.0417	**0.0356**	**0.0386**

For known correlation coefficient pattern I: Constant, the median of MAEs from the DCC model was 0.0152 much lower than the medians of MAEs from other three estimators, indicating that in this case the DCC model performs better than the others.

For known correlation type II, the centered rolling correlation coefficient estimator provide the least median of MAEs, indicating the centered rolling correlation coefficient estimator performes better than the others for the Step correlation pattern.

In case of known correlation type III: Periodic and type IV: Non-periodic, the NCSV-TVC estimator provide lower median of MAEs than the other three estimators. This indicated that the NCSV-TVC estimator provided the most accurate estimated time-varying correlation coefficients.

Following the results of the Monte Carlo simulation II, it capably implied that the NCSV-TVC estimator provided the estimated time-varying correlation coefficients that were effectively fitted to correlation coefficients pattern that was frequently varied and was suitable estimator for applying to investigate co-movement patterns of the ASEAN-5 stock market indexes.

3.4 The use of model selection criteria for selecting the number of knots of the natural cubic spline function for estimating financial volatility and its application on the bilateral exchange rates and the effective exchange rates of ASEAN-5

Although the natural cubic spline function with subjectively selected number of knot was able to estimate financial volatility and capably reveal changing patterns of the estimated volatilities, the result from the Monte Carlo simulation I addressed that number of knots was a critical issue on estimating the natural cubic spline volatility following the NCSV method. Moreover, the subjectively selected number of knots was entirely depended on professional judgement. An inappropriate number of knots might effect to the goodness of fits of the volatility model. Therefore, this study tried to investigate on the use of four model selection criteria for selecting number of knots of the natural cubic spline function in estimating financial volatility following the NCSV method.

This study conducted the Monte Carlo simulation III to compare performances of four model selection criteria, consisting of Akaike's information criteria, Bayesian information criteria, Generalized cross-validation, and Modified generalized cross-validation, in selecting number of knots for the natural cubic spline function in the NCSV method. Ten sets of the simulated returns series according to ten types of pre-specified volatilities were generated and were applied by the natural cubic spline function with a number of knots in possible range varying from 3 to 42 knots. Since each simulated returns series has 1,500 observations, the interval size between knots of the natural cubic spline function thus varies from approximately 40 observations to 750 observations per interval. The averages of *RMSE*, *AIC*, *BIC*, *GCV* and *MGCV* according to the estimated volatility following the NCSV method were calculated. Table 3.4 presented these values categorized by assigned number of knots and five types of pre-specified volatility as follows.

The least average of *RMSE* determined the benchmark number of knots for each pre-specified volatility. The first five groups of simulated returns were generated by 60 trading days rolling standard deviation (A, B, C, D, E), mostly required more number of knots for their best fitted volatility models than the simulated returns generated by 120 trading days rolling standard deviation (F, G, H, I, J). It implies that if true volatility is high fluctuated, it will require more knots to model the natural cubic spline volatility.

Table 3.4 The number of knots selected by model selection criteria for the natural cubic spline volatility models of the ASEAN-5 bilateral and effective exchange rates

Simulated series	***RMSE***	***AIC***	***BIC***	***GCV***	***MGCV***
A: T-60	**25**	**25**	**25**	**25**	**25**
B: S-60	**28**	27	9	**28**	11
C: M-60	**31**	**31**	12	28	12
D: I-60	**29**	27	23	27	23
E: P-60	**26**	23	9	**26**	9
F: T-120	**25**	22	13	16	16
G: S-120	**17**	16	9	**17**	11
H: M-120	**15**	**15**	11	**15**	11
I: I-120	**22**	19	12	17	17
J: P-120	**10**	**10**	9	**10**	**10**

Regarding the benchmark number, a proper knot selection criterion has to select neither too many nor too few numbers of knots than the benchmark number. Among the ten groups of simulated returns, *GCV* selects the number of knot identical to the benchmark number for six groups (A, B, E, G, H, J). For the other four groups (C, D, F, G), *GCV* selects a less number of knots, but it is not much different from the benchmark. Likewise, *AIC* selects the number of knot identical to the benchmark for four groups of simulated returns (A, C, H, J).

In contrast, there is only one group of simulated returns (A) that *BIC* identifies the number of knots identical the benchmark number. For the rest groups, the number of knot indicated by *BIC* is much less to the benchmark number. *MGCV* is a little better than the *BIC*. It selects the same number of knots as the benchmark number for two groups of simulated returns (A, J). The *BIC* and *MGCV* do not perform well for the groups of simulated returns with more fluctuated pre-specified volatility (60 trading days rolling standard deviation). However, they tend to select a number nearer to the benchmark for the groups of simulated returns generated by less fluctuated pre-specified volatility (120 trading days rolling standard deviation). Following the simulation results, *GCV* is the most preferred criterion, while the second-best criterion is the *AIC*. In contrast, the *BIC* provides a small number of knots for most cases.

To determine the proper natural cubic spline function for estimating financial volatilities of ASEAN-5 exchange rates which had 5,219 observations per series, a set of pre-specified number knots is assigned from 3 to 130. Consequently, the size of the interval between knots varies from approximately 40 observations to 2,609 observations per interval. Table 3.5 showed the optimum number of knot selected by each model selection criteria for the natural cubic spline volatility estimation of the ASEAN-5 exchange rates.

The number of knots for the volatility estimation models indicated by *BIC* is relatively fewer than the number obtained by the other three criteria. Likewise, *MGCV* often selects a number of knots identical to a number chosen by *BIC*. Therefore, *BIC* and *MGCV* tend to indicate an under-fitted model. *GCV* and *AIC* select an equivalent number of knots in some cases. However, the behavior in knot selection of *GCV* is more consistent than *AIC*, because in some cases, *AIC* assigns too large number of knots for the volatility estimation model.

Table 3.5 The number of knots selected by model selection criteria for the natural cubic spline volatility estimation models of the ASEAN-5 bilateral and effective exchange rates

Exchange rates	*AIC*	*BIC*	*GCV*	*MGCV*
ASEAN-5 Bilateral exchange rates (BER)				
THB	120	49	**108**	83
SGD	120	38	**120**	31
MYR	88	88	**88**	44
IDR	129	82	**78**	57
PHP	123	44	**28**	28
ASEAN-5 Effective exchange rates (EER)				
THB	129	43	**83**	83
SGD	120	31	**96**	31
MYR	129	33	**117**	33
IDR	129	59	**75**	57
PHP	123	28	**31**	28

Regarding this comparison, the *GCV* is likely to provide an accurate number of knots for estimating the natural cubic volatility of the ASEAN-5 exchange rates. Since the effective exchange rates are less volatile, in some cases *GCV* designates a smaller number of knots than a number of knots of the bilateral exchange rates' volatility estimation models. The volatilities of ASEAN-5 exchange rates estimated by the natural cubic spline volatility function with a number of knots selected by *GCV* are showed in Figure 3.7.

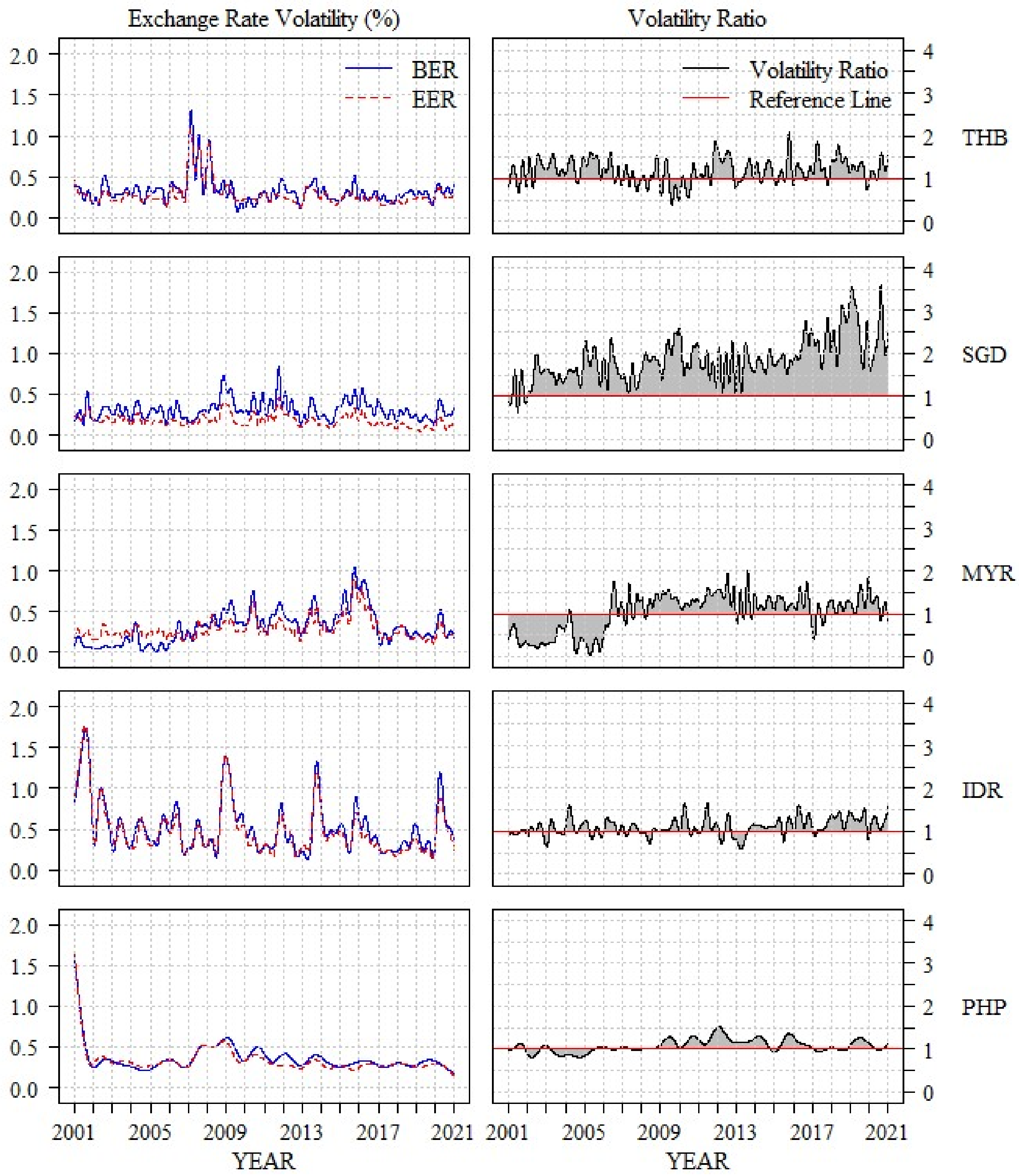


Figure 3.7 The estimated natural cubic spline volatilities of the ASEAN-5 exchange rates and the volatility ratio between bilateral and effective exchange rates.

Graphs in the right column illustrate the comparison of the bilateral exchange rate volatilities (BER) and the effective exchange rate volatilities (EER) of the ASEAN-5 currencies in the same axis, while graphs in the right column show the volatility ratio, the ratio of the bilateral exchange rate volatilities over the effective exchange rate volatilities of the corresponding currencies.The dynamic patterns of the bilateral exchange rate volatility of the investigated currencies were not entirely different from the pattern of the same currencies' effective exchange rate volatility. The BER and EER volatilities of IDR were more fluctuated than the other currencies, while the exchange rate volatilities of SGD and PHP indicated that these two currencies were more stable than the other currencies.

The BER and EER of THB were most fluctuated during the global financial crisis period. The exchange rate volatilities of THB were more than 10 percent in that period, and then diminished to less than 5 percent after the crisis. This showed the exchange rate of THB was stable after the global financial crisis until now. Moreover, the exchange volatilities of THB after the global financial crisis remains lower than the level in the period before the crisis. The EER volatility of MYR was obviously higher than its BER volatility in the period before 2005, that Malaysia adopted the pegging to USD exchange rate policy. After the floating exchange rate policy was implement, the BER and EER volatilities of MYR relatively had the same movement. Furthermore, both exchange rates volatilities of MYR enormously increased during 2014-2016 and drop sharply at the beginning of 2017.

The reference lines in the right graphs of Figure 3.7 indicated the volatility ratio equal to one. For the period that the exchange rate volatility ratios of the ASEAN-5 currencies are higher than the reference line, the bilateral exchange rates of the ASEAN-5 currencies were more fluctuated than its corresponding effective exchange rates. The volatility ratios of the ASEAN-5 currencies were mostly higher than the reference line, especially SGD; its bilateral exchange rate volatility is almost twice higher than its effective exchange rate volatility. The average volatility ratios of THB, SGD, MYR, IDR and PHP are 1.18, 1.85, 1.03, 1.10 and 1.07 respectively.

Chapter 4

Discussions and conclusions

This chapter was organized into three sections. The first section discussed on the comparative performances of the proposed methods in estimating financial volatility and time-varying correlation, and the empirical analysis on the ASEAN-5 financial time series. The second section consequently concluded overall findings of this study. Finally, the third section addressed limitation of this study and recommendation for further studies.

4.1 Discussions

Following the NCSV method, the volatilities of the ASEAN-5 financial time series estimated by the natural cubic spline function with a number of quantile knots, where interval between knots was approximately one trading-year, were sightly different to the GARCH-NCS outcomes from the smoothing GARCH (1,1) volatility method. Although previous study addressed that the GARCH (1,1) model was the most popular volatility model for examining financial volatility (Ledoit *et al.*, 2003), the comparative results from the Monte Carlo simulation I in this study showed that the NCSV method provided estimated volatilities that better revealed the changing pattern of financial volatility than the smoothing GARCH (1,1) volatility method. Therefore, it implied that the estimated volatilities by the natural cubic spline volatility model was better in revealing volatility's changing pattern of the ASEAN-5 financial time-series as well.

The natural cubic spline volatilities exhibited that the ASEAN-5 stock indexes were enormously fluctuated during 2007-2009 which was the period of the global financial crisis. The natural cubic spline volatilities of Thai baht in both exchange rates were also highest during the period of global financial crisis similar to Indonesian rupiah, contrast to Singapore dollar which was less fluctuated. The finding was consistent to Klyuev and Dao (2017) which also showed that the volatility of ASEAN-5 exchange rates were high during the crisis especially Thai baht and Indonesian rupiah.

Furthermore, the patterns of changes in time-varying correlation obtained by the NCSV-TVC estimator exhibited high time-varying correlation coefficients after 2007. These stronger linkages indicated a higher integration in the financial markets of ASEAN. Furthermore, the patterns of change in time-varying correlation between the SET index and the others are more fluctuated than the other patterns which are consistent with the findings in Prukumpai and Sethapramote (2018).

Although the DCC model performed well in case of estimating the time-varying correlation that had constant pattern, the Monte Carlo simulation II demonstrated that the NCSV-TVC estimator and the centered rolling correlation estimator provide better estimates in case of the time-varying correlation's pattern was inconstant. This finding corresponded to Adams *et al.* (2017) which also reported that the DCC model is not a suitable choice when the pattern of changes in correlation is not constant or has structure breaks.

For the investigation of the use of model selection criteria in selecting a number of quantile knots for the natural cubic spline function in estimating financial volatility following the NCSV method, the Monte Carlo simulation III showed that the *GCV* was a preferred criterion. Regarding a number of knots selected by the *GCV* criterion, number of observations in the interval between knots for the natural cubic spline function for estimating financial volatilities of the ASEAN-5 bilateral and effective exchange rates were varying between 44 and 193 observations per interval. These number were much smaller than the number assigned by *BIC* for spline-GARCH model in Engle and Rangel (2008) for the volatilities of the ASEAN-5 stock indexes.

Note that the functional form of the spline function used in this study was a natural cubic spline function. It was more flexible than the quadratic spline function of spline-GARCH model. Thus, the natural cubic spline function in the NCSV method required a more number of knots to provide better volatility's changing patterns. Comparing to 22 knots which were subjectively selected, the estimated volatility by the natural cubic spline function with a number of knots selected by model selection criterion provided more in depth changing patterns of financial volatility.

It was clear that *BIC* provided a smaller number of quantile knots than *GCV* in case of applying the NCSV method. However, it could not conclude that *GCV* was superior than *BIC* in other cases. So it required to compare *GCV* and *BIC* in the other environment, such as in selecting a number of knot for the spline-GARCH model.

Moreover, the natural cubic spline volatilities with a number of quantile knots selected by the *GCV* criterion, showed that the bilateral exchange rates to USD of Malaysian ringgit were less stable between 2015 to 2016, since the depreciation of Malaysian ringgit to the world currencies in October 2015 (Quadry *et al.*, 2017). Malaysia increased their money supply by lowering its interest rate to absorb the exchange rate shock which induced the depreciation in their currencies (Kaur *et al.*, 2019).

Additionally, the volatility of Malaysian ringgit bilateral exchange rates during 2001-2005 was less than the volatility of the effective exchange rates. Since in that period, Malaysia adopted the exchange rate policy that pegged to United States dollar. So its bilateral exchange rates remained constant, while its effective exchange rate was more volatile. After 2005 all ASEAN-5 employed manage floating exchange rate policy (Klyuev and Dao, 2017), the comparison between the estimated volatilities of the bilateral and effective exchange rates showed that the effective exchange rates were obviously more stable than the bilateral exchange rates. This reflected the typical characteristic of the effective exchange rates, which were able to absorb the uncertain exchange rate policies of their trade partners (Thuy and Thuy, 2019).

Furthermore, Singapore dollar was more capable to confront the uncertainty in the international trade and investment than the other currencies. This is because the stability of the USD affects the volatility of the bilateral exchange rates. Several studies are likely to eliminate the influence of the USD instability by employing the effective exchange rate volatility rather than the bilateral exchange rate volatility in order to examine the real stability of the currency (Al-Abri and Baghestani, 2015; Kaur *et al.*, 2019; Thuy and Thuy, 2019).

4.2 Conclusions

This study applied the natural cubic spline function as a method to investigate financial stability and financial integration of the ASEAN during 2001-2020 through financial volatilities and time-varying correlation coefficients of the ASEAN-5 financial time series.

Initially, the natural cubic spline function was proposed for estimating financial volatility that indicated financial stability following the first objective of this study. The NCSV method utilized the natural cubic spline function and maximum log-likelihood estimation. With 22 quantile knots, where an interval between knots was approximately one trading-year, the NCSV method was applied to estimate financial volatility of the ASEAN-5 financial time series during two decades of interests including the stock market indexes, the bilateral exchange rate to USD, and the effective exchange rates. The estimated natural cubic spline volatilities of the investigated time series were less fluctuated than the GARCH (1,1) volatility. In addition, these estimated volatilities well exhibited the inclusive changing patterns of financial volatilities during the period of interests indicating the uncertain financial stability of the ASEAN in that period. Volatilities of the ASEAN-5 stock exchange markets, especially Indonesia stock exchange, were found enormously high during the global financial crisis in 2007-2009, similar to the exchange rate volatility of Thai baht and Indonesian rupiah. While, the least fluctuated exchange rates volatilities of Singapore dollar indicated that this currency was the most stable currency in this region. Moreover, the Monte Carlo simulation conducted in this study showed that the NCSV method provided the estimated volatility that was better in revealing changing pattern of financial volatility than the smoothing GARCH (1,1) volatility method suggested by Awalludin and Saelim (2016).

For the second objective, this study also demonstrated a method to estimate time-varying correlation coefficient coefficients for the ASEAN-5 stock market indexes by employing the NCSV method and the indirect covariance concept. According to the estimated time-varying correlation coefficients, it showed that since ASEAN had an initiative to establish ASEAN economic community in 2007, the estimated time-varying correlation coefficients among ASEAN-5 stock exchange markets were higher than the unconditional correlation coefficient level during 2001-2006. The time-varying correlation coefficients rapidly increased during the global financial crisis in 2007 and continuously remained high level during 2009-2012 indicating interdependences among the ASEAN-5 stock exchange markets which was an evidence of financial integration in the ASEAN economic community. Regarding the comparison to the existing time-varying correlation estimator, the results of the Monte Carlo simulation showed that the time-varying correlation estimator using the NCSV method was better to capture the uncertain patterns of change in time-varying correlation than the dynamic conditional correlation model and provided the estimates of time-varying correlation coefficients for the whole period of interest different to both rolling correlation coefficient estimator.

To achieve the last objective, this study investigated the use of model selection criteria as the alternative way to select a number of quantile knots instead of the subjective selection. The comparative results from the Monte Carlo simulation showed that among four model selection criteria investigated in this study, the Generalized Cross-Validation (*GCV*) was the most preferred model selection criterion for selecting a number of quantile knots for the natural cubic spline function for estimating financial volatility in the NCSV method. While, Bayesian Information Criteria (*BIC*) and Modified Generalized Cross-Validation (*MGCV*) had a tendency to determine a too small number of quantile knots. In case of Akaike's Information Criteria (*AIC*), it often designated a number of quantile knots that provided the estimated volatility which was quite comparable to the pre-specified volatility. However, it had a tendency to select a too many number of knots for the natural cubic spline function for estimating financial volatilities of the empirical datasets.

After applying Generalized Cross-Validation (*GCV*) to select a number of quantile knots for estimating the natural cubic spline volatilities of the ASEAN-5's bilateral exchange rates and effective exchange rates, the designated number of quantile knots for each natural cubic spline function was different corresponding to the behavior of each exchange rate series, varied from 28 knots to 120 knots. The interval between 28 knots was approximately 10 trading-months, while the interval between 120 knots was about 2 trading-months. For the exchange rate that had moderately frequent changing pattern such as Philippines peso, it needed a small number of quantile knots for estimating the natural cubic spline volatility. While the exchange rate that had highly frequent changing pattern, such as Singapore dollar, it needed to estimate the natural cubic spline volatility with a greater number of quantile knots. Moreover, the natural cubic spline function with a number of quantile knots selected by model selection criteria provided more details of volatility's changing pattern than the natural cubic spline function with quantile knots, that their interval between knots was approximately one trading-year.

Additionally, the estimated exchange rate volatilities of the ASEAN-5 currencies revealed inconstant dynamic pattern of the ASEAN-5 exchange rate volatilities. The bilateral exchange rates of the ASEAN-5 currencies obviously were more volatile than the effective exchange rates, especially the bilateral exchange rate of the Singapore dollar, which is almost twice greater than its effective exchange rate. It was a clear evidence showing the stability of the Singapore dollar indicated by its effective exchange rate volatility. Moreover, this difference also obviously indicated the influence of the United States dollar on the volatility of the bilateral exchange rate.

Moreover, in case of Malaysia ringgit the bilateral exchange rate volatility of this currency during pegging exchange rate policy was found lower than the effective exchange rate volatility at the same period. However, after adopting floating exchange rate policy, the bilateral exchange rate volatility gradually increased to the relatively same level of the effective exchange rate volatility. This exhibited the influence of exchange rate policy to currency's stability observed by the volatilities of the bilateral exchange rate and the effective exchange rate.

4.3 Limitations and recommendations for further studies

This study showed that the NCSV method was a recommended method for estimating financial volatility. The natural cubic spline volatility was obviously showed an inclusive changing pattern of financial volatility which indicated time-varied financial stability. Moreover, the NCSV method was practical to estimate time-varying correlation coefficients which revealed co-movement which showed independence between a pair of financial time series, the evidence of financial integration. However, there were some limitations corresponding to methodology used in this study.

First of all, the NCSV method proposed in this study assumed that financial returns had a normal distribution. This assumption was also hold for many volatility models, such as the GARCH model (Poon and Granger, 2003). However, following the stylized facts of financial returns (Cont, 2001), many financial volatility model alternatively determinded the fat-tail distribution for the financial return. So further studies are required to apply the natural cubic spline volatility estimation method with the other types of distribution.

Employing the quantile knots, which knots' locations were changed with respect to the change of a number of knot, was practical for estimating the natural cubic spline volatility in this study. However, this procedure might not assign a most preferred number of knots or a most preferred knots' location for the investigated financial time series. Following Wongsai *et al.* (2017), there were another way to assign knots for the natural cubic spline function by applying non equi-spaced knots to obtain better natural cubic spline fitted values. Further studies are required to examine the influence of non equi-spaced knots on financial volatility estimation following the NCSV method.

Lastly, following the scope of this study there were only four model selection criteria investigated in this study. It might conclude that the Generalized Cross-Validation was more suitable than the other three criteria. However, rather than these four frequently used criteria, there were other criteria that were utilized for selecting number of knots of a spline function in previous studies such as Akaike's information criteria corrected or *AICc* (Montoya *et al.,* 2014), Mallow's *CP* criterion (Lee, 2003) or Hannan-Quinn information criteria (Lee *et al.*, 2018). Therefore, in order to obtain more general conclusion on the use of model selection criteria for selecting a number of quantile knots for the natural cubic spline function, further comparative studies are required to demonstrate the natural cubic spline volatility estimation method with a more coverage of model selection criteria.

References

Adams, Z., Fuss, R., and Gluck, T. (2017). Are correlation constant? Empirical and theorical results on popular correlation models in finance. *Journal of Banking and Finance, 84*(2017), 9-24.

Ahmed, F., and Singh, V. K. (2016). Financial integration among RCEP (ASEAN+6) economies: Evidences from stock and forex markets. *South Asian Journal of Management, 23*(1), 164-188.

Al-Abri, A., and Baghestani, H. (2015). Foreign investment and real exchange rate volatility in emerging Asian countries. *Journal of Asian Economics, 37*, 34-47.

ASEAN Secretariat. (2020). *ASEAN Statistical Yearbook 2020*. Jakarta.

Audrino, F., and Buhlmann, P. (2009). Splines for financial volatility. *Journal of the Royal Statistical Society, 71*(3), 655-670.

Awalludin, S. A., and Saelim, R. (2016). Modelling the Volatility and Assessing the Performance of the Model. The 20th International Annual Symposium on Computational Science and Engineering, Kasetsart University, Bangkok, July 27-29, 2016.

Awalludin, S. A., Ulfah, S., and Soro, S. (2018). Modelling the stock price returns volatility using GARCH (1,1) in some Indonesia stock prices. *Journal of Physics: Conference Series, 948* (2018). Doi: 10.1088/1742-6596/984/1/012068

Bank for International Settlements. (2021a, February 14). Effective exchange rate indices. Retrieved from https://www.bis.org/statistics/full_webstats_eer_d_dataflow_csv_row.zip

Bank for International Settlements. (2021b, February 14). US dollar exchange rates. Retrieved from https://www.bis.org/statistics/full_webstats_xru_current_d_dataflow_csv_row.zip

Bank of Thailand. (2021, February 14). SET & External Stock Market Indexes. Retrieved from https://www.bot.or.th/App/BTWS_STAT/statistics/BOTWEBSTAT.aspx?reportID=132&language=ENG

Bentes, S. R. (2015). Forecasting volatility in gold returns under the GARCH, IGARCH and FIGARCH frameworks: New evidence. *Physica A, 438*, 355–364.

Billio, M., and Caporin, M. (2010). Market linkages, variance spillovers and correlation stablility: Empirical evidence of financial contagion. *Computational Statisitcs and Data Analysis, 54*(2010), 2443-2458.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics, 31*(3), 307-327.

Breiman, L. (1993). Fitting additive models to regression data: Diagnostics and alternative views. *Computational Statistics and Data Analysis, 15*, 13-46.

Chen, G., Abraham, B., and Bennett, G. W. (1997). Parametirc and non-parametric modelling of time series: An empirical study. *Environmetrics, 8*, 63-74.

Chiang, T. C., Jeon, B. N., and Li, H. (2007). Dynamic correlation analysis of financial contagion: Evidence from Asian markets. *Journal of International Money and Finance, 26*, 1206-1228.

Click, R. W., and Plummer, M. G. (2005). Stock market integration in ASEAN after the Asian financial crisis. *Journal of Asian Economics, 16*(1), 5-28.

Conrad, C., and Kleen, O. (2020). Two are better than one: Volatility forecasting using multiplicative component GARCH-MIDAS models. *Journal of Applied Econometrics, 35*(1), 19-45.

Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. *Quantitative Finance, 1*(2001), 223-236.

Craven, P., and Wabha, G. (1979). Smooting noisy data with spline functions: Estimating the correct degree of smooting by method of generalized cross-validation. *Numerische Mathematik, 31*(4), 377-403.

Dimitriou, D., and Kenourgios, D. (2013). Financial criese and dynamic linkage among international currencies. *Journal of International Financial Markets, Institutions and Money, 26*, 319-332.

Ding, L., Huang, Y., and Pu, X. (2014). Volatility linkage across global equity markets. *Global Finance Journal, 25*(2), 71-89.

Ding, Z., Granger, C. W., and Engle, R. F. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance, 1*, 83-106.

Do, G. Q., Mcleer, M., and Sriboonchitta, S. (2009). Effects of international gold market on stock exchange volatility: evidence from asean emerging stock markets. *Economics Bulletin, 29*(2), 599-610.

Elton, E. J., and Gruber, M. J. (1997). Modern portfolio theory, 1950 to date. *Journal of Banking and Finance, 21*(11-12), 1743-1759.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica, 4*(50), 987-1008.

Engle, R. F. (2002). Dynamic conditional correlation. *Journal of Business and Economic Statistics, 3*(20), 339-350.

Engle, R. F. (2004). Risk and volatility: Econometric models and financial practice. *American Economic Review, 3*(94), 405-420.

Engle, R. F., and Rangel, J. G. (2008). The Spline-GARCH model for low-frequency volatility and its global macroeconomic causes. *The Review of Financial Studies, 21*(3), 1187-1222.

Engle, R. F., and Russell, J. R. (1998). Autoregressive conditional duration: A new model for irregularly spaced transaction data. *Econometrica, 66*(5), 1127-1162.

Engle, R. F., and Sheppard, K. (2001, October). Theoretical and Empirical properties of Dynamic Conditional Correlation Mulitvariate GARCH. NBER Working Papar No.8554, pp. 1-43.

Engle, R. F., Ghysels, E., and Sohn, B. (2013). Stock market volatility and macroeconomic fundamentals. *The Review of Economics and Statistics, 95*(3), 776-797.

Erb, C. B., Harvey, C. R., and Viskanta, T. E. (1994). Forecasting international equity correlations. *Financial Analysts Journal, 50*(6), 32-45.

Fabozzi, F. J., Gupta, F., and Markowitz, H. M. (2002). The legacy of modern portfolio theory. *The Journal of Investing, 11*(3), 7-22.

Fama, E. F. (1965). Random walks in stock market prices. *Financial Analysts Journal, 51*(1), 55-59.

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The Journal of Finance, 25*(2), 383-417.

Fama, E. F., and French, K. R. (1988). Permanent and temporary components of stock prices. *The Journal of Political Economy, 96*(12), 246-273.

Farida. (2016). Investigating the Performance of the GARCH (1, 1) Model. M.S. Thesis, Prince of Songkla University.

Figlewski, S. (1997). Forecasting Volatility. *Financial markets institutions and instruments, 6*(1), 1-88.

Forbes, K. J., and Rigobon, R. (2002). No contagion, only interdependence: Measuring stock market comovement. *The Journal of Finance, 57*(5), 2223-2261.

Friedman, J. H. (1991). Multivariate adaptive regression splines. *The Annals of Statistics, 19*(1), 1-67.

Gavranic, K., and Miletic, D. (2016). US dollar stability and the global currency reserves. *Eurasian Journal of Economics and Finance, 4*(3), 14-24.

Goetzmann, W. N., Li, L., and Rouwenhorst, K. G. (2005). Long-term global market correlation. *Journal of Business, 78*(1), 1-38.

Hansen, P. R., and Lunde, A. (2005). A forecast comparison of volatility models: Does anything beat a GARCH (1,1). *Journal of Applied Econometrics, 20*(7), 873-889.

Harvey, B. (2017). Evaluating financial integration and cooperation in the ASEAN. *Michigan Business & Entrepreneurial Law Review, 7*(1), 119-158.

Huber, P. J. (1964). Robust estimation of a location parameter. *The Annals of Mathematical Statistics, 35*(1), 73-101.

Kabigting, L., and Hapitan, R. B. (2011). ASEAN 5 stock markets, currency risk and volatility spillover. *Economics Bulletin, 10*(3), 63-84.

Karali, B., & Power, G. J. (2013). Short- and long-run determinants of commodity price volatility. *American Journal of Agricultural Economics, 95*(3), 724-738.

Kaur, T. A., Manual, V., and Eeswaran, M. (2019). The volatility of the Malaysian ringgit: Analyzing its impact on economic growth. *International Journal of Recent Technology and Engineering, 7*(5S), 82-88.

Klyuev, V., and Dao, T.N. (2017). No more clubbing, the evolution of exchange rate behaviour in the ASEAN-5 countries. *Journal of Southeast Asian Economies, 34*(2), 233-265.

Ledoit, O., Santa-Clara, P., and Wolf, M. (2003). Flexible multivariate GARCH modeling with an application to international stock markets. *The Review of Economics and Statistics, 85*(3), 735-747.

Lee, T. C. M. (2003). Smoothing parameter selection for smoothing splines: A simulation study. *Computational Statistics and Data Analysis, 42*(1), 139-148.

Lee, C. L., Stevenson, S., and Lee, M. L. (2018). Low-frequency volatility of real estate securities and macroeconomic risk. *Accounting and Finance, 58*(2018), 311-342.

Lewis, P. A., and Stevens, J. G. (1991). Nonlinear modeling of time series using Multivariate Adaptive Regression Splines (MARS). *Journal of the American Statistical Association, 86*(41), 864-877.

Liu, S., Tang, T., McKensie, A. M., and Liu, Y. (2015). Low-frequency volatility in China's gold futures market and its macroeconomic determinants. *Mathematical Problems in Engineering*, 2015. Doi: 10.1155/2015/646239

Mandelbrot, B. B. (1963). The Variation of Certain Speculative Prices. *The Journal of Business, 36*(4), 394-419.

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance, 7*(1), 77-91.

McMillan, D. G. (2019). Cross-asset relations, correlation and economic implications. *Global Finance Journal, 41*, 60-78.

Montoya, E. L., Ulloa, N., and Miller, V. (2014). A simulation study comparing knot selection methods with equally spaced knots in a penalized regression spline. *International Journal of Statistics and Probability, 3*(3), 96-110.

Ponziani, R. M. (2019). Foreign exchange volatility modeling of Southeast Asian major economies. *Journal of Economics, Business and Accountancy Ventura, 22*(2), 283-297.

Poon, S. H. (2005). A Practical Guide to Forecasting Financial Market Volatility. John Wiley & Sons Inc, New York, U.S.A., pp. 1-17.

Poon, S. H., and Granger, C. W. (2003). Forecasting volatility in financial markets: A review. *Journal of Economic Literature, 2*(41), 478-539.

Prukumpai, S., and Sethapramote, Y. (2018). Stock market integration in the ASEAN-5. *Applied Economics Journal, 25*(1), 15-34.

Quadry, M. O., Mohamad, A., and Yusof, Y. (2017). On the Malaysian ringgit exchange rate determination and recent depreciation. *International Journal of Economics, Management and Accounting, 25*(1), 1-26.

Rey, S., and Nivoix, S. (2018). Dynamics of Tokyo electric power company and the Nikkei: 1985 to 2016 including the Fukushima disaster. *Journal of Economic Integration, 33*(1), 979-1010.

Rillo, A. D. (2018). ASEAN financial integration: Opportunities, risks and challenges. *Public Policy Review, 14*(5), 901-923.

Saejiang, S., Pornwiriyamongkol, W., and McNeil, D. (2001). Time series analysis of banking share returns in Thailand. *Songklanakarin Journal of Science and Technology, 23*(3), 443-448.

Shimizu, S. (2014). ASEAN financial and capital markets: policies and prospects of regional integration. *Pacific Business and Industries, 14*(54), 2-36.

Silverman, B. W. (1985). Some aspects of the spline smoothing approach to non-parametric regression curve fitting. *Journal of the Royal Statistical Society. Series B (Methodological), 47*(1), 1-52.

Smith, P. L. (1982). Curve fitting and model with splines using statistical variable selection techniques. Hampton, Virginia: National Aeronautics and Space Administration. Retrieved from https://ntrs.nasa.gov/citations/19830005637

Staszczak, D. E. (2015). Global instability of currencies: reasons and perspectives according to the state-corporation hegemonic stability theory. *Revista de Economia Politica, 35*(1), 174-198.

Suits, D. B., Mason, A., and Chan, L. (1978). Spline function fitted by standard regression methods. *The Review of Economics and Statistics, 60*(1), 132-139.

Syllignakis, M. N., and Kouretas, G. P. (2011). Dynamic correlation analysis of financial contagion: Evidence from the central and eastern European markets. *International Review of Economics and Finance, 20*, 717-732.

Thuy, V. N., and Thuy, D. T. (2019). The impact of exchange rate volatility on exports in Vietnam: A bounds testing approach. *Journal of Risk and Financial Management, 12*(6), 1-14.

Tiwari, A. K., Mutascu, M. I., and Albulescu, C. T. (2016). Continuous wavelet transform and rolling correlation of European stock markets. *International Review of Economics and Finance, 42*, 237-256.

Tri, H. T., Nga, V. T., and Duong, V. H. (2019). The determinants of foreign direct investment in ASEAN: New evidence from financial integration factor. *Business and Economic Horizons, 15*(2), 292-303.

Wahba, G. (1975). Smoothing Noisy data with spline function. *Numerische Mathematik, 24*(5), 383-393.

Wang, G.J., and Xie, C. (2013). Cross-correlation between Renminbi and four major currencies in the Renminbi currency basket. *Physic A, 392*, 1418-1428.

Wongsai, N., Wongsai, S., and Huete, A. R. (2017). Annual seasonality extraction using the cubic spline function and decadal trend in temporal daytime MODIS LST data. *Remote Sensing, 9*(12), 1254. Doi: 10.3390/rs9121254

Yin, K., Liu, Z., and Liu, P. (2017). Trend analysis of global stock market linkage based on a dynamic conditional correlation network. *Journal of Business Economics and Management, 18*(4), 779-800.

Appendix I

Article I: “The time-varying correlation estimator using the natural cubic spline volatility”

Advances and Applications in Statistics
© 2020 Pushpa Publishing House, Prayagraj, India
http://www.pphmj.com
http://dx.doi.org/10.17654/AS062010079
Volume 62, Number 1, 2020, Pages 79-95 **ISSN: 0972-3617**

THE TIME-VARYING CORRELATION ESTIMATOR USING THE NATURAL CUBIC SPLINE VOLATILITY

Jetsada Laipaporn and Phattrawan Tongkumchum

Faculty of Humanities and Social Sciences
Prince of Songkla University
Pattani Campus, Thailand
e-mail: jetsada.l@psu.ac.th

Faculty of Science and Technology
Prince of Songkla University
Pattani Campus, Thailand
e-mail: phattrawan@gmail.com

Abstract

The time-varying correlation is an indicator used by financial analysts to assess the co-movement of financial time series. This study introduces the time-varying correlation estimator using the natural cubic spline volatility for estimating time-varying correlation. This estimator reveals the pattern of change in correlation and the simulation showed that it also provides more accurate estimates of the time-varying correlation than the rolling correlation estimator and the dynamic conditional correlation model. Furthermore, this estimator is practical for assessing the co-movement of the ASEAN-5 stock indices. The empirical result showed that the degree of time-varying correlation of the ASEAN-5 stock markets after 2007 was higher than the previous period indicating the stronger linkages and more integration of the ASEAN-5 stock markets.

Received: March 15, 2020; Accepted: May 1, 2020
2010 Mathematics Subject Classification: 62P20, 91B84.
Keywords and phrases: co-movement, rolling correlation, DCC model.

1. Introduction

Time-varying correlation is an indicator for assessing the relationship between two observed time series. Ding et al. [6] and Meric et al. [19] employed a time-varying correlation for revealing the inter-dependent behaviour and the co-movement pattern between the financial series. Time-varying correlation is useful for identifying the linkage between the stock markets. The high degree of time-varying correlation of two stock markets indicates the strong linkage between them [13] and this linkage reflects how much the markets are integrated [15].

Since time-varying correlation is likely to increase during a highly volatile period [23], declines occur during the bull markets and rises in the bear markets [13]. Consequently, some studies have applied this uncertain behaviour of time-varying correlation to signal the financial contagion caused by the financial crisis [3].

As time-varying correlation reflects the co-movement of financial time series, it has become a critical factor in the portfolio selection model [20, 8, 11]. Since investing in the stocks having a strong co-movement pattern raises the risk of an investment portfolio, investors thus often try to diversify their portfolio by distributing their investment into stocks or stock markets that have a low degree of time-varying correlation. However, there is no ideal approach to estimate the time-varying correlation [18].

The most straightforward approach is the rolling correlation estimator. This estimator is practically used for investigating the evolution of time-varying correlation. By changing the size of the rolling window, it capably highlights the pattern of correlation changes both in long-term and short-term of the market dynamics [2, 30]. Moreover, the literature shows that this estimator is the best suited for use in portfolio management [7]. Unfortunately, the rolling correlation estimator uses sub-sample data. Thus, it is impossible to assess the time-varying correlation at every point in time for the whole observed period.

Another approach for assessing time-varying correlation is the dynamic conditional correlation or DCC model. This model was introduced by Engle [9] and is widely used in the academic world. The DCC model provides the robustness analysis of time-varying correlation by allowing conditional asymmetries in both volatility and correlation and overcomes the heteroscedasticity problem [5]. Contrast to rolling correlation estimator, the DCC model uses the full sample for estimating [29]. Thus, it capably provides the estimates at every point in time for the whole investigated period [3]. Although the estimation of the DCC model is less complicated and easier to estimate than the other multivariate models [10], this estimator cannot still display the precise pattern of changes in correlation which can be easily obtained by the rolling method [18].

This study aims to introduce a new approach for estimating time-varying correlation by employing the natural cubic spline volatility and indirect covariance. The natural cubic spline volatility is estimated by maximum likelihood estimation [16, 17, 12], while the indirect covariance is easily calculated by the statistical concept of "variance of a sum and variance of a difference of random variable" [27, 24].

In order to assess the performance of this new estimator, this study employs the Monte Carlo simulation to compare the estimation accuracy to the two existing approaches, the rolling correlation estimator and the DCC model. Furthermore, this study empirically uses the new estimator for assessing the pattern of the co-movement among the ASEAN-5 stock indices during 2000-2018 and uses R program for statistical analysis.

The remainder of this article is structured as follows: Section 2 presents the time-varying correlation estimators used in this study. The application of Monte Carlo simulation to compare the performance among four correlation estimators is presented in Section 3. The empirical analysis on the pattern of the ASEAN-5 stock market co-movement is given in Section 4, and the conclusion made in this study is presented in Section 5.

2. The Time-varying Correlation Estimator

In this section, the time-vary correlation estimators used in this study are presented as follows:

2.1. The rolling correlation estimator

The rolling correlation is the Pearson's correlation of asset returns in the time-specified window, which roll step by step throughout the interest period. This estimator is simple and prominent in applied finances. Studies such as Engle [9], Wang and Xie [28] and Tiwari et al. [26] have employed this estimator as a benchmark to compare the performance against the more sophisticated estimators. The rolling correlation $(\hat{\rho}_{xy,t})$ between two return series, r_x and r_y over the time span from $t - s$ to $t - 1$, where t is time, and s is the size of the time-specified window, is expressed as follows:

$$\hat{\rho}_{xy,t} = \frac{\sum_{i=t-s}^{t-1} (r_{x,i} - \bar{r}_x)(r_{y,i} - \bar{r}_y)}{\sqrt{\sum_{i=t-s}^{t-1} (r_{x,i} - \bar{r}_x)^2 \sum_{i=t-s}^{t-1} (r_{y,i} - \bar{r}_y)^2}}. \tag{1}$$

$\bar{r}_x$ and $\bar{r}_y$ are the mean of each return series which is approximately zero. This estimator is known as the backward rolling correlation estimator or historical correlation estimator since this estimator used data between time span, $t - s$ to $t - 1$, to estimate the correlation at time t.

However, the study of Rey and Nivoix [22] alternatively used the centred rolling correlation estimator instead of the backward correlation estimator, because it provides a more accurate result. This estimator is formulated as follows:

$$\hat{\rho}_{xy,t} = \frac{\sum_{i=t-0.5s}^{t+0.5s} (r_{x,i} - \bar{r}_x)(r_{y,i} - \bar{r}_y)}{\sqrt{\sum_{i=t-0.5s}^{t+0.5s} (r_{x,i} - \bar{r}_x)^2 \sum_{i=t-0.5s}^{t+0.5s} (r_{y,i} - \bar{r}_y)^2}}. \tag{2}$$

To estimate the correlation at time t where s is the size of the time-specified window, the centred rolling correlation estimator differently estimates the sub-sample between t minus half of s and t plus half of s. For the simulation and the empirical part, this study thus employed both types of the rolling correlation estimator, the backward rolling correlation estimator and the centred rolling correlation estimator.

2.2. The dynamic conditional correlation model

The DCC model assumes that a series of financial returns are conditionally multivariate normal with zero expected value and covariance matrix H_t. The covariance matrix H_t is parameterized as

$$H_t = D_t R_t D_t. \tag{3}$$

The matrix D_t is the $n \times n$ diagonal matrix of time-varying standard deviation derived from the univariate GARCH models $(\sqrt{h_{i,t}})$ on the ith diagonal, $i = 1, 2, 3, \ldots, n$ and n is the number of series. The matrix R_t is the matrix of time-varying correlation.

Engle [9] proposed the two-step procedure for the DCC model estimation. The first step is estimating the univariate GARCH model for each return series. The second step is applying the standardized residuals or volatility-adjusted returns $(\varepsilon_{i,t} = r_{i,t}/\sqrt{h_{i,t}})$ to estimate the parameters of the conditional correlation as follows:

$$R_t = (diag(Q_t))^{1/2} Q_t (diag(Q_t))^{1/2}. \tag{4}$$

The element of Q_t is given by $q_{xy,t}$, where $x, y = 1, 2, 3, \ldots, n$ and $x \neq y$,

$$q_{xy,t} = (1 - \alpha - \beta)\bar{q}_{xy} + \alpha\varepsilon_{x,t-1}\varepsilon_{y,t-1} + \beta q_{xy,t-1}. \tag{5}$$

The element $\bar{q}_{xy}$ is the unconditional covariance between standardized residuals of series, x and y. The parameters α and β are the news and the decay coefficients respectively.

The $(diag(Q_t))^{1/2} = diag(1/\sqrt{q_{11,t}}, \ldots, 1/\sqrt{q_{nn,t}})$. Then time-varying correlation $(\hat{\rho}_{xy,t})$ in the matrix R_t has the form as below:

$$\hat{\rho}_{xy,t} = \frac{q_{xy,t}}{\sqrt{q_{xx,t}q_{yy,t}}}. \tag{6}$$

Denote the parameters in D_t by θ, and the additional parameters in R_t by ϕ. Then the log-likelihood can be rewritten as the sum of a volatility part and a correlation part:

$$L(\theta, \phi) = L_V(\theta) + L_C(\theta, \phi). \tag{7}$$

The two-step procedure for the DCC model estimation is by first maximizing the likelihood $L_V(\theta)$ to find $\hat{\theta} = \arg\max\{L_V(\theta)\}$ and then using $\hat{\theta}$ as given for maximizing the correlation part, $\max_{\phi}\{L_C(\hat{\theta}, \phi)\}$. The $L_V(\theta)$ is equivalent to the sum of univariate GARCH likelihoods which are separately maximizing maximized.

2.3. The time-varying correlation estimator using the natural cubic spline volatility

Suppose there are two returns series, x and y. The correlation between x and y or ρ_{xy} can be formulated as follows:

$$\rho_{xy} = \frac{\operatorname{cov}(x, y)}{\sqrt{\operatorname{var}(x)\operatorname{var}(y)}}. \tag{8}$$

However, the covariance between x and y or $\operatorname{cov}(x, y)$ can be indirectly derived by using the identity "variance of a sum and variance of a difference of random variables" described in Tsay [27] and Selove [24], as follows:

$$\operatorname{var}(x + y) = \operatorname{var}(x) + \operatorname{var}(y) + 2\operatorname{cov}(x, y), \tag{9}$$

$$\operatorname{var}(x - y) = \operatorname{var}(x) + \operatorname{var}(y) - 2\operatorname{cov}(x, y), \tag{10}$$

$$\operatorname{cov}(x, y) = (\operatorname{var}(x + y) - \operatorname{var}(x - y))/4. \tag{11}$$

This identity continuously holds for time-varying covariance. So, time-varying covariance between return series x and y can be acquired by the variance of $x + y$ and $x - y$.

Let $s_{x,t}$, $s_{y,t}$, $s_{x+y,t}$ and $s_{x-y,t}$ be the standard deviations or volatilities of x, y, $x + y$ and $x - y$, respectively. Then the time-varying correlation between return series x and y $(\rho_{xy,t})$ can be rewritten as the following formula:

$$\rho_{xy,t} = \frac{S^2_{x+y,t} - S^2_{x-y,t}}{4\sqrt{S^2_{x,t}S^2_{y,t}}}. \tag{12}$$

This new approach employed the natural cubic spline volatility model to estimate the volatility of those the series mentioned above. Following Laipaporn and Tongkumchum [16], the returns series of x at time t $(r_{x,t})$ is assumed as the product of time-varying volatility or time-varying standard deviation $(s_{x,t})$ and random noise $(\varepsilon_{x,t})$ which is normally distributed with a zero mean and a unit standard deviation:

$$r_{x,t} = s_{x,t}\varepsilon_{x,t}. \tag{13}$$

The time-varying volatility $(s_{x,t})$ is modelled as the natural cubic spline function with respect to time (t) as follows:

$$s_{x,t} = \alpha + \beta t + \sum_{k=1}^{p-2}\theta_k\left\{(t - t_k)^3_+ - \frac{t_p - t_k}{t_p - t_{p-1}}(t - t_{p-1})^3_+ + \frac{t_{p-1} + t_k}{t_p - t_{p-1}}(t - t_p)^3_+\right\}. \tag{14}$$

The equi-spaced knots are placed at t_k, where k is 1, 2, 3, ..., p and p is the number of knots. This function is applied to the absolute values of the return series $(r_{x,t})$, which are the proxies of time-varying standard deviation

[14]. Consequently, the parameters of this function $(\alpha, \beta$ and $\theta_k)$ are obtained by maximizing the log-likelihood function as follows:

$$L = \sum_{t=1}^{n}\left\{-\log(s_{x,t}) - \frac{r_t^2}{2s_t^2}\right\}. \tag{15}$$

By using the Newton-Raphson method, the derivatives of the likelihood functions according to each parameter are obtained by the simple algebra [12].

3. The Monte Carlo Simulation

This study applied the Monte Carlo simulation to compare the performance among four correlation estimators which are (1) the rolling correlation estimator, (2) the centred rolling correlation estimator, (3) the DCC model and (4) the time-varying correlation estimator using the natural cubic spline volatility.

This simulation comprises of three steps. The first one is to generate two returns series (x and y) of 4,000 simulated daily returns or approximately 16 years, with zero mean, known volatility $(\sigma_{x,t}$ and $\sigma_{y,t})$ and known correlation $(\rho_{xy,t})$ as follows:

$$\begin{pmatrix} r_{x,t} \\ r_{y,t} \end{pmatrix} \sim n\left[0, \begin{pmatrix} \sigma_{x,t}^2 & \rho_{xy,t}\sigma_{x,t}\sigma_{y,t} \\ \rho_{xy,t}\sigma_{x,t}\sigma_{y,t} & \sigma_{y,t}^2 \end{pmatrix}\right], \tag{16}$$

where the four patterns of the known correlation $(\rho_{xy,t})$ are pre-determined, consisting of constant pattern, step pattern, periodic pattern and non-periodic pattern. The examples of the absolute returns with known volatility x and y and the four known correlation patterns are shown in Figures 1 and 2, respectively.

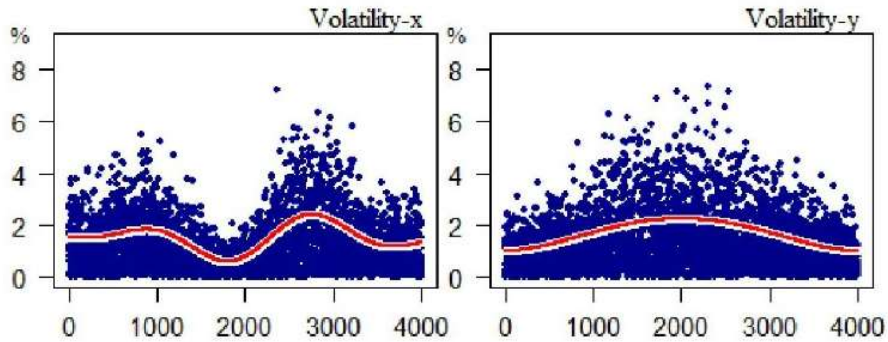


Figure 1. Examples of absolute returns (points) with their known volatility (dotted lines).

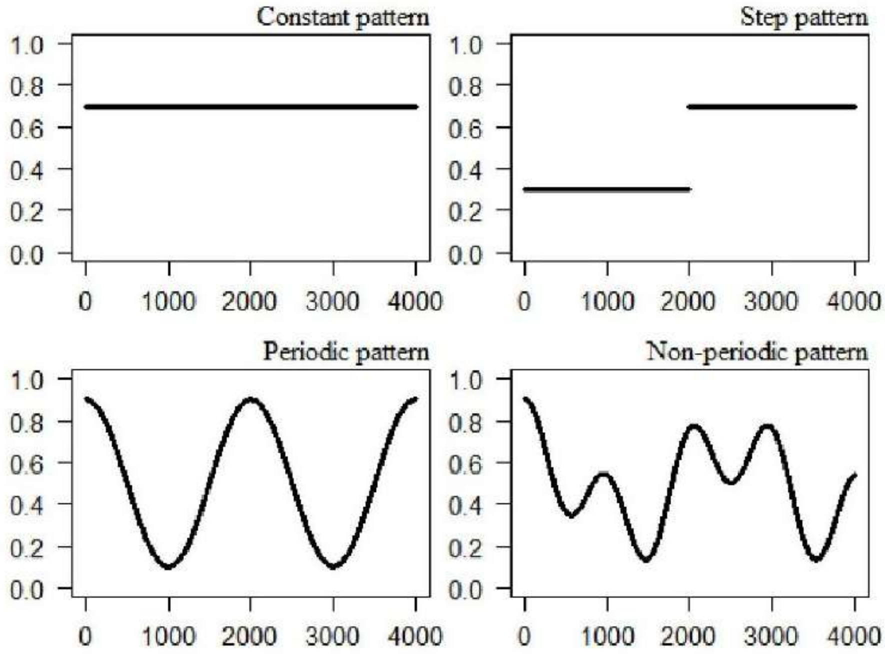


Figure 2. The four correlation patterns $(\rho_{xy,t})$.

The second step of the simulation is to estimate the correlation of the four approaches. The selection of the 250-trading day was based on previous research [19, 28, 26, 18]. The 250 trading-day which is almost one trading-year is capable of capturing the short-term pattern of the time-varying correlation, for the size of the rolling window and thus employs the same condition for the equi-space between knots for the time-varying correlation estimator using the natural cubic spline volatility.

Since the correlation patterns are pre-specified, the third step is to compare the accuracy of these estimators by using the mean absolute error (MAE) as follows:

$$\mathrm{MAE}_k = \frac{\sum_{t=1}^{n} | \rho_{xy,t} - \hat{\rho}_{xy,t} |}{n}, \tag{17}$$

where $\rho_{xy,t}$ and $\hat{\rho}_{xy,t}$ are the known correlation and the estimated correlation obtained by the estimators, k is the type of correlation estimator and n is the number of time-varying correlation values which is equal to 4,000. The estimator with a lower MAEs has a better performance than others.

This study processed the simulation 200 times for each known correlation pattern and applied four estimators which are the backward rolling correlation estimator (R), the centred rolling correlation estimator (C), the DCC model (D) and the time-varying correlation estimator using the natural cubic spline volatility (S) to estimate correlation. As the results, the boxplots result of MAEs of each estimator for each correlation patterns are shown in Figure 3.

Figure 3 shows that in the case of the constant correlation patterns, the DCC model (D) performs better than others. In contrast to the other cases, the centred rolling correlation estimator (C) and the time-varying correlation estimator using the natural cubic spline volatility (S) provide more accurate estimates than the other two especially in the case of periodic and non-periodic patterns.

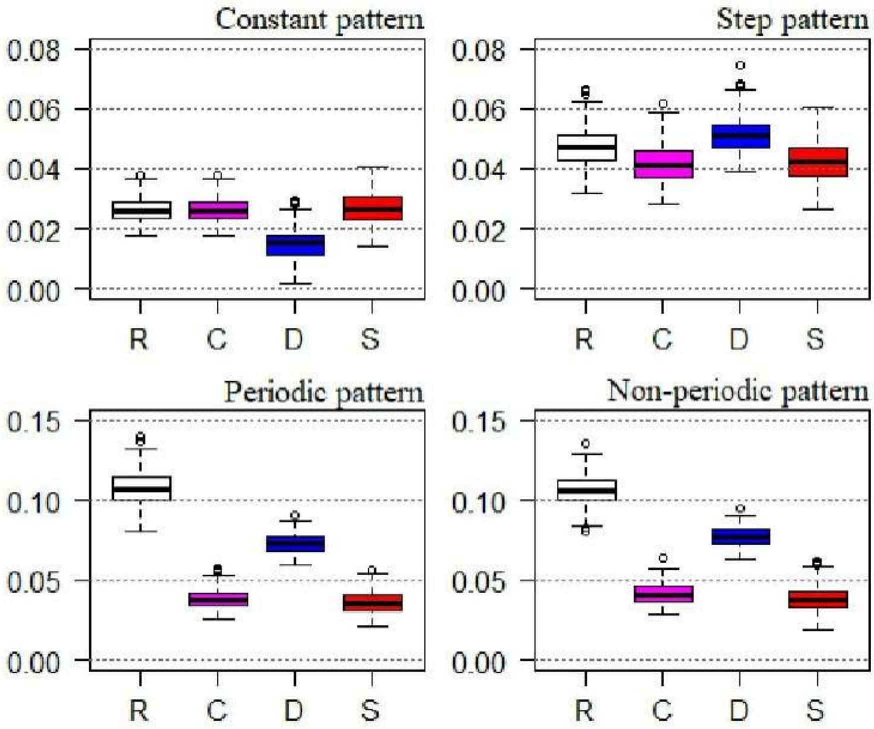


Figure 3. The boxplots of MAEs of each estimator for each correlation pattern.

4. The Empirical Analysis

After the financial crisis in 1997 and the global financial crisis between the year 2007-2008, ASEAN initiated the ASEAN free trade area (AFTA) in 2010 to increase inter-regional trades and strengthen the regional economy according to the ASEAN charter officially signed in November 2007. Following the establishment of the European Union community, in 2015, ASEAN then advanced tighter cooperation in all aspects to establish a single ASEAN community (AC). Furthermore, ASEAN constructed three pillars of the ASEAN community: Economic Community (AEC), Political-Security Community (APSC) and Socio-Cultural Community (ASCC). They also established the connectivity through the physical connectivity, the regulation connectivity and the people to people connectivity to ensure that ASEAN'S are properly connected.

Due to the AEC blueprints and the ASEAN connectivity, the financial sectors in this region have become more integrated than ever before. Singapore, the financial hub for ASEAN as well as ASIA, Malaysia and Thailand have well established domestic market while Indonesia and the Philippines stock markets are still developing [25]. However, the development of the AEC through the ASEAN connectivity likely affects the linkages among ASEAN stock market. Thus, this study chose the co-movement pattern of the ASEAN-5 stock markets as an issue for the empirical analysis part.

The indices of the ASEAN-5 stock markets used in this study comprised the Stock Exchange of Thailand Index (SET), the Straits Times Index (STI), the Bursa Malaysia index (KLSE), the Indonesia Stock Exchange index (JKSE), and the Philippine Stock Exchange index (PSEI). Those indices on January 1, 2001 and December 31, 2018 were obtained from Yahoo finance except the SET index, which was obtained from the Stock Exchange of Thailand. The descriptive statistics of their daily logged returns series are summarized in Table 1.

Table 1. Descriptive statistics of the daily returns of the ASEAN-5 stock index

Descriptive statistics	SET	STI	KLSE	JKSE	PSEI
Minimum	–0.16063	–0.09095	–0.09978	–0.11306	–0.08698
Mean	0.00017	–0.00009	0.00007	0.00040	0.00037
Median	0.00047	0.00016	0.00031	0.00010	0.00030
Maximum	0.10577	0.07530	0.04502	0.07362	0.16177
Variance	0.00017	0.00012	0.00006	0.00018	0.00016
Std. deviation	0.01312	0.01130	0.00814	0.01347	0.01279
No. of obs.	4075	4075	4075	4075	4075

Again, the four estimators were applied for estimating the time-varying correlation among those ASEAN-5 stock indices. The size of the window of the rolling methods and the equi-space of the natural cubic spline volatility were equal to 250 trading-day as well. The result obtained from the estimated correlation coefficients among the ASEAN-5 stock markets by four estimators is shown in Figure 4.

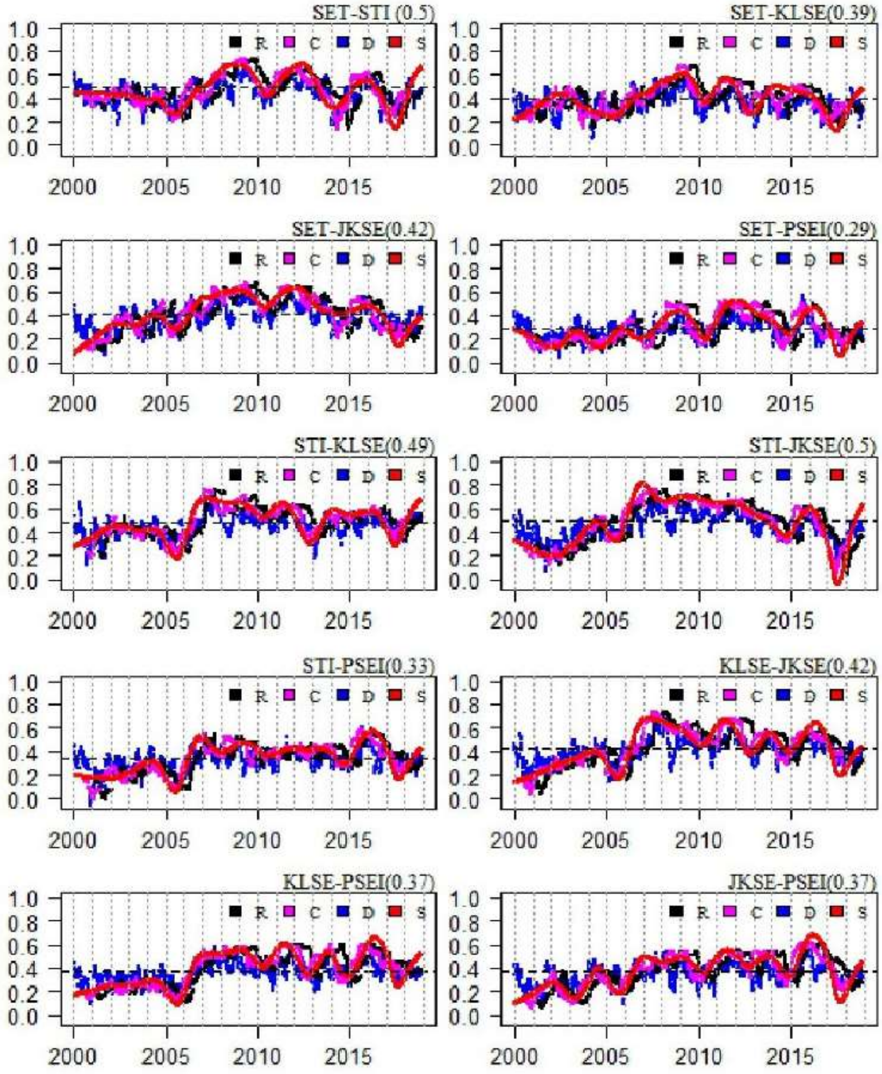


Figure 4. The time-varying correlations among the ASEAN-5 stock markets.

The values in the parenthesis of the left legend in each plot are pairwise unconditional correlation coefficients among ASEAN-5 stock markets. These unconditional correlation coefficients revealed that the linkages between the PSEI, SET and STI are not reliable as each other's linkages since the unconditional correlation coefficients between the PSEI and these two indices are quite low, equaling 0.29 and 0.33, respectively. In contrast to the linkage between the STI and the other indices except for the PSEI, which are quite strong as the correlation coefficients range from 0.37 to 0.5.

Among the four time-varying correlation estimators, only two estimators, namely, the DCC model (D) and the estimator using the natural cubic spline volatility (S) provided the estimated values for the whole period of interest. The correlation estimator using the natural cubic spline volatility (S) provided a more precise pattern of change in time-varying correlation than the other estimators, while the backward rolling correlation estimator (R) provided the estimates were ahead of the others.

Compared to the unconditional correlation coefficients, the pattern of change in correlation coefficients among the ASEAN-5 stock markets was capably separated into two parts. In the beginning, the time-varying correlation coefficient before 2006 of all pairwises was mostly lower than the level of unconditional correlation coefficients. The correlation coefficients between 2006-2007 raised sharply in the cases of STI-KLSE, STI-JKSE, STI-PSEI, KLSE-JKSE and KLSE-PSEI, while the correlation coefficients of SET-STI, SET-KLSE, SET-JKSE, SET-PSEI and JKSE-PSEI gradually increased. The time-varying correlation coefficients among ASEAN-5 stock market after 2007 were mostly higher than the level of unconditional correlation. However, there was a decline in 2017, which was followed by an increase to the unconditional level.

5. The Conclusion

Although the DCC model performed well when the correlation's pattern is constant, the simulation showed that the time-varying correlation estimator using the natural cubic spline volatility and the centre rolling correlation estimator provide better estimates in case of inconstant patterns. Our finding corresponds to Adams et al. [1] which also reported that the DCC model is not a suitable choice when the pattern of changes in correlation is not constant or has structure breaks.

The patterns of changes in time-varying correlation coefficients of the ASEAN-5 stock markets obtained by the time-varying correlation estimator using the natural cubic spline volatility showed that after 2007, the ASEAN-5 stock markets had stronger linkages, indicating a higher integration in the financial markets of ASEAN. Furthermore, the patterns of change in time-varying correlation between the SET index and the others are more fluctuated than the other patterns which are consistent with the findings in Prukumpai and Sethapramote [21] as well as Chitkasame and Tansuchat [4].

Consequently, the time-varying correlation estimator using the natural cubic spline volatility is practical for correlation analysis, because it capably captures the uncertain patterns of change in time-varying correlation and provides the estimates for the whole period of interest which is different to the rolling approaches.

Acknowledgement

We would like to thank Emeritus Professor Dr. Don McNeil for his support and guidance.

References

[1] Z. Adams, R. Fuss and T. Gluck, Are correlation constant? Empirical and theoretical results on popular correlation models in finance, Journal of Banking and Finance 84 (2017), 9-24.

[2] M. Billio and M. Caporin, Market linkages, variance spillovers and correlation stability: empirical evidence of financial contagion, Comput. Statist. Data Anal. 54 (2010), 2443-2458.

[3] T. C. Chiang, B. N. Jeon and H. Li, Dynamic correlation analysis of financial contagion: evidence from Asian markets, Journal of International Money and Finance 26 (2007), 1206-1228.

[4] T. Chitkasame and R. Tansuchat, An analysis of contagion effect on ASEAN stock market using multivariate Markov switching DCC GARCH, Thai J. Math. 17 (2019), 135-152.

[5] D. Dimitriou and D. Kenourgios, Financial crises and dynamic linkage among international currencies, Journal of International Financial Markets, Institutions and Money 26 (2013), 319-332.

[6] L. Ding, Y. Huang and X. Pu, Volatility linkage across global equity markets, Global Finance Journal 25 (2014), 71-89.

[7] C. B. Erb, C. R. Harvey and T. E. Viskanta, Forecasting international equity correlations, Financial Analysts Journal 50 (1994), 32-45.

[8] E. J. Elton and M. J. Gruber, Modern portfolio theory, 1950 to date, Journal of Banking and Finance 21 (1997), 1743-1759.

[9] R. Engle, Dynamic conditional correlation, J. Bus. Econom. Statist. 3 (2002), 339-350.

[10] R. F. Engle and K. Sheppard, Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH, NBER Working Paper No. 8554, 2001, pp. 1-43.

[11] F. J. Fabozzi, F. Gupta and H. M. Markowitz, The legacy of modern portfolio theory, The Journal of Investing 11 (2002), 7-22.

[12] F. Farida, N. Makaje, P. Tongkumchum, A. Phon-On and J. Laipaporn, Natural cubic spline model for estimating volatility, International Journal on Advanced Science, Engineering and Information Technology 4 (2018), 1085-1090.

[13] K. J. Forbes and R. Rigobon, No contagion, only interdependence: measuring stock market comovements, The Journal of Finance 57 (2002), 2223-2261.

[14] S. Figlewski, Forecasting volatility, Financial Markets Institutions and Instruments 6 (1997), 1-88.

[15] W. N. Goetzmann, L. Li and K. G. Rouwenhorst, Long-term global market correlation, Journal of Business 78 (2005), 1-38.

[16] J. Laipaporn and P. Tongkumchum, Maximum likelihood estimation of non-stationary variance, International Statistical Institute Regional Statistics Conference, Bali, 2017.

[17] J. Laipaporn and P. Tongkumchum, The use of information criteria for selecting number of knots in natural cubic spline volatility estimation, International Conference on Applied Statistics (ICAS), Bangkok, 2018.

[18] D. G. McMillan, Cross-asset relations, correlation and economic implications, Global Finance Journal 41 (2019), 60-78.

[19] G. Meric, C. Lentz, W. Smeltz and I. Meric, International evidence on market linkage after the 2008 stock market crisis, The International Journal of Business and Finance Research 6 (2012), 45-57.

[20] H. Markowitz, Portfolio selection, The Journal of Finance 7 (1952), 77-91.

[21] S. Prukumpai and Y. Sethapramote, Stock market integration in the ASEAN-5, Applied Economics Journal 25 (2018), 15-34.

[22] S. Rey and S. Nivoix, Dynamics of Tokyo electric power company and the Nikkei: 1985 to 2016 including the Fukushima disaster, Journal of Economic Integration 33 (2018), 979-1010.

[23] M. N. Syllignakis and G. P. Kouretas, Dynamic correlation analysis of financial contagion: evidence from the central and eastern European markets, International Review of Economics and Finance 20 (2011), 717-732.

[24] S. L. Sclove, A Course on Statistics for Finance, CRC Press, Boca Raton, 2013.

[25] S. Shimizu, ASEAN financial and capital markets: policies and prospects of regional integration, Pacific Business and Industries 14 (2014), 2-36.

[26] A. K. Tiwari, M. I. Mutascu and C. T. Albulescu, Continuous wavelet transform and rolling correlation of European stock markets, International Review of Economics and Finance 42 (2016), 237-256.

[27] R. S. Tsay, An Introduction to Analysis of Financial Data with R, Wiley, New Jersey, 2012.

[28] G.-J. Wang and C. Xie, Cross-correlation between Renminbi and four major currencies in the Renminbi currency basket, Phys. A 392 (2013), 1418-1428.

[29] K. Yin, Z. Liu and P. Liu, Trend analysis of global stock market linkage based on a dynamic conditional correlation network, Journal of Business Economics and Management 18 (2017), 779-800.

Appendix II

Article II: "Estimating the natural cubic spline volatilities of the ASEAN-5 exchange rates"

Print ISSN: 2288-4637 / Online ISSN 2288-4645
doi:10.13106/jafeb.2021.vol8.no3.0001

Estimating the Natural Cubic Spline Volatilities of the ASEAN-5 Exchange Rates*

Jetsada LAIPAPORN[1], Phattrawan TONGKUMCHUM[2]

Received: November 20, 2020 Revised: January 25, 2021 Accepted: February 03, 2021

Abstract

This study examines the dynamic pattern of the exchange rate volatilities of the ASEAN-5 currencies from January 2006 to August 2020. The exchange rates applied in this study comprise bilateral and effective exchange rates in order to investigate the influence of the US dollar on the stability of the ASEAN-5 currencies. Since a volatility model employed in this study is a natural cubic spline volatility model, the Monte Carlo simulation is consequently conducted to determine an appropriate criterion to select a number of quantile knots for this model. The simulation results reveal that, among four candidate criteria, Generalized Cross-Validation is a suitable criterion for modeling the ASEAN-5 exchange rate volatilities. The estimated volatilities showed the inconstant dynamic patterns reflecting the uncertain exchange rate risk arising in international transactions. The bilateral exchange rate volatilities of the ASEAN-5 currencies to the US dollar are more variable than their corresponding effective exchange rate volatilities, indicating the influence of the US dollar on the stability of the ASEAN-5 currencies. The findings of this study suggest that the natural cubic spline volatility model with the quantile knots selected by Generalized Cross-Validation is practical and can be used to examine the dynamic patterns of the financial volatility.

Keywords: Bilateral Exchange Rate, Effective Exchange Rate, Model Selection, Generalized Cross-Validation, Knots

JEL Classification Code: C13, C14, C22, G15

1. Introduction

Generally, exchange rate volatility indicates an uncertain fluctuation in relative price of one currency to other currencies (Laipaporn & Tongkumchum, 2017). It reflects the exchange rate risk in international trade and investment transactions (Kennedy & Nourzad, 2016; Teulon, Guesmi, & Mankai, 2014). Several studies have reported that the exchange rate volatility has a negative impact on the expansion of international trade and the economic growth (Upadhyaya, Dhakal, & Mixon, 2020; Tan, Duong, & Chuah, 2019; Purwono, Mucha, & Mubin, 2018; Soleymani, Chua, & Hamat, 2017; Al-Abri & Baghestani, 2015; AbuDalu, Ahmed, Almasaled, & Elgazoli, 2014) and also affects the stability of the capital markets (Campa, 2020; Dang, Le, Nguyen, & Tran, 2020). The time-varying correlation between a pair of the exchange rate volatilities also illustrates a link between currencies, which is evidence of international financial integration in international financial markets (Liu, Wang, & Sriboonchitta, 2019; Singh & Ahmed, 2016). Furthermore, central banks typically apply exchange rate volatility as a primary indicator for monitoring currency's stability (Klyuev & Dao, 2017).

Previous studies have mostly focused on investigating only two types of exchange rate. The first is referred as bilateral exchange rate, a relative price of one currency to another, usually the US dollar, a major currency in the world economy (Kennedy & Nourzad, 2016; Teulon, Guesmi, & Mankai, 2014). The second is an effective exchange rate, an index indicating the average of a currency's bilateral exchanges, weighted by its trading volumes in the reference year (Upadhyaya, Dhakal, & Mixon, 2020; Tan, Duong, & Chuah, 2019).

After the financial crisis in 1997 and the global financial crisis in 2007, the Association of Southeast Asian Nations

*Acknowledgements:
We would like to thank Emeritus Professor Dr. Don McNeil for his support and guidance.
[1]First Author and Corresponding Author. Lecturer, Economics Program, Faculty of Humanities and Social Sciences, Prince of Songkla University, Thailand [Postal Address: 181, Charoen Pradit Road, Muang District, Pattani Province, 94000, Thailand]
Email: jetsada.l@psu.ac.th
[2]Assistant Professor, Department of Mathematics and Computer Sciences, Faculty of Science and Technology, Prince of Songkla University, Thailand. Email: phattrawan.t@psu.ac.th

(ASEAN) set up an initiative to establish a free trade area in order to eliminate trade barriers and support regional integration (Ahmed & Singh, 2016). Consequently, the blueprint of the ASEAN community has been declared as a masterplan to establish a single market of goods and services as well as capitals and skilled labors (Ponziani, 2019; Rillo, 2018). Since then, ASEAN has become a safety area against sudden capital outflow (Harvey, 2017) and more attractive to foreign direct investment, especially the ASEAN-5 countries including Thailand, Singapore, Malaysia, Indonesia, and the Philippines (Tri, Nga, & Duong, 2019).

The financial infrastructure of the ASEAN-5 economy has been steadily changing over the past fifteen years. Moreover, after the global financial crisis, the US dollar became less stable (Gavranic & Miletic, 2016) and caused instability in the world's monetary system (Staszczak, 2015). This situation possibly affected the exchange rate volatility of the ASEAN-5 currencies. Therefore, this study aims to apply the natural cubic spline volatility model to explore the dynamic patterns of the exchange rate volatilities of the ASEAN-5 currencies both in terms of the bilateral exchange rates and the effective exchange rates from January 2006 to August 2020.

The natural cubic spline volatility or NCSV model has been proposed in the study of Laipaporn and Tongkumchum (2017). Though, this model is practical to reveal the dynamic patterns of the estimated volatility (Farida, Makaje, Tongkumchum, Phonon, & Laipaporn, 2018), it still needs to identify an appropriate number of knots in order to influence the model's goodness of fit (Laipaporn & Tongkumchum, 2018).

Previous studies have usually applied a user-specified number of knots to the NCSV model. However, in this study, the Monte Carlo simulation will be used to find a proper data-driven criterion to select a number of knots among candidate criteria, including the Akaike's Information Criteria (AIC), Bayesian Information Criteria (BIC), General Cross-Validation (GCV), and Modified General Cross-Validation (MGCV), and then employ the most appropriate criterion to specify the number of knots of the NCSV model for estimating the ASEAN-5 exchange rate volatilities.

The remainder of this article is organized as follows. Section 2 and section 3 present the background and details of the methodology used in this study. The simulation results on the knot selection criteria and the empirical results on the ASEAN-5 exchange rates' volatilities are stated and discussed in Section 4. The conclusions made from this study are presented in Section 5.

2. Literature Review

Volatility is not directly measured like weight and height but it is usually calculated or estimated from its proxy, returns on the financial asset's price or financial index, according to statistical formulas or statistical models (Laipaporn & Tongkumchum, 2017). A spline is a function that many studies have employed to estimate volatility because its continuous piece-wise polynomials are flexible to capture the cyclical pattern of financial volatility. The Spline Generalized Autoregressive Conditional Heteroscedasticity or Spline-GARCH model introduced by Engel and Rangel (2008) and the Generalized Autoregressive Conditional Heteroscedasticity Mixed Data Sampling or GARCH-MIDAS model proposed by Engle, Ghysels and Sohn (2013) are examples of volatility model that utilizes spline function. They used the spline function in a quadratic polynomial form as a part of their model to capture the dynamics of low-frequency volatility and investigate the relationship between low-frequency volatility and macroeconomic variables.

Similarly, the NCSV model also utilizes a natural cubic spline function to estimate financial volatility. A natural cubic spline function is another functional form of a spline, which is piecewise cubic polynomials that are linear beyond the extreme knots (Laipaporn & Tongkumchum, 2017). Recently, Laipaporn and Tongkumchum (2020) employed the NCSV model to estimate the volatilities of the ASEAN-5 stock index and used these estimated volatilities to construct the time-varying correlations in order to illustrate the patterns of co-movement among ASEAN-5 stock market index. Likewise, Farida, Makaje, Tongkumchum, Phon-on, and Laipaporn (2018) also applied the NCSV model to estimate the volatility of crude oil price. The study found a cyclical pattern of the volatility dynamics identical to the pattern obtained by the other volatility model.

However, one critical issue of utilizing spline function for volatility modeling lies in knot selection. Knots are the connectors between the continuous piece-wise polynomials of the spline function. The flexibility of the spline function depends on the number of knots used to compile the function. Consequently, an excessive number of knots might lead to an over-fitted volatility model, on the other hand, an inadequate number of knots tends to provide the under-fitted model (Laipaporn & Tongkumchum, 2018). Additionally, a spline function with many knots is not guaranteed to provide a more fitted model (Breiman, 1993).

Laipaporn and Tongkumchum (2020) and Farida et al. (2018) applied a user-specified number of knots in their respective studies. They subjectively selected an appropriate number of knots concerning the data investigated in their studies. By applying the equi-spaced knots, Laipaporn and Tongkumchum (2020) set an interval between knots at 250 trading-day per interval and then assigned a number of knots according to that interval and a number of observations. Likewise, Farida et al. (2018) set the length between knots at almost 200 trading-days and consequently determined the number of knots for their NCSV model.

In contrast, previous studies that used the spline-GARCH model and the GARCH-MIDAS model alternatively determined the number of knots by utilizing information criteria such as Akaike's Information Criteria (AIC) and Bayesian Information Criteria (BIC) (Conrad & Kleen, 2020; Lee, Stevenson, & Lee, 2018; Engle, Ghysels, & Sohn, 2013; Engle & Rangel, 2008).

Similarly, Laipaporn, and Tongkumchum (2018) also employed both the AIC and BIC criteria for selecting a number of knots for the NCSV model. They found that AIC performed well to the simulated dataset but failed to provide the appropriate knots for the NCSV model in the case of the empirical data. However, the number of equi-spaced knots using in Laipaporn and Tongkumchum (2018) is exponentially increasing by 2^{n-1} where n is an increasing step. This procedure is different from the other studies, which usually increase the number of knots one at a time.

Based on prior studies, there are other approaches that have been used to determine the number of knots in a spline function. Montoya, Ulloa, and Miller (2014) compared various knots selection criteria and found that generalized cross-validation or GCV introduced by Craven and Wabha (1979) is more suitable method in selecting the number of knots for the penalized regression spline model. Likewise, Chen, Abraham, and Bennett (1997) and Lewis and Stevens (1991) utilized the modified generalized cross-validation or MGCV proposed by Friedman (1991) for selecting the number of knots of the multivariate adaptive regression or MARS model. They found that this approach provided a parsimonious time series model that exhibited a cyclical pattern of the time series data.

Hence, this study investigates more candidate criteria other than the AIC and BIC to find the most appropriate approach for modeling the NCSV model and alternatively increasing the number of quantile knots of the NCSV model one at a time.

3. Research Methods and Materials

Two dataset were used in this study. The first dataset is the daily returns of the ASEAN-5 bilateral exchange rates and their effective exchange rates from January 2006 to August 2020. The daily returns series are the logarithm returns calculated using their corresponding exchange rates, which were retrieved from the website of Bank of International Settlements. They comprised of the exchange rates of Thai baht (THB), Singapore dollar (SGD), Malaysian ringgit (MYR), Indonesian rupiah (IDR), and Philippine peso (PHP).

The second dataset is the simulated returns series. it is generated for the Monte Carlo simulation in order to find a proper criterion to select an appropriate number of knots for the NCSV model. By supposing that true volatility is known, this study generated the simulated daily returns series as a random noise with zero mean and pre-specified volatility (σ_t) using the following equation.

$$r_t = n(0, \sigma_t) \tag{1}$$

Ten types of pre-specified volatilities were determined as the rolling standard deviation of the daily returns of ASEAN-5 bilateral exchange rates with two different rolling windows which are 60 and 120 trading days per window. Note that the wider rolling windows provide less fluctuated rolling standard deviations.

Based on ten types of pre-specified volatility, ten groups of 500 series of 1,500 simulated daily returns were generated and used as a dataset used for the Monte-Carlo simulation. An example of simulated returns series in absolute term and its corresponding pre-specified volatility are shown in Figure 1.

As introduced in Laipaporn and Tongkumchum (2017), the NCSV model is based on the assumption that the time series of the financial returns (r_t) is the product of time-varying volatility (s_t) and random noise (ε_t), which is normal distributed with a zero mean and a unit standard deviation as follows.

$$r_t = s_t \varepsilon_t \tag{2}$$

The time-varying volatility (s_t) is parameterized as the natural cubic spline function with respect to time (t = 1, 2, 3, …, T) as the succeeding equation.

$$s_t = \alpha + \beta_t + \sum_{k=1}^{p-2} \theta_k \left(\begin{array}{c} (t-t_k)_+^3 - \dfrac{t_p - t_k}{t_p - t_{p-1}} (t - t_{p-1})_+^3 \\ + \dfrac{t_{p-1} - t_k}{t_p - t_{p-1}} (t - t_p)_+^3 \end{array} \right) \tag{3}$$

The total number of observations is equal to T, p is a number of knots k where k = 1, 2, 3, …, p. Each knot is placed at t_k which is a quantile order k of time t in the interval [1, T].

To estimate the parameters of the NCSV model, the natural cubic spline function is applied to the absolute values of the returns series ($|r_t|$), which are the proxies of the daily volatility (Figlewski, 1997). Consequently, the parameters of this function (α, β and θ_k) are estimated by maximizing the log-likelihood function (L) as follows.

$$L = \sum_{t=1}^{T} \left(-\log(s_t) - \frac{(|r_t|)^2}{2s_t^2} \right) \tag{4}$$

This study set three steps to select the NCSV model with the optimal number of knot based on previous literature works. The first step is to determine a possible range of a number of knots p. A number of knots p usually begins with three as the lower limit of the possible range.

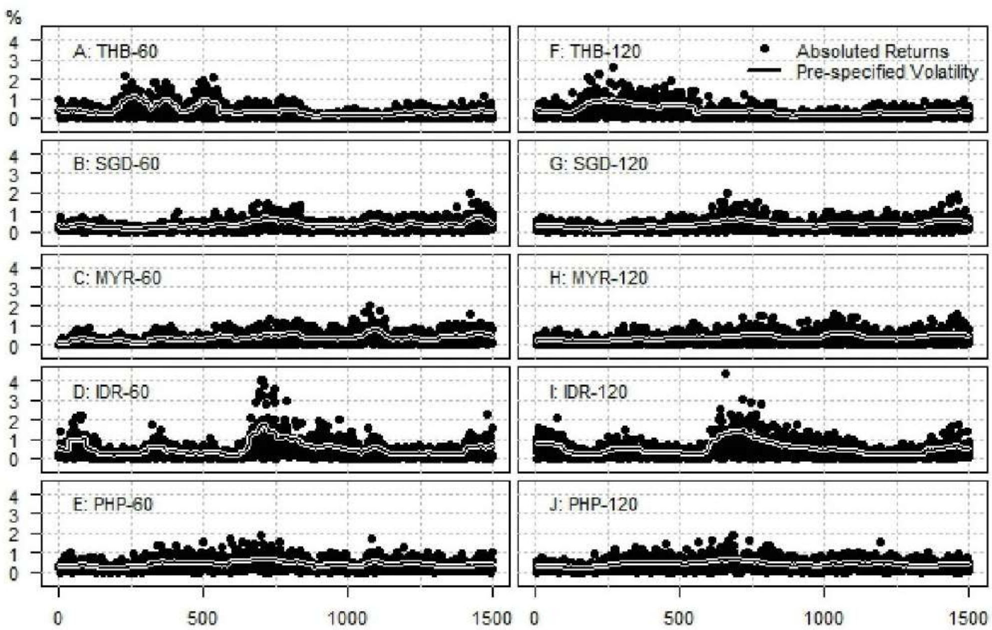


Figure 1: Examples of Simulated Daily Returns in Absolute Term and their Corresponding Pre-Specified Volatilities

The three knots include two boundary knots at the first and the last observation and one interior knot in the middle. To ensure that there are at least 40 observations which is the number of observations in one quarter between each knot. The upper limit of the number of knots is therefore set at $T/p < 40$.

The second step is to estimate the NCSV model's parameters with the number of knots in the possible range. The third step is to apply four candidate criteria: AIC, BIC, GCV and MGCV to the NCSV model obtained from the second step using the following formula.

$$\mathrm{AIC} = -L + 2p \quad (5)$$

$$\mathrm{BIC} = -L + p\log(n) \quad (6)$$

$$\mathrm{GCV} = \frac{T^{-1}\sum_{t=1}^{T}\left(|r_t| - s_t\right)^2}{\left(1 - \frac{(p-1)}{T}\right)^2} \quad (7)$$

$$\mathrm{MGCV} = \frac{T^{-1}\sum_{t=1}^{T}\left(|r_t| - s_t\right)^2}{\left(1 - \frac{(p+1)+dp}{T}\right)^2} \quad (8)$$

Note that d in the MGCV formula is a parameter representing the cost of the increased knot in the spline function. The larger number of d tends to signify a fewer number of knots. This study sets d equal to 2 following the recommendation in Friedman (1991). The NCSV model with the least value of each criterion, consequently indicates the optimal number of knots for that criterion.

For the simulated return datasets, the root mean square error (RMSE) of the pre-specified volatility (σ_t) and the estimated volatility (s_t) estimated by the NCSV model is calculated as the following equation.

$$\mathrm{RMSE} = \sqrt{\frac{\left(\sigma_t - s_t\right)^2}{T}} \quad (9)$$

The NCSV model with the least RMSE indicates the optimal number of knot corresponding to each pre-specified volatility. Accordingly, this optimal number became the benchmark number of knots for the performance comparison among four criteria in the Monte Carlo simulation. A candidate criterion that specifies the number of knots closest to the benchmark number is the most appropriate knot selection criterion for estimating the NCSV model of the ASEAN-5 currencies.

4. Results and Discussion

For the Monte Carlo simulation, the simulated returns series according to ten types of pre-specified volatilities were applied to the NCSV model with a number of knots in the possible range varying from 3 to 42 knots. Since each returns series has 1,500 observations, the interval size between knots of the NCSV model thus varies from nearly 40 observations to 750 observations per interval.

Figure 2 shows graphs plotting the number of knots in the possible range and the averages of RMSE, AIC, BIC, GCV and MGCV obtained from the NCSV models.

According to each criterion, the vertical line and the number at the corner of each graph indicate the least averages and an optimal number of knots. The least average values of RMSE specify the benchmark number of knots for each group of simulated returns. The first five groups of simulated returns were generated by 60 trading days rolling standard deviation (A, B, C, D, E), mostly require more number of knots for the NCSV model than the simulated returns generated by 120 trading days rolling standard deviation (F, G, H, I, J). It implies that if true volatility is high fluctuated, it will require more knots to model the NCSV.

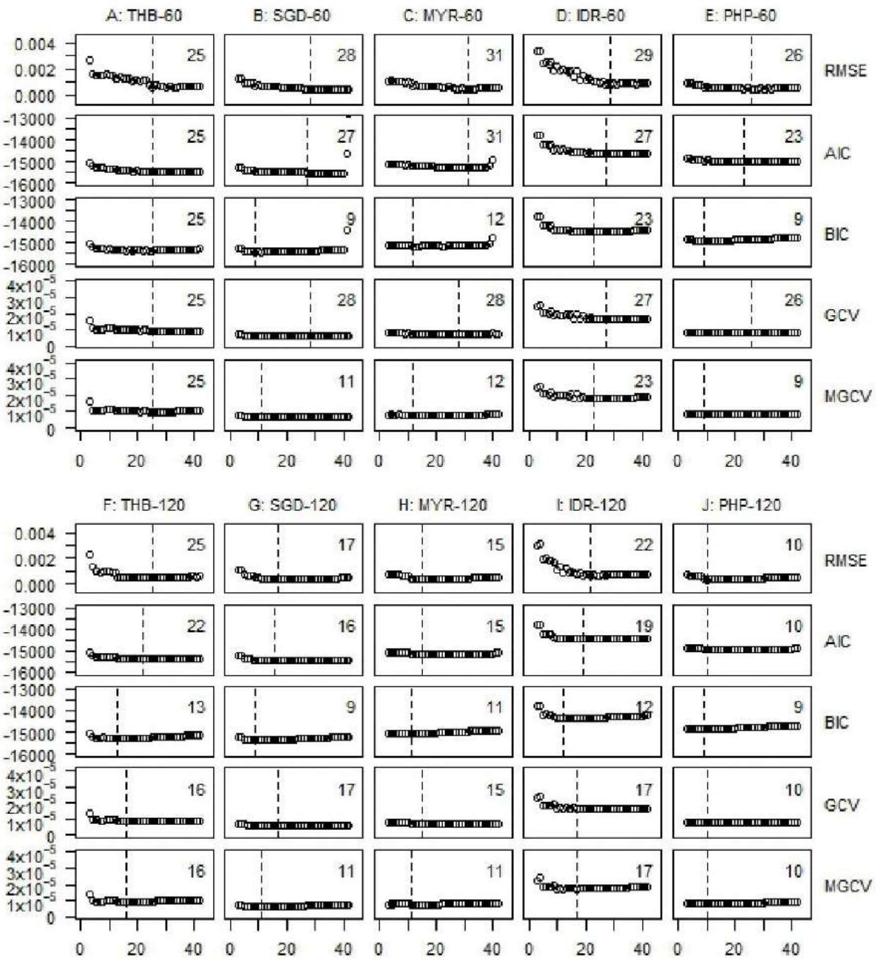


Figure 2: The Number of Knots and the Average Values of RMSE, AIC, BIC, GCV and MGCV from the NCSV Models by Ten Groups of the Simulated Returns

Regarding the benchmark number, a proper knot selection criterion has to select neither too many nor too few numbers of knots than the benchmark number. Among the ten groups of simulated returns, GCV selects the number of knot identical to the benchmark number for six groups (A, B, E, G, H, J). For the other four groups (C, D, F, G), GCV selects a less number of knots, but it is not much different from the benchmark. Likewise, AIC selects the number of knot identical to the benchmark for four groups of simulated returns (A, C, H, J).

In contrast, there is only one group of simulated returns (A) that BIC identifies the number of knots identical the benchmark number. For the rest groups, the number of knot indicated by BIC is much less to the benchmark number. MGCV is a little better than the BIC. It selects the same number of knots as the benchmark number for two groups of simulated returns (A, J). The BIC and MGCV do not perform well for the groups of simulated returns with more fluctuated pre-specified volatility (60 trading days rolling standard deviation). However, they tend to select a number nearer to the benchmark for the groups of simulated returns generated by less fluctuated pre-specified volatility (120 trading days rolling standard deviation).

Following the simulation results, GCV is the most preferred criterion, while the second-best criterion is the AIC. In contrast, the BIC provides a small number of knots for most cases. This result is similar to Laipaporn and Tongkumchum (2018), which found the AIC to be a more preferred criterion than the BIC in the case of simulated data. Note that the number of knots in this study increases one at a time. This contrasts with Laipaporn and Tongkumchum (2018) study where an increase in the number of knots occurred exponentially from 3 to 7, 15, 31, 63, 127, etc. Although increasing the number of knots one at a time is a time-consuming process, this simulation demonstrates that this approach is entirely accurate in determining the number of knots for the NCSV model.

For the empirical part, the series of 3,773 daily returns on the bilateral exchange rates and the effective exchange rates of the ASEAN-5 currencies are presented in Figure 3.

Mean values of these daily returns series are tested and they are not significantly different from zero. Among the five currencies, IDR has the broadest range of daily returns series. The differences between the minimum and maximum value of its bilateral and effective exchange rates are nearly 13 and 12 percentage points, respectively. These two series also have the highest kurtosis at 15.7 and 19.7 points. Whereas, SGD has less varied daily returns series than the others. The standard deviations of its bilateral and effective exchange rates are 0.35 and 0.19, respectively. The skewness of all series is relatively small. They are close to zero in the range between –0.61 and 0.28.

To determine the proper NCSV models of ASEAN-5 exchange rates, a set of pre-specified number knots is assigned from 3 to 96. The size of the interval between knots varies from nearly 40 observations to 1,886 observations per interval. The values of AIC, BIC, GCV and MGCV of the NCSV models of the ASEAN-5 exchange rates and the possible range number of knots are displayed in Figure 4.

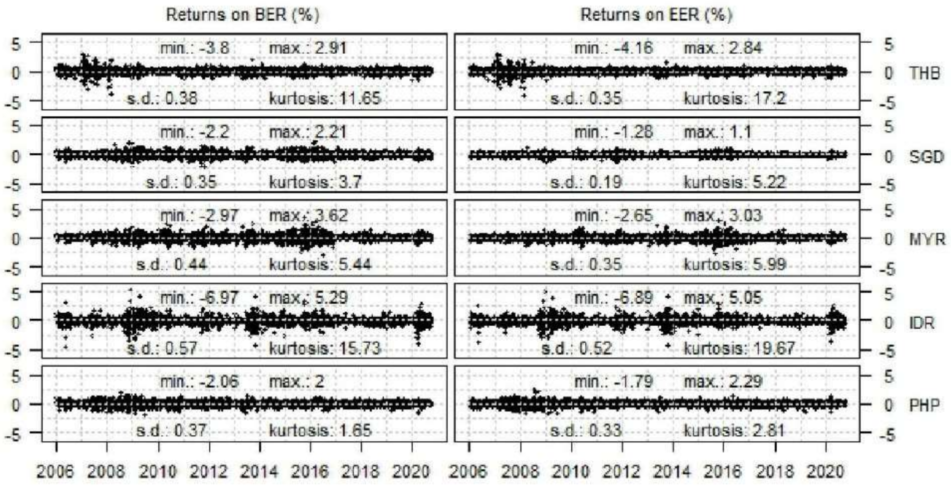


Figure 3: Returns on the Bilateral Exchange Rates (BER) and the Effective Exchange Rates (EER) of the ASEAN-5 Currencies

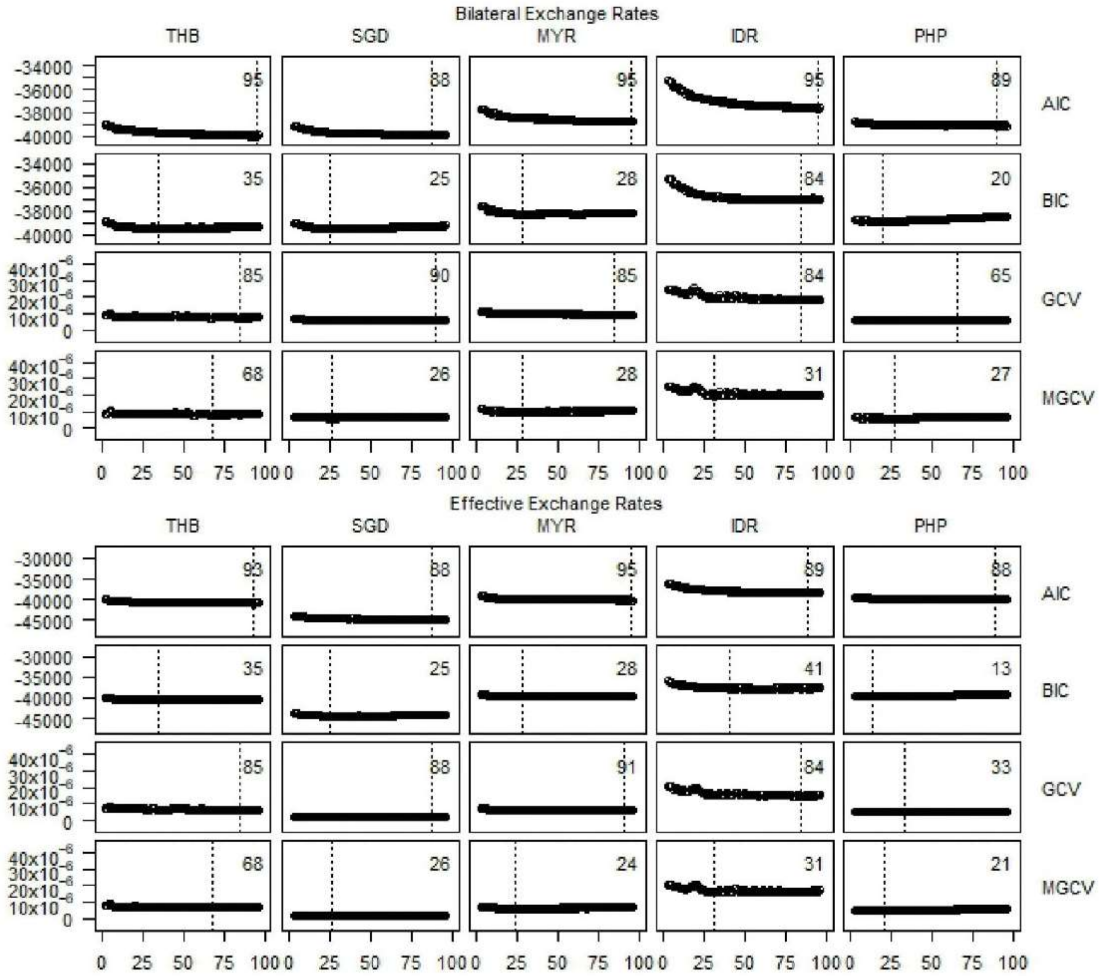


Figure 4: The AIC, BIC, GCV and MGCV from the NCSV Models of Daily Returns of ASEAN-5 Currencies Currencies

The number of knots for the NCSV models of ASEAN-5 exchange rates indicated by BIC is relatively fewer than the number obtained by the other three criteria. Likewise, MGCV often selects a number of knots identical to a number chosen by BIC. Therefore, BIC and MGCV tend to indicate an under-fitted model. GCV and AIC select an equivalent number of knots in some cases. However, the behavior in knot selection of GCV is more consistent than AIC, because in some cases, AIC assigns too large number of knots for the NCSV model. Regarding this comparison, the GCV is likely to provide an accurate number of knots for modeling the natural cubic volatility of the ASEAN-5 exchange rates.

Since the effective exchange rates are less volatile, in some cases GCV designates a smaller number of knots than a number of knots of the bilateral exchange rates' volatility models. The intervals between knots according to the number of knots selected by GCV in this study vary from 42 to 118 trading days. These intervals are much smaller than the intervals assigned by the same criterion in Engle and Rangel (2008) for the volatilities of the ASEAN-5 stock index. Note that the functional form of the spline function used in this study is a natural cubic spline function, which is different from the quadratic spline function used in Engle and Rangel (2008). The natural cubic spline is more flexible than the quadratic spline. Consequently, it needs a more number of knots to fit the volatility model.

The volatilities of ASEAN-5 exchange rates estimated by the natural cubic spline volatility models with a number of knots selected by GCV are shown in Figure 5. Graphs in the left column illustrate the comparison of the bilateral exchange rate volatilities (BER) and the effective exchange rate volatilities (EER) of the ASEAN-5 currencies in the same axis, while graphs in the right column show the volatility ratio, the ratio of the bilateral exchange rate volatilities over the effective exchange rate volatilities of the corresponding currencies.

The dynamic patterns of the bilateral exchange rate volatility of the ASEAN-5 currencies are not entirely different from the pattern of the same currencies' effective exchange rate volatility. The bilateral and effective exchange rate of IDR is more volatile than the other currencies, while the exchange rate volatilities of SGD and PHP indicate that these two currencies are more stable than the other currencies. The finding is similar to Ponziani (2019) and Klyuev and Dao (2017).

The bilateral and effective exchange rates of THB are most volatile during the global financial crisis period. The exchange rate volatilities of THB were more than 10 percent in that period, and then diminished to less than 5 percent after the crisis. This shows the exchange rate of THB is stable after the global crisis until now. The exchange rates of MYR are less stable between 2015 to 2016 since the MYR depreciation to the world currencies in October 2015 (Quadry, Mohamad, & Yusof, 2017). Malaysia increases their money supply by lowering its interest rate to absorb the exchange rate shock (Kaur, Manual, & Eeswaran, 2019).

The reference lines in the left graphs of Figure 5 indicates the volatility ratio equal to one. As shown in Figure 5, in the period that the exchange rate volatility ratios of the ASEAN-5 currencies are higher than the reference line, the bilateral exchange rates of the ASEAN-5 currencies are more volatile than its corresponding effective exchange rates. The volatility ratios of the ASEAN-5 currencies are mostly higher than the reference line, especially SGD; its bilateral exchange rate volatility is almost twice higher than its effective exchange rate volatility. The average volatility ratios of THB, SGD, MYR, IDR and PHP are 1.17, 1.99, 1.24, 1.12 and 1.11 respectively.

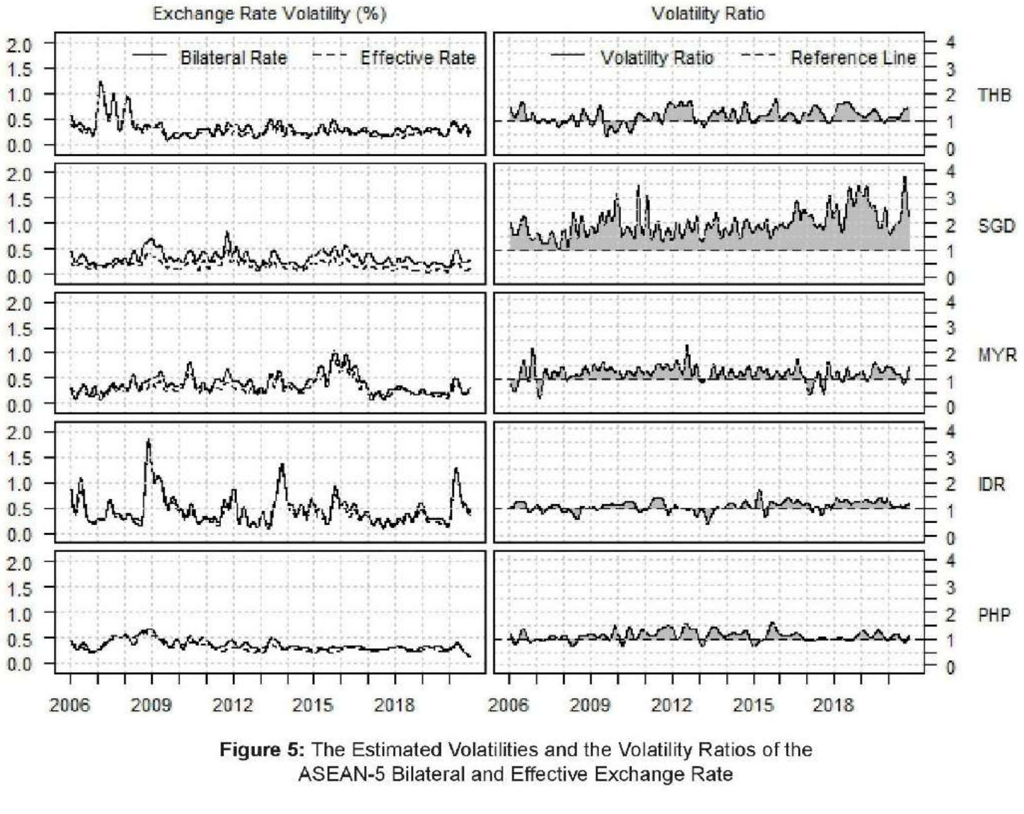


Figure 5: The Estimated Volatilities and the Volatility Ratios of the ASEAN-5 Bilateral and Effective Exchange Rate

Since the stability of the effective exchange rate reflects its typical characteristic, which can absorb the uncertain exchange rate policies of its trade partners (Thuy & Thuy, 2019), the SGD was more capable to confront the uncertainty in the international trade and investment than the other currencies. This is because the stability of the US dollar affects the volatility of the bilateral exchange rates. Several studies are likely to eliminate the influence of the US dollar instability by employing the effective exchange rate volatility rather than the bilateral exchange rate volatility in order to examine the real stability of the currency (Kaur, Manual, & Eeswaran, 2019; Thuy & Thuy, 2019; Al-Abri & Baghestani, 2015).

5. Conclusions

To estimate the exchange rate volatility of the ASEAN-5 currencies, this study applied the natural cubic spline model with various data-driven knot selection criteria comprised of the AIC, BIC, GCV and MGCV. This study further employed the Monte Carlo simulation to find the most appropriate knot selection criteria. The simulation showed that GCV is the most preferred since it assigns a number of knots closest to the benchmark number. The BIC and MGCV tend to determine a smaller number than the other criteria in simulated datasets and empirical datasets. For the simulated dataset, AIC performs well. It often selects an identical number of knot to the benchmark knots. However, it selects too much number of knots for the empirical datasets.

Additionally, the exchange rate volatilities of the ASEAN-5 currencies, estimated using the natural cubic spline model with a number of knots selected by GCV, revealed the inconstant dynamic pattern of the ASEAN-5 exchange rate volatilities. The effective exchange rates of the ASEAN-5 currencies have less variation than the bilateral exchange rates, especially the bilateral exchange rate of the Singapore dollar, which is almost twice larger than the effective rate. It is clear evidence showing the stability of the Singapore dollar and the influence of the US dollar on the variation of the bilateral exchange rate of the ASEAN-5 currencies.

References

AbuDalu, A., Ahmed, E. M., Almasaled, S. W., & Elgazoli, A. I. (2014). The real effective exchange rate impact on ASEAN-5 economic growth. *International Journal of Economics & Management Sciences, 3*(2), 1–11. https://doi.org/10.4172/2162-6359.1000174

Ahmed, F., & Singh, V. K. (2016). Financial integration among RCEP (ASEAN+6) economics: Evidences from stock and forex markets. *South Asian Journal of Management, 23*(1), 164–188. Retrieved November 1, 2020 from: http://search.ebscohost.com/login.aspx?direct=true&db=bth&AN=115318412&site=ehost-live

Al-Abri, A., & Baghestani, H. (2015). Foreign investment and real exchange rate volatility in emerging Asian countries. *Journal of Asian Economics, 37*, 34–47. https://doi.org/10.1016/j.asieco.2015.01.005

Breiman, L. (1993). Fitting additive models to regression data: Diagnostics and alternative views. *Computational Statistics & Data Analysis, 15*, 13–46. https://doi.org/10.1016/0167-9473(93)90217-H

Campa, A. C. Jr. (2020). Capturing the short-run and long-run causal behavior of Philippine stock market volatility under vector error correction environment. *Journal of Asian Finance, Economics and Business, 7*(8), 41–49. https://doi.org/10.13106/jafeb.2020.vol7.no8.041

Chen, G., Abraham, B., & Bennett, G. W. (1997). Parametric and non-parametric modelling of time series: An empirical study. *Environmetrics, 8*, 63–74. https://doi.org/10.1002/(SICI)1099-095X(199701)8:1<63::AID-ENV238>3.0.CO;2-B

Conrad, C., & Kleen, O. (2020). Two are better than one: Volatility forecasting using multiplicative component GARCH-MIDAS models. *Journal of Applied Econometrics, 35*(1), 19–45. https://doi.org/10.1002/jae.2742

Craven, P., & Wabha, G. (1979). Smoothing noisy data with spline functions: Estimating the correct degree of smoothing by method of generalized cross-validation. *Numerische Mathematik, 31*(4), 377–403. https://doi.org//10.1007/BF01404567

Dang, V. C., Le, T. L., Nguyen, Q. K., & Tran, D. Q. (2020). Linkage between exchange rate and stock prices: Evidence from Vietnam. *Journal of Asian Finance, Economics and Business. 7*(12), 95–107. https://doi.org/10.13106/jafeb.2020.vol7.no12.095

Engle, R. F., & Rangel, J. G. (2008). The spline-GARCH model for low-frequency volatility and its global macroeconomic causes. *The Review of Financial Studies, 21*(3), 1187–1222. Retrieved November 1, 2020 from: https://www.jstor.org/stable/40056848

Engle, R. F., Ghysels, E., & Sohn, B. (2013). Stock market volatility and macroeconomic fundamentals. *The Review of Economics and Statistics, 95*(3), 776–797. https://doi.org/10.1162/REST_a_00300

Farida, Makaje, N., Tongkumchum, P., Phon-on, A., & Laipaporn, J. (2018). Natural cubic spline model for estimating volatility. *International Journal on Advanced Science, Engineering and Information Technology, 4*(8), 1085–1090. https://doi.org/10.18517/ijaseit.8.4.3107

Figlewski, S. (1997). Forecasting volatility. *Financial Markets Institutions and Instruments, 6*(1), 1–88. https://doi.org/10.1111/1468-0416.00009

Friedman, J. H. (1991). Multivariate adaptive regression splines. *The Annals of Statistics, 19*(1), 1–67. https://doi.org/ 10.1214/aos/1176347963

Gavranic, K., & Miletic, D. (2016). US dollar stability and the global currency reserves. *Eurasian Journal of Economics and Finance, 4*(3), 14–24. https://doi.org/10.15604/ejef.2016.04.03.002

Harvey, B. (2017). Evaluating financial integration and cooperation in the ASEAN. *Michigan Business & Entrepreneurial Law Review, 7*(1), 119–158. Retrieved November 1, 2020, from: https://repository.law.umich.edu/cgi/viewcontent.cgi?article=1067&context=mbelr

Kaur, T. A., Manual, V., & Eeswaran, M. (2019). The volatility of the Malaysian ringgit: Analyzing its impact on economic growth. *International Journal of Recent Technology and Engineering, 7*(5S), 82–88. Retrieved November 1, 2020, from: https://www.ijrte.org/wp-content/uploads/papers/v7i5s/Es2129017519l9.pdf

Kennedy, K., & Nourzad, F. (2016). Exchange rate volatility and its effect on stock market volatility. *International Journal of Human Capital in Urban Management, 1*(1), 37–46. https://doi.org/10.7508/ijhcum.2016.01.005

Klyuev, V., & Dao, T.-N. (2017). No more clubbing, the evolution of exchange rate behaviour in the ASEAN-5 countries. *Journal of Southeast Asian Economies, 34*(2), 233–265. https://doi.org/10.1355/ae34-2a

Laipaporn, J., & Tongkumchum, P. (2017). Maximum likelihood estimation of non-stationary variance. In: *Proceedings of International Statistical Institue Regional Statistics Conference 2017 (ISI RSC 2017)*, (pp. 774–780). Bali.

Laipaporn, J., & Tongkumchum, P. (2018). The use of information criteria for selecting number of knots in natural cubic spline volatility estimation. In *Proceedings of International Conference of Applied Statistics 2018* (pp. 161–164). Bangkok: Faculty of Sciences and Technologies, Prince of Songkla University, Pattani campus.

Laipaporn, J., & Tongkumchum, P. (2020). The time-varying correlation estimator using the natural cubic spline volatility. *Advances and Applications in Statistics, 62*(1), 79–95. http://doi.org/10.17654/AS062010079

Lee, C. L., Stevenson, S., & Lee, M. -L. (2018). Low-frequency volatility of rea estate securities and macroeconomic risk. *Accounting and Finance, 58*(2018), 311–342. https://doi.org/10.1111/acfi.12288

Lewis, P. A., & Stevens, J. G. (1991). Nonlinear modeling of time series using Multivariate Adaptive Regression Splines (MARS). *Journal of the American Statistical Association, 86*(41), 864–877. https://doi.org/ 10.1080/01621459.1991.10475126

Liu, J., Wang, M., & Sriboonchitta, S. (2019). Examining the interdependence between the exchange rates of China and ASEAN countries: A canonical vine copula approach. *Sustainability, 11*, 1–20. https://doi.org/10.3390/su11195487

Montoya, E. L., Ulloa, N., & Miller, V. (2014). A simulation study comparing knot selection methods with equally spaced knots in a penalized regression spline. *International Journal of Statistics and Probability, 3*(3), 96–110. https://doi.org/ 10.5539/ijsp.v3n3p96

Ponziani, R. M. (2019). Foreign exchange volatility modeling of Southeast Asian major economies. *Journal of Economics, Business and Accountancy Ventura, 22*(2), 283–297. https://doi.org/10.14414/jebav. v22i2.1912

Purwono, R., Mucha, K., & Mubin, M. K. (2018). The dynamic of Indonesia's current account deficit: Analysis of the impact of exchange rate volatility. *Journal of Asian Finance, Economics and Business, 5*(2), 25–33. https://doi.org/10.13106/jafeb.2018.vol5.no2.25

Quadry, M. O., Mohamad, A., & Yusof, Y. (2017). On the Malaysian ringgit exchange rate determination and recent depreciation. *International Journal of Economics, Management and Accounting, 25*(1), 1–26. Retrieved November 1, 2020 from: https://journals.iium.edu.my/enmjournal/index.php/enmj/article/view/366

Rillo, A. D. (2018). ASEAN financial integration: Opportunities, risks and challenges. *Public Policy Review, 14*(5), 901–923. Retrieved November 1, 2020, from: https://www.mof.go.jp/english/pri/publication/pp_review/ppr14_05_04.pdf

Singh, V. K., & Ahmed, F. (2016). Econometric analysis of financial co-integration of least developed countries (LDCS) of Asia and the Pacific. *China Finance Review International, 6*(2), 208–227. https://doi.org/10.1108/CFRI-06-2015-0056

Soleymani, A., Chua, S. Y., & Hamat, A. C. (2017). Exchange rate volatility and ASEAN-4's trade flows: is there a third country effect?, *International Economics and Economic Policy, 14*, 91–117. https://doi.org/10.1007/s10368-015-0328-9

Staszczak, D. E. (2015). Global instability of currencies: reasons and perspectives according to the state-corporation hegemonic stability theory. *Revista de Economia Politica, 35*(1), 174–198. https://doi.org/10.1590/0101-31572015v35n01a10

Tan, K. G., Duong, L. N., & Chuah, H. Y. (2019). Impact of exchange rates on ASEAN's trade in the era of global value chains: An empirical assessment. *Journal of International Trade & Economic Development, 28*(7), 873–901. https://doi.org/10.1080/09638199.2019.1607532

Teulon, F., Guesmi, K., & Mankai, S. (2014). Regional stock market integration in Singapore: A multivariate analysis. *Economic Modelling, 43*, 217–224. https://doi.org/10.1016/j.econmod.2014.07.045

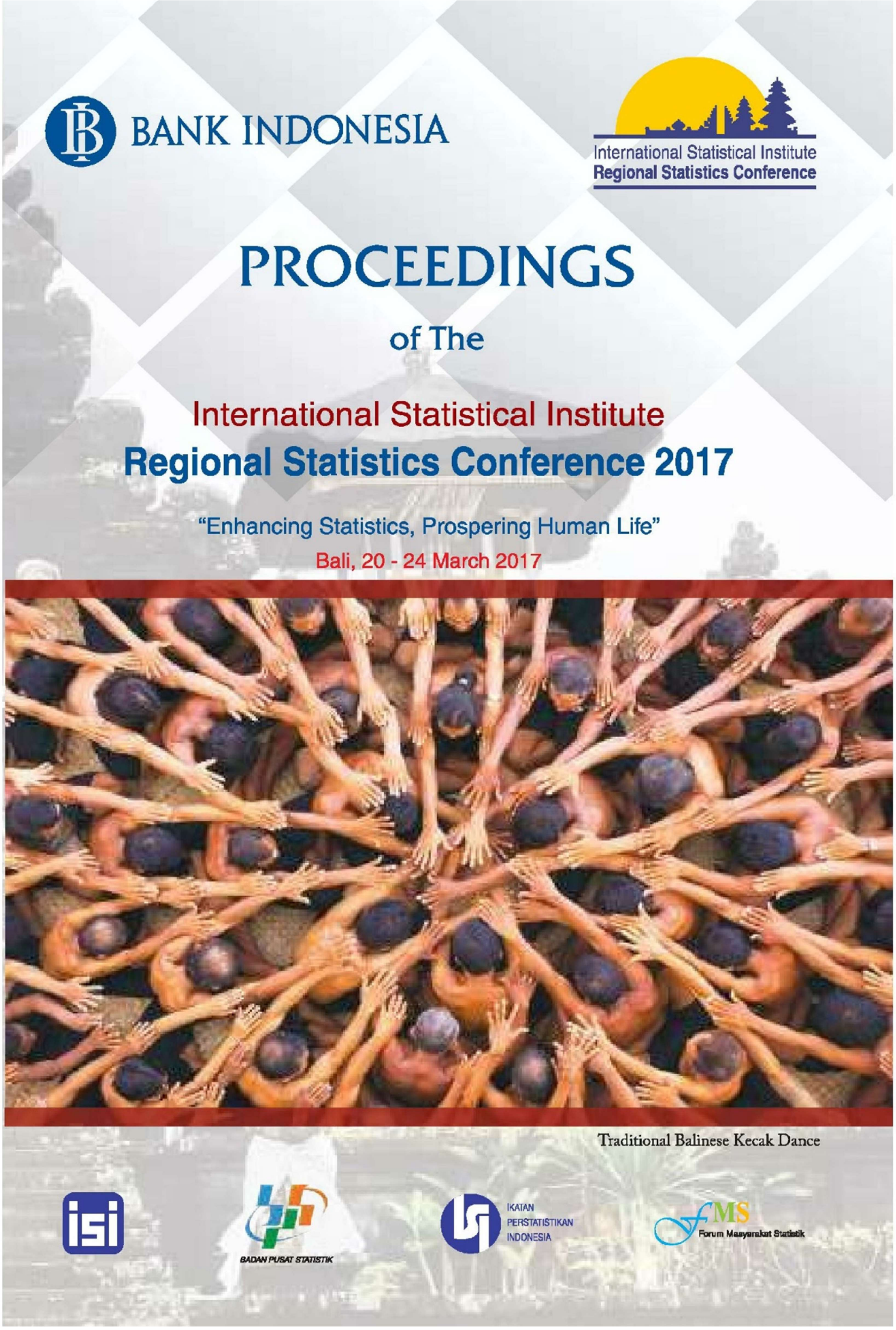
Thuy, V. N., & Thuy, D. T. (2019). The impact of exchange rate volatility on exports in Vietnam: A bounds testing approach. *Journal of Risk and Financial Management, 12*(6), 1–14. https://doi.org/10.3390/jrfm12010006

Tri, H. T., Nga, V. T., & Duong, V. H. (2019). The determinants of foreign direct investment in ASEAN: New evidence from financial integration factor. *Business and Economic Horizons, 15*(2), 292–303. https://doi.org/10.15208/beh.2019.18

Upadhyaya, K. P., Dhakal, D., & Mixon, F. G. (2020). Exchange rate volatility and exports: some new estimates from the ASEAN-5. *Journal of Developing Areas, 54*(1), 65–73. https://doi.org/10.1353/jda.2020.0004

Appendix III

Proceeding paper I: “Maximum likelihood estimation of non-stationary variance”


BANK INDONESIA
International Statistical Institute
Regional Statistics Conference
PROCEEDINGS
of The
International Statistical Institute
Regional Statistics Conference 2017
"Enhancing Statistics, Prospering Human Life"
Bali, 20 - 24 March 2017
Traditional Balinese Kecak Dance
isi
BADAN PUSAT STATISTIK
IKATAN PERSTATISTIKAN INDONESIA
FMS
Forum Masyarakat Statistik

Proceedings 2nd ISI Regional Statistics Conference, 20-24 March 2017, Indonesia (Session CPS25)

Maximum Likelihood Estimation of Non-stationary Variance

Jetsada Laipaporn*
Prince of Songkla University, Pattani, Thailand – laipaporn.j@gmail.com
Phattrawan Tongkumchum
Prince of Songkla University, Pattani, Thailand – phattrawan@gmail.com

Abstract

The general autoregressive conditional heteroscedasticity – GARCH (1, 1) model is widely used for estimating non-stationary variance of financial time series, but it gives results that need smoothing in order to show structural changes over the period of interest. Its estimation also requires iterative optimization of a function involving derivatives that need to be computed numerically, and is not guaranteed to converge. We consider an alternative method using maximum likelihood to estimate parameters in a natural cubic spline function. This method gives similar results to GARCH (1, 1) when applied to exchange rates of currencies in term of special drawing right (SDR) for Thailand, Singapore, Malaysia and Indonesia over the last two decades, and simulation studies suggest that it is more accurate than GARCH (1, 1).

Keywords: Heteroscedasticity; natural cubic spline function; GARCH (1, 1); exchange rates.
Journal of Economic Literature (JEL) Classification: C13; C14; C22; G15

1. INTRODUCTION

The non-stationary variance or heteroscedastic condition is critical issue in financial modeling. With respect to this condition, most of financial analysts and economists apply General Autoregressive Conditional Heteroscedasticity or GARCH model introduced by Bollerslev (1986) to estimate the financial volatility. The analysts usually apply the GARCH model to daily returns by using maximum likelihood estimation. Unfortunately these estimates of daily volatility are fluctuated and unclear to illustrate the fundamental movement of the non-stationary variance.

Generally, the natural cubic splines (NCS) function is practical to extract the non-stationary variance, because the continuous piecewise cubic polynomials is efficient in capturing the flexible trend among noisy data (see Hastie *et al.*, 2009). However, Fitting the NCS function to the estimates of the GARCH model will be a good estimate of the non-stationary variance, if only the GARCH model is satisfactory to exhibit the behavior of the observed returns variation.

Therefore, this study proposes an alternative approach to estimate the non-stationary variance of the exchange rates in term of special drawing rights (SDR) for 4 currencies comprised of Thai baht, Singapore dollar, Malaysia ringgit and Indonesia rupiah, and the generated returns with known signal from Monte Carlo simulation by using a maximum likelihood estimation to fit a NCS function directly to the absolute returns and compares to NCS fits to the GARCH (1,1) which is adopted as the representative of the GARCH model.

The next parts of this paper are organized as follows. The second section informs the term of exchange rate data used for calculating returns and how to generate returns from the Monte Carlo simulation. The third section provides methodologies of two approaches to estimate non-stationary variance. The estimated results are reported and discussed in the fourth section. The last section concludes this study.

2. DATA

Two kinds of returns series are employed. The first one is the returns from the exchange rates which are officially used in the international market. The second one is generated by Monte Carlo simulation. The details of these data are described as follows.

2.1. Exchange Rate

Basically, many literatures employ the effective exchange rate to investigate the impact of its volatility on the national economy rather than using the bilateral exchange rate (for detail discussion, see Clark *et al.*, 2004 and McKenzie, 1999), because the effective rate efficiently reflects the uncertainty of the national currency pricing. The effective exchange rate is the average of a national currency relative to an index or basket of the bilateral exchange rate weighted by the trade volumes. These trade volumes indicate the relative importance of that each bilateral rate to the valuation of the national currency (for detail, see Turner *et al.*, 1993).

Calculating the effective exchange rate is comprehensive and need sufficient data for producing the index. However, there is an alternative way to price the national currency. Eugenio (2016) introduces to use the commodity's price in term of the Special Drawing Rights units (SDR). Hence, the SDR is the weighted average of major national currencies traded in the world market. The price in term of SDRs subsequently becomes the international price of that commodity. By this concept, the exchange rates of national currency in term of SDRs are employed as the international price relative to the major currencies of that national currency.

This paper employs the exchange rate of four national currencies in term of SDRs per currency unit which are obtained from the website of the International Monetary Fund or IMF (http://www.imf.org/external/np/fin/ert/GUI/Pages/CountryDataBase.aspx comprise of Indonesia rupiah (IDR), Malaysian ringgit (MYR), Singapore dollar (SGD) and Thai baht (THB) during January 3rd 1994 to December 30th 2016. These exchange rates are plotted in Figure 1.

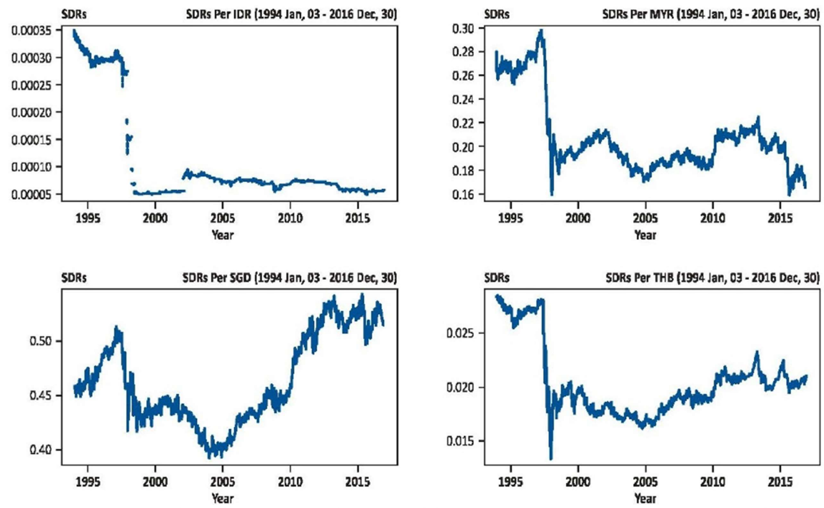


Figure 1. The Exchange Rate of Indonesia Rupiah (IDR), Malaysian Ringgit (MYR), Singapore Dollar (SGD) and Thai Baht (THB) During 1994-2016

The daily variation of the exchange rate is measured by daily returns (u_t) which is a rate of the proportional increasing or decreasing of the exchange rate from preceding day as this following equation,

$$u_t = \frac{E_t - E_{t-1}}{E_{t-1}} \qquad (1)$$

E_t and E_{t-1} are exchange rate on the present day and the preceding day, respectively. Figure 2 graphs the returns of Thai baht, Singapore dollar, Malaysia ringgit and Indonesia rupiah during given period.

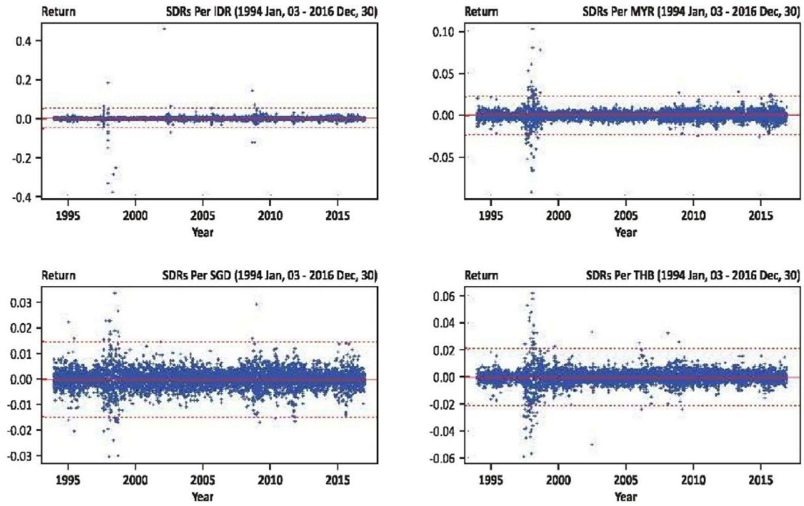


Figure 2. Returns on one Trading Day of Indonesia Rupiah (IDR), Malaysian ringgit (MYR), Singapore Dollar (SGD) and Thai Baht (THB) During 1994-2016

2.2. Generated Returns from Monte Carlo Simulation

The generated returns are simulated following the stylist facts of financial asset returns as discussed in Cont (2001) and Engle (2003). The generated returns are separated into two group based on different assumption.

The first group comprises of four datasets of the returns. Each dataset contains 5,000 random data with zero mean and inconstant variance with typical known signal. These returns are consequently generated as

$$u_t=\mu+s_t z_t \tag{2}$$

The u_t is a return on day t which is a function of expected return (μ) and residual as white noises (z_t) with a known signal (s_t). Since the simulated returns have zero mean, so the expected return (μ) is equal to zero.

The second group is assumed the same as the first group, except the white noises (z_t) become the fat-tailed residual (ε_t) following this equation

$$u_t=\mu+s_t \varepsilon_t \tag{3}$$

The fat-tailed residuals (ε_t) are the transformation of white noises (z_t) on day t. This transformation is to stretch the tails of the white noise's distribution between two critical points, -c and c with the stretching factor equal to a. The transformation is followed this formula

$$\varepsilon_t=\begin{cases} c+a(z_t-c) & ,c > z_t \\ z_t & ,-c < z_t < c \\ -c+a(z_t+c) & ,z_t < -c \end{cases} \tag{4}$$

c and a are equal to 1.25 and 2, respectively.

This study applies four known signals (s_t) represented four different movement of non-stationary variance in the long-run. These signals are constructed as a function of time (t) as shown in Figure 3, consisted of constant (s_0), linear (s_1), quadratic (s_2), and cubic (s_3) signals.

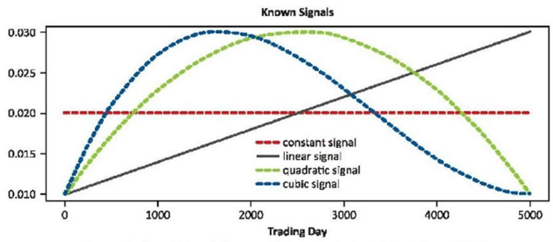


Figure 3. The Four Types of Known Volatility Signal (s_t)

3. METHODOLOGY

This paper utilizes two approaches for estimating the non-stationary variance using NCS function. The first approach is fitting NCS function to the estimated volatility of the GARCH (1, 1) model. Meanwhile, the second approach is fitting NCS function directly to the absolute value of the observed returns. The details of each approach are described as follows.

3.1. NCS Fits to GARCH (1, 1) Estimates.

This approach begins with estimating the daily volatility by using the GARCH (1, 1) model and then fitting NCS function to the estimated volatility for non-stationary variance.

The GARCH (1, 1) assumes that the observed returns (u_i) are independent and the distribution of returns is normal with mean equal to zero and conditional on its variance σ_i^2. This model is usually parameterized as

$$\sigma_i^2 = (1 - \alpha - \beta)V_L + \alpha u_{i-1}^2 + \beta\sigma_{i-1}^2 \qquad (5)$$

V_L is a long term variance which is constant over the period of interest. u_i and σ_i^2 are observed return and estimated conditional variance on day i, respectively. The parameter α is a measure of the influence of the most recent return value and β is a smoothing constant (for detail, see Bollerslev, 1986; Engle, 2001 and Brooks, and 2008).

This model can be fitted to the observed returns by maximizing the likelihood of the n observations. Using the formula for the probability density function of this normal distribution, the likelihood (L) is thus

$$L = \prod_1^n \left[\frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(\frac{u_i^2}{\sigma_i^2}\right) \right] \qquad (6)$$

where the parameters α and β in the GARCH (1, 1) model are substituted for maximizing the likelihood function by using the Newton-Raphson method with Marquardt damping factor followed this iteration process,

$$\theta_{j+1} = \theta_j - dH_j^{-1} \times w_j \qquad (7)$$

At iteration j, θj and ωj are 2×1 vectors containing estimates of α and β and their first derivatives, respectively. H_j is the corresponding 2×2 matrix of the second derivatives. The Marquardt damping factor (d) is constant and in the range between 0 and 1. This factor is designed to decrease the changes at each iteration and thus prevent overshooting maximum values, which are constrained within the conditions, $0 < \alpha < 1$, $0 < \beta < 1$ and $0 < \alpha+\beta < 1$. Confidence intervals for the parameters α and β can be obtained by using the statistical theory of maximum likelihood estimators.

After fitting the GARCH (1, 1) model, the squared root of the estimated conditional variance (σ_i^2) becomes the daily volatility (σ_i) which is used for estimating the non-stationary variance.

The non-stationary variance is estimated in term of the daily deviations (σ_i) which are assumed as a NCS function. This function is express as following equation,

$$\sigma_i = a + bi + \sum_{k=1}^{p} c_k \left(i - t_k \right)_+^3 \tag{8}$$

where i denotes time period (day). σ_i is estimated deviation on day i. The p knots are placed at $t_1<t_2<\ldots<t_p$. Function $(i - t_k)_+$ is a plus function that equal to $i - t_k$ for $i > t_k$ and 0 for otherwise. Since cubic spline function is linear in the distant past and future, the coefficients of quadratic and cubic are 0 for $i < t_1$ and $i > t_p$. To satisfy these constraints, the cubic spline functions becomes

$$\sigma_i = a + bi + \sum_{k=1}^{p-2} c_k \left[\left(i - t_k \right)_+^3 - \frac{t_p - t_k}{t_p - t_{p-1}} \left(i - t_{p-1} \right)_+^3 + \frac{t_{p-1} - t_k}{t_p - t_{p-1}} \left(i - t_p \right)_+^3 \right] \tag{9}$$

The parameters a, b and c_k (k = 1, 2... p-2) are obtained by fitting the NCS function with least square estimation (Venables *et al.*, 2002).

3.2. NCS fits to absolute returns

This approach estimates the non-stationary variance by fitting the NCS function directly to the absolute value of the returns series. The returns are assumed independent and normally distributed with zero mean and inconstant standard deviation σ_i (i = 1, 2, 3... n).

Since the absolute returns have a long memory property that keeps effect of fluctuation continuing persistent over a long time period, so they can be used as the proxy of daily standard deviation (for details, see Ding *et al.*, 1993).

Note that the parameters of natural cubic spline function can be estimated by using the ordinary least squares method, but it may not give preferable results, because the returns have inconstant variance. So the maximum likelihood method is alternatively applied (Greene, 2002). The log likelihood function is followed this equation,

$$L = \sum_{i-1}^{n} \left[-\log(\sigma_i) - \frac{u_i^2}{2\sigma_i^2} \right] \tag{10}$$

The daily deviation σ_i is estimated as equation (9) on day i and u_i is return on day i. The parameters of NCS function is obtained by maximizing the log likelihood function with the Newton-Raphson method with Marquardt damping factor. The iteration process is followed the equation (7) where θ_j is p × 1 matrix that contains the estimate of parameters a, b and c_k (k = 1, 2... p-2). The vector of first derivative and second derivative of likelihood functions according to each parameter expressed as simple algebraic are contained in p × 1 matrix (ω_j) and p × p matrix (H_j), respectively. The standard errors of parameters a, b and c_k can be obtained from the square root of - *diagonal* (H^{-1}).

4. RESULT AND DISCUSSION

Following two estimating approaches described above, the non-stationary variances in term of daily deviation of four exchange rates are plotted in Figure 4. These deviations are estimated by fitting a NCS function with 45 equi-space knots. The space between each pair of knots equals to 130 days which are trading days in a half year.

Using maximum likelihood method to fit the NCS to the absolute value of the returns of these four exchange rates is remarkably better to trace the variation of returns series than fitting the NCS to the daily volatility. Moreover, as shown at the legend of each graph, fitting NCS function to the absolute returns provides the higher likelihood value than the other approach, especially in case of Indonesia Rupiah (IDR).

The estimate daily deviations of daily volatility of Indonesia Rupiah (the top left panel of Figure 4) are much higher than the absolute value of the actual daily returns during low variation period and lower during high variation period. This evidence shows that the GARCH (1, 1) model is inappropriate to estimate the daily volatility of the Indonesia Rupiah.

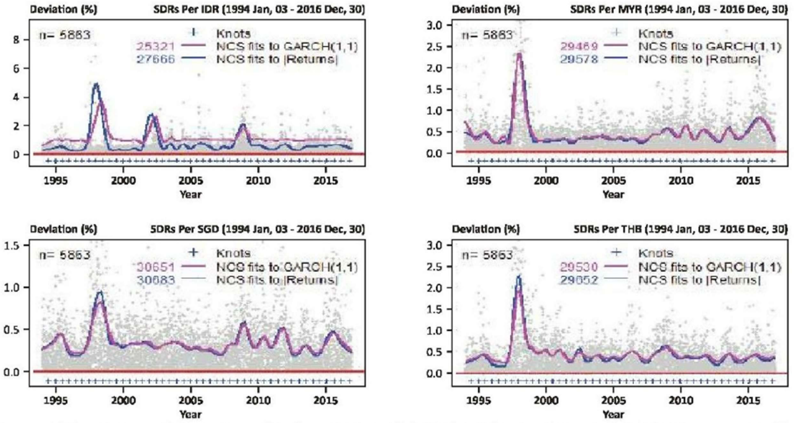


Figure 4. The Estimated Deviation of Indonesia Rupiah (IDR), Malaysian Ringgit (MYR), Singapore Dollar (SGD) and Thai Baht (THB) During 1994-2016

The same as the real data, the NCS function are fitted to two groups of generated returns by using a half year equi-space knots. The daily deviations of these two groups which are the generated returns with white noise residual and fat-tailed residual are shown in Figures 5 and 6, respectively. These estimated results show that the both estimating approached are efficient to trace the known signal of the non-stationary variance. These estimated results show the same typical trends as the specified signals.

Again, both graphic and the likelihood values shown at the legend of each graph indicate that using maximum likelihood method to fit the NCS function to the absolute returns provides the daily deviations which are fitted to the variation of daily returns better than fitting the NCS function to the GARCH (1, 1) estimates, especially in case of the generated returns with fat-tailed residuals.

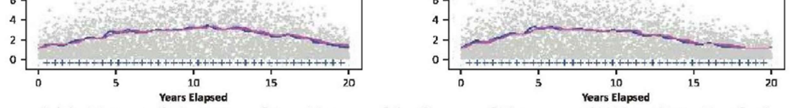


Figure 5. The Estimated Deviation of Four Dataset of the Generated Returns with White Noise Residuals and Each Known Signal

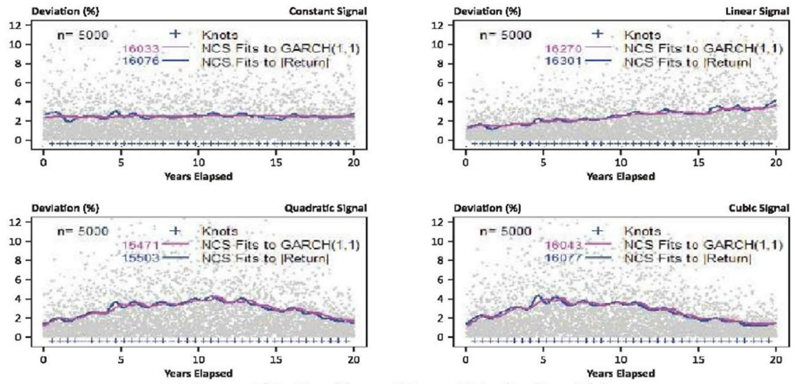


Figure 6. The Four Types of Known Volatility Signal (s_t)

5. CONCLUSIONS

This paper investigates two approaches for estimating the non-stationary variance of the financial time series data. The graphical results show that these two approaches are efficiency to trace the structural variation of daily returns, but the likelihood values show that using maximum likelihood estimation for fitting the NCS function to the absolute returns provides more accurate estimated deviations than fitting the NCS function to the estimates of GARCH (1, 1). Furthermore, this paper also shows that this maximum likelihood estimation of non-stationary variance can be utilized as the baseline for comparing to the other financial volatility models.

6. ACKNOWLEDGEMENTS

Most of all, we would like to thank emeritus professor Don McNeil for his continuous advice and encouragement.

REFERENCES

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics* **31**, 307-327.

Brooks, C. (2008). *Introductory econometrics for finance.* Cambridge: Cambridge University Press.

Clark, P. B., Tamirisa, N., Wei, S.-J., Sadikov, A., & Zeng, L. (2004). *A New look at exchange rate volatility and trade flows.* Washington, D.C.

Cont, R. (2001). Empirical properties of assets returns: stylized facts and statistical issues. *Quantitative Finance,* **1**, 223-236.

Ding, Z., Granger, C. W., & Engle, R. F. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance,* **1**, 83-106.

Engle, R. (2001). GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics. *Journal of Economic Perspectives,* **15** (4), 157-168.

Engle, R. (2003). Risk and Volatility: Econometric Models and Financial Practice. *Nobel Lecture, December 8, 2003.*

Eugenio, D. B. (2016). Volatile volatility: Conceptual and measurement issues related to price trends and volatility. In *Food price volatility and its implications for food security and policy, eds.* (pp. 35-57).

Greene, W. (2002). *Econometric Analysis.* New Jersey: Prentice Hall.

Hastie, T., Tibshirani, R., & Friedman, J. (2009). *The Elements of Statistical Learning: Data Mining, Inference, and Prediction.* New York: Springer-Verlag.

McKenzie, M. D. (1999). The impact of exchange rate volatility on international trade flows. *Journal of the economic survey,* **13** (1), 71-106.

Turner, P., & Van 't dack, J. (1993). Measuring international price and cost competitiveness. *BIS Economic Papers,* **39**.

Venables, W. N., & Ripley, B. D. (2002). *Modern Applied Statistics with S.* New York: Springer-Verlag.

Appendix IV

Proceeding paper II: "The use of information criteria for selecting number of knots in natural cubic spline volatility estimation"


INTERNATIONAL CONFERENCE ON APPLIED STATISTICS 2018
"Improving Data Value & Data Quality using Data Science"
ICAS 2018
Conference Proceedings
liver cancer
24-26 October 2018
Centra by Centara, Government Complex Hotel
& Convention Centre Chaeng Watthana Bangkok

INTERNATIONAL CONFERENCE ON APPLIED STATISTICS (ICAS) 2018, 24-26 October 2018
Code O51

The Use of Information Criteria for Selecting Number of Knots in Natural Cubic Spline Volatility Estimation

Jetsada Laipaporn* and Phattrawan Tongkumchum

Prince of Songkla University, Pattani, Thailand
*Corresponding Email: laipaporn.j@gmail.com
Email: phattrawan@gmail.com

ABSTRACT

The low-frequency volatility has been used as the indicator of the change in the financial market stability due to the macroeconomic situations. Previous studies estimated this volatility by applying spline function to the series of financial assets' returns and using an information criterion to specify the optimum number of knots. However, some studies especially in case applying natural cubic spline function to estimate the low-frequency volatility mostly selected the number of knots subjectively. Therefore, this study tried to compare the performance of two widely used information criteria, Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC), for selecting the number of knots of natural cubic spline volatility model. The results of the Monte Carlo simulation found that BIC selected the less number of knot and under-parameterized the model while the empirical results shown that AIC was likely to use too many number of knots and over-parameterized the model.

Keywords: Volatility; Natural Cubic Spline Function; Number of knots; Information criteria

1 INTRODUCTION

Financial volatility is one of key indicators of market stability. The higher volatility indicates that the index of the stock market has a wider range of changing and potential losses are higher. Therefore, some investors may not be able to withstand that risk and decide to delay their investment. During low volatility, the stock market index also changes but slightly. The market is more stable and the value of possible losses is lower. So investment banks can reduce their reserve requirement due to reduced risk. According to the influences of volatility in the stock market on investment, most investors consider volatility to be an important information for their decision-making.

Financial volatility cannot be measured directly like weight or height. Thus, there are so many approaches to estimate volatility through it proxy, return series. Those approaches were different to each other due to the objective of each study (Poon, 2005). Among those studies, they found that spline function is a suitable function for modeling the low-frequency volatility (Farida *et al.*, 2018; Laipaporn & Tongkumchum, 2017; Engle & Rangel, 2008). Those study assumed that the financial volatility was hypothetically divided into two parts. The low-frequency volatility was defined as the slow-moving part of financial volatility, indicating the long-run change of market stability (Engle & Rangel, 2008). Additionally, it governed the cyclical moving of the financial volatility (Awalludin & Saelim, 2016) and related to the change of the macroeconomic factors (Engle & Rangel, 2008). Though, it differed to the other part, the high-frequency volatility that indicated the change of the index according to the most recent information.

To apply spline function for estimating volatility, the previous studies have shown that using the too many number of knots might provide overfitted model (Engel & Rangel, 2008). Therefore, it needed to define an appropriated number of knots that provided the most explainable volatility model. Engel and Rangel (2008) used Bayesian Information Criterion (BIC) for selecting the number of knots of the exponential quadratic spline function in low-frequency volatility estimation, differed to Liu et al. (2015) which used Akaike's Information Criterion (AIC) with the same function. Some studies, such as Farida et al. (2018), Laipaporn and Tongkumchum (2017) and Awalludin and Saelim (2016), subjectively chose the suitable number of knots of the natural cubic spline functions for their volatility model.

This study tried to apply the information criterion for selecting the number of knots of the natural cubic spline volatility model followed Laipaporn and Tongkumchum (2017). The performances of two widely used information criteria, AIC and BIC, were assessed by the Monte Carlo simulation to identify which information criteria was suitable for specifying the number of knots in natural cubic spline volatility model. Furthermore, the empirical results of using these two information criteria for selecting the natural cubic spline volatility model among various number of knots of two stock market index, Stock Exchange of Thailand index(SET) and Strait Time index (STI), during 1997-2017 was also presented.

This paper is organized as follows. Section 2 describes data and methodology used in this study. Section 3 and section 4 informs the Monte Carlo simulation result and the empirical results of the natural cubic spline volatility of two stock market index respectively. The last section is a conclusion.

2 METHODS

This study might divide into 2 parts. The first is Monte Carlo simulation. This simulation was conducted to compare the performance between two information criteria in selecting number of knots in natural cubic spline volatility model given the true volatility was previously identified. The information criteria which provided the volatility model that better fitted to the given volatility was indicated as the preferred criteria. Another part of this study was applying the information criteria for specifying the natural cubic spline volatility model of each stock market index. The details of data and methodology were described as follows.

2.1 *Data*

2.1.1 *Simulated returns series*

This study assumed that the daily returns series $(R_t^{i,j})$ had zero mean. Each series which contained 5,000 daily returns indexed by trading day (t), was simulated as the multiplicative combination of two components, the known volatility and the noise series, which parameterized by the following formula.

$$R_t^{i,j} = \sigma_t^j \epsilon_t^i \qquad (1)$$

According to Engel and Rangel (2008), the low-frequency volatility was the unconditional volatility, which was not constant but gradually changed by the time. Consequently, three kinds of pre-specified volatility (σ_t^j), which included the low fluctuated volatility, the moderate fluctuated volatility and the high fluctuated volatility, were assumed as an additive combination of a single sinusoidal function of time, trading day (t), followed Saejiang *et al.* (2001). j identified the kind of these volatilities. Besides that, the 100 series of random noise ϵ_t^i, all were assumed having fat-tailed distribution, were generated by transforming the white noise, z_t^i, followed Huber (1964) as this formula.

$$\epsilon_t^i = \begin{cases} c + a(z_t^i - c) \\ z_t^i \\ -c + a(z_t^i + c) \end{cases} \qquad (2)$$

The constant values c and a were 1.25 and 2.5, respectively. Totally there were 300 simulated returns series in this simulation. These three pre-specified volatilities and three absolute returns series from 300 simulated returns series with respect to each pre-specified volatility are shown in Figure 1.

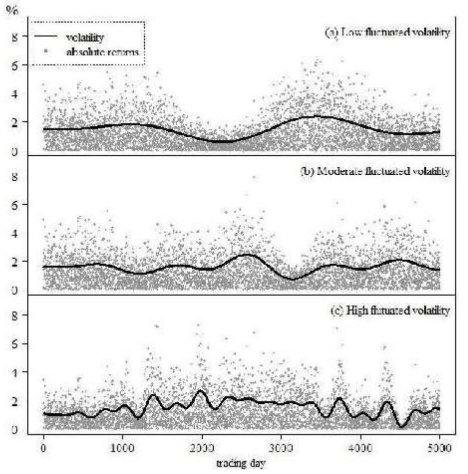


Figure 1: three kinds of pre-specified volatility with the example of the absolute returns series of each volatility

2.1.2 Returns series of two stock markets index

The index of two stock markets, SET and STI, during 1997-2017 were obtained from yahoo finance website. The daily returns of each stock market index were calculated as following equation.

$$R_t = \log \frac{I_t}{I_{t-1}} \tag{3}$$

Where R_t is the log return and I_t is the daily stock index at time t. Time series plots of daily stock index and their corresponding daily absolute returns are shown in Figure 2.

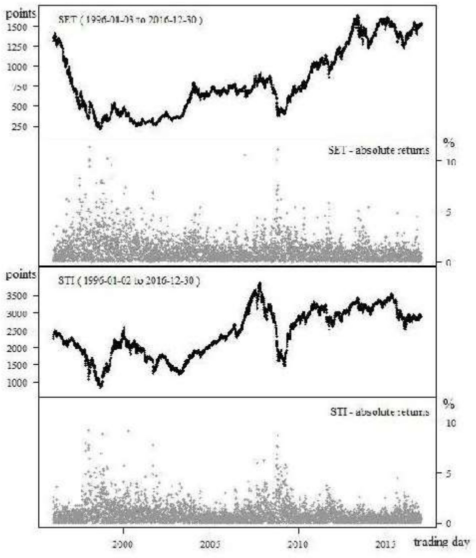


Figure 2: Time series plots of the Stock Exchange of Thailand index and the Strait Time index with their daily absolute returns

2.2 Natural cubic spline volatility model

Spline function has been an attractive and flexible non-parametric method for curve estimation (Silverman, 1985). This function has been used for approximating the shape of curvilinear function without the necessity of pre-specifying the mathematical form of the function (Suits et al., 1978). The natural cubic spline function was a spline function that was linear in the distant past and future and practically fitted to the dependent variable for extracting the variation pattern of that series (Wahba, 1975). In financial context, the natural cubic spline function has been widely used as an interpolation technique to estimate yield curve of the financial assets (Hastie et al., 2009; Greene, 2002; Engle & Russell, 1998).

Laipaporn and Tongkumchum (2017) used natural cubic spline function for modeling volatility. This model supposed that the returns series had two multiplicative components The first component was the conditional volatility (S_t) which was modeled by the natural cubic spline function with equi-spaced knots and the second component was white noises (Z_t). Thus, the returns and volatility models were parameterized as follows.

$$R_t = S_t Z_t \tag{4}$$

$$S_t = a + bt + \sum_{k=1}^{p} c_k (t - t_k)_+^3 \tag{5}$$

where t denoted time which $t_1 < t_2 < ... < t_p$ were specified knots and an additive term, $(t - x)_+$ was $t - x$ for $t > x$ and zero otherwise. Since this spline function was linear outside the boundary knots, t_1 and t_p, the coefficients of quadratic and cubic were 0 for $t < t_1$ and $t > t_p$. To satisfy these constraints, the cubic spline functions in equation 5 became

$$S_t = a + bt + \sum_{k=1}^{p} c_k \left[(t - t_k)_+^3 - \frac{t_p - t_k}{t_p - t_{p-1}} (t - t_{p-1})_+^3 + \frac{t_{p-1} - t_k}{t_p - t_{p-1}} (t - t_p)_+^3 \right] \tag{6}$$

The parameters of this function were estimated by maximizing the log likelihood function with respect to the returns series. The log likelihood function (L) was defined as follows,

$$L = \sum_{t=1}^{n} \left[-\log(s_t) - \frac{R_t^2}{2S_t^2} \right] \tag{7}$$

where S_t was natural cubic spline volatility following the equation 6 and R_t was the return on day t.

2.3 The set of the number of equi-spaced knots

Number of knots effected the estimated natural cubic spline volatility. Increasing number of equi-spaced knots provided the volatility model that was more fitted to the returns series. Successively, the estimated volatility with respect to that model became more varied. However, in order to prevent overfitting, the volatility model needed an optimal number of knots. To obtain that number, first was setting the set of possible number and second was electing the appropriate number by the selection criteria.

The natural cubic spline function required two boundary knots. Thus, the first member in the set of possible number was 3. With 3 knots, the whole returns series were separated into 4 parts. To increase the knots from 3 knots to the next one, the additional knots were put in the middle of each separated parts. So the next number in the set was 3+4 = 7. By repeating this procedure, the members in the set of number of knot consequently included 3, 7, 15, 31, 63 and so on.

2.4 The number of knots selection criteria

Increasing number of knots was increasing more parameters in the natural cubic spline volatility model. It made the volatility model more sensitive to the changes of daily returns. Generally, root mean squared error (RMSE) was used to indicate the goodness of fit of the model. It shown how well the estimates was fitted to the observed data. Therefore, this study employed RMSE for selecting number of knots that made the natural cubic spline volatility model most fitted to pre-specified volatility not to the simulated returns and the number of knots that gave the least RMSE was chosen as the preferred number. RMSE was calculated as follows.

$$RMSE = \sqrt{\frac{\sum_t^n(\sigma_t - S_t)^2}{n}} \quad (8)$$

where σ_t was the pre-specified volatility and S_t was the estimated one.

In case of modeling unknown volatility, it could not apply RMSE to indicated the optimum number of knots like the case of simulation. The other criteria that have been used for specifying spline model in previous studies were AIC and BIC (Engel & Rangel, 2008; Liu et al., 2015). Both AIC and BIC were broadly used for specifying parsimonious model from the set of candidate models (Burnham and Anderson, 2002). These two criteria were formulated as follows.

$$AIC = -L + 2P \quad (9)$$

$$BIC = -L + P\log(n) \quad (10)$$

where L is maximum likelihood value. P is number of parameters and n is number of observations (Hastie et al., 2009; Venables & Ripley, 2002). The first term of two criteria was likelihood value which indicated the goodness of fit of the model whereas the second term was penalized term with respect to the number of parameters in the model. The weight of penalized term of AIC was constant but the weight of BIC was not constant. It was higher when number of observation increased (Burnham & Anderson, 2002).

RMSE as well as AIC and BIC were calculated after each estimation in Monte Carlo simulation, while only AIC and BIC were calculated in empirical study. The lowest value of RMSE AIC and BIC indicated the best approximate model according to each criterion.

3 MONTE CARLO SIMULATION RESULTS

Since each returns series in this simulation included 5,000 daily returns, subsequently the maximum number in the set of number of knots was 127. This number was obtained by trying to add more knots until it did not provide the lower value of RMSE, AIC and BIC. So the number of knots in the set included 3, 7, 15, 31, 63 and 127 knots.

All 300 returns series were used for estimating natural cubic spline volatility six times with six different number of knots, so each returns series had six candidate models and six values of RMSE, AIC and BIC. Figure 3 is the boxplots that summarized the values of RMSE, AIC and BIC obtained by this simulation. These three statistics were grouped by the kind of pre-specified volatility of the simulated returns series.

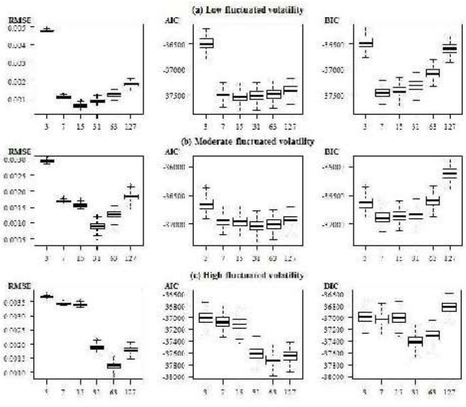


Figure 3: boxplot summarized the values of AIC, BIC and RMSE of the natural cubic spline volatility models with respect to the pre-specified volatility and the number of knots

RMSE values precisely shown that the simulated returns series generated from the same pre-specified volatility used the same number of knots for the appropriate natural cubic spline volatility model. The optimum number of knots for low, moderate and high fluctuated volatility possibly were 15, 31 and 63, respectively. This evidence shown that the more fluctuate volatility needed the more knots for natural cubic spline volatility model.

Comparing to AIC and BIC, the results shown that the optimum number of knots indicated by AIC were more likely to the number indicated by RMSE, differed to BIC which provide the smaller number than the other two criteria.

After identifying the number of knots that provided the least values of each criteria, RMSE and AIC specified the same number of knots for all simulated returns series which were 15, 31 and 63 for low, moderate and high fluctuated volatility, respectively. While BIC gave several different number. Most of number indicated by BIC were less than the number provided by the other two criteria.

These results implied that the number of knots indicated by BIC provided the estimated volatility that was underfitted to the pre-specified volatility when compared to RMSE, whereas AIC specified the well fitted model. Details of comparison shown in Table 1.

Table 1: The number of knots the provided the least criterion's value classified by kinds of pre-specified volatility and knot selection criteria (number of simulated returns series shown in the brackets)

Criteria	Low fluctuated volatility	Moderate fluctuated volatility	High fluctuated volatility
RMSE	15 (100)	31 (100)	63 (100)
AIC	15 (100)	31 (100)	63 (100)
BIC	7 (90)	7 (96)	31 (100)
	15 (10)	15 (2)	
		31 (2)	

4 RESULTS

The number of daily returns of SET and STI were 5135 and 5252, respectively. Likewise, the maximum number in the set of number of knots was 127 and there were six candidate volatility models for both series. Only AIC and BIC were calculated and compared. Their values were shown in Figure 4.

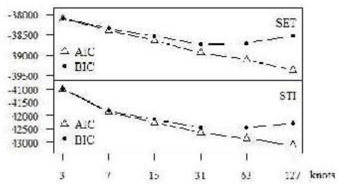


Figure 4: the values of AIC and BIC of the natural cubic spline volatility models of SET and STI with respect to the number of knots

BIC values were likely to lower when applying the more number of knots to the model. The natural cubic spline volatility model with 31 knots provided the lowest value of BIC for both series. However, BIC values became greater for the volatility model with the number of knots greater than 31. These BIC values still behaved like the values in Monte Carlo simulation. They could specify the appropriate model from the set of candidate models. Contrast to AIC, their values were smaller as increasing the number of knots. They differed to the AIC values in simulation which had a lowest value as a point for identifying the appropriate model from the set of candidate models.

Figure 5 and Figure 6 shown the low-frequency volatilities with their corresponding absolute returns series for both SET and STI series. These volatilities estimated by the natural cubic spline volatility model with 31 knots indicated by BIC, 127 knots indicated by AIC and 63 knots which subjectively selected.

The same as simulation results, BIC was likely to under-parameterize the model and the volatilities estimated by the model specified by BIC poorly traced the daily returns variation. Meanwhile, the number of knots indicated by AIC seem to provide too fluctuated estimated volatility. The model specified by AIC became over-parameterization.

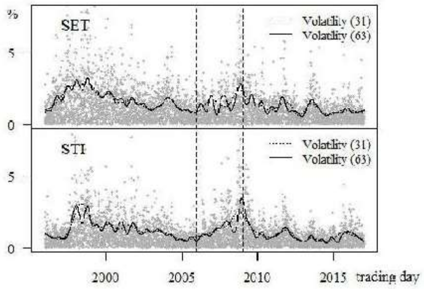


Figure 5: the low-frequency volatilities of SET and STI in case of applying 31 and 63 knots with the natural cubic spline volatility model and their corresponding absolute returns series

The estimated volatility by the model with 63 knots was higher fluctuated than the volatility estimated by the model with 31 knots but it was better to explain the variation of daily returns than another one especially in the period during 2006-2008 as shown in Figure 5.

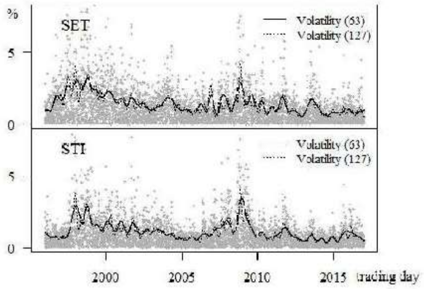


Figure 6: the low-frequency volatilities of SET and STI in case of applying 63 and 127 knots with the natural cubic spline volatility model and their corresponding absolute returns series

The estimated volatility by the model with 63 knots was less fluctuated than the volatility estimated by the model with 127 knots but their capability to trace the changes of return variation were not significantly different as seen in Figure 6. Therefore, the natural cubic spline volatility models with 63 knots were selected from the set of candidate models as the parsimonious model for estimating low-frequency volatility for both SET and STI.

5 CONCLUSIONS

The results of the Monte Carlo simulation precisely shown that BIC under-parameterized the model. This criterion specified the natural cubic spline volatility model with the number of knot that provided under estimated volatility. The empirical results also shown that the low-frequency volatilities of SET and STI estimated by the model which specified by BIC were less capability to trace the variation of their daily returns.

AIC seemly performed well in Monte Carlo simulation. The volatility models for each simulated returns series selected by AIC from the set of candidate models were fitted well to the pre-specified volatility. But empirical results shown that AIC provided too fluctuated low-frequency volatility and over-parameterized volatility model.

This study concluded that using BIC as number of knots selection criteria made the natural cubic spline volatility model under-parameterized. However, it needed more consideration for using AIC as a criterion for specifying the number of knots of natural cubic spline volatility model.

ACKNOWLEDGEMENTS

The authors would like to thank Emeritus Professor Don McNeil for his kindly suggestions.

REFERENCES

Awalludin, S.A. & Saelim, R. 2016. Modelling the volatility and assessing the performance of the model. The 20th International Annual Symposium on Computational Science and Engineering, Faculty of Science, Kasetsart University, Bangkok, July 27-29, 2016.

Engle, R.F. & Rangel, J. G. 2008. The Spline-GARCH model for low-frequency volatility and its global macroeconomic causes. *The Review of Financial Studies*, 21(3), 1187-1222.

Engle, R.F. and Russell, J. R. 1998. Autoregressive conditional duration: a new model for irregularly spaced transaction data. *Econometrica*, 66(5), 1127-1162.

Farida, Makaje N., Tongkumchum P., Phon-On A. & Laipaporn J. 2018. Natural cubic spline model for estimating volatility, *International Journal on Advanced Science, Engineering and Information Technology*, 8(4). DOI:10.18517/ijaseit.8.4.3107

Greene, W.H. 2002. *Econometric Analysis*. Prentice Hall, U.S.A, pp. 199-200.

Hastie, T., Tibshirani, R. & Friedman, J. 2009. The Elements of Statistical Learning: Data Mining, Inference, and Prediction. Springer-Verlag, New York, pp.141-148.

Huber, P.J. 1964. Robust estimation of a location parameter. *The Annals of Mathematical Statistics*, 35(1), 73-101.

Laipaporn, J. and Tongkumchum, P. 2017. Maximum likelihood estimation of non-stationary variance. 2nd ISI Regional Statistics Conference, 20-24 March 2017. Bali, Indonesia.

Liu, S., Tang, T., McKensie, A.M. & Liu, Y. 2015. Low-frequency volatility in China's gold futures market and its macroeconomic determinants. *Mathematical Problems in Engineering*. Article ID 646239. Doi:10.1155/2015/646239

Poon, S.H. 2005. *A Practical Guide to Forecasting Financial Market Volatility*. John Wiley and Sons, U.K., pp. 1-17.

Saejiang, S., Pornwiriyamongkol, W. & McNeil, D. 2001. Time series analysis of banking share returns in Thailand. *Songklanakarin Journal of Science and Technology*, 23(3), 443-448.

Silverman, B.W. 1985. Some aspects of the spline smoothing approach to non-parametric regression curve fitting. *Journal of the Royal Statistical Society. Series B (Methodological)*, 47(1), 1-52.

Suits, D.B., Mason, A. & Chan, L. 1978. Spline function fitted by standard regression methods. *The Review of Economics and Statistics*, 60(1), 132-139.

Venables, W.N. & Ripley, B.D. 2002. *Modern Applied Statistics with S. Springer-Verlag New York*, pp. 414-418.

Wahba, G. 1975. Smoothing noisy data with spline function. *Numerische Mathematik*, 24(5), 383-393

Vitae

Name: Mr. Jetsada Laipaporn

Student ID: 5820330008

Educational Attainment:

Degree	**Name of institution**	**Year of Graduation**
B.A. (Economics)	Thammasat University	2001
M.A. (Economics)	Thammasat University	2008

Work-Position and Address:

Position: Lecturer

Address Faculty of Humanities and Social Sciences,

Prince of Songkla University, Pattani Campus

List of Publication and Proceeding:

Publication:

Laipaporn, J., and Tongkumchum, P. 2020. The time-varying correlation estimator using the natural cubic spline volatility. *Advances and Application in Statistics*, 62(1): 79-95.

Laipaporn, J., and Tongkumchum, P. 2021. estimating the natural cubic spline volatilities of the ASEAN-5 exchange rates. *Journal of Asian Finance, Economics and Business*, 8(3): 1-10.

International Conference:

Laipaporn, J., and Tongkumchum, P. Maximum likelihood estimation of non-stationary variance. The 2nd International Statistical Institute Regional Statistics Conference, Bali, Indonesia, 20-24 March 2007.

Laipaporn, J., and Tongkumchum, P. The use of information criteria for selecting number of knots in natural cubic spline volatility estimation. International Conference on Applied Statistics, Bangkok, Thailand, 24 – 26 October 2018.